A STOCHASTIC MODEL FOR THE EOLIAN SALTATION AND SUSPENSION TRANSPORT

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ABSTRACT

A model for the simulation of the transport of heavy particles by a turbulent boundary layer is established. The resolution of the lagrangian equation of the solid particle motion (in suspension or saltation) requires the knowledge of the instantaneous velocity of the surrounding fluid. This velocity is determined by a continuous stochastic equation. An appropriate lagrangian correlation timescale is considered in order to include the influences of gravity and inertia of the solid particle. The model consists of the aerodynamic entrainment of particles by the turbulent structures of the flow, the computation of the decorrelation between the solid and the fluid trajectories, the determination of the driving fluid velocity along the particle trajectory, the computation of the particle trajectory, the impact of grains on the bed and the estimate of new ejected sand or dust particles (splash function). Comparisons of the results from our simulation with different wind tunnel experiments are in a good agreement.

INTRODUCTION

The entrainment, transport and deposition of dust-sized sediment can have a severe impact on the natural environment and human activity. For a sufficiently strong wind, dust particles can be entrained by aerodynamic forces or by the impact forces of saltating grains which return to the soil and interact energetically. This last phenomenon is considered as the main source of dust particle entrainment and erosion (Shao et al., 1993). Eolian sediment transport must be considered as a stochastic process because of two main reasons (Anderson, 1987, 1991). The particles are transported by turbulent fluctuations of the wind and the impacts of saltating particles with soil result in subsequent ejections whose number and velocities may be known only in a probabilistic sense.

An aeolian saltation motion can be schematized as being comprised of four linked processes : the aerodynamic entrainment of saltating particles, the grain trajectories, the impact of the particles with the soil (the splash process), the trajectories of the ejected and rebounded particles and finally, the wind field modification.

The aim of this paper is to develop a stochastic lagrangian model for the eolian saltation and suspension transport. The stochastic approach is a more physical approach because physics that give rise to individual trajectories are easily accounted for. However, this requires a knowledge of the probability distribution of all characteristics of ejections and impact with the bed. We present a stochastic model for saltating particle transport, including the modelisation of the lift-off, the splash process and the rebound and ejection of particles. No wind field modification is considered here.

THE GLOBAL MODEL

The stochastic transportation model

For heavy sand particles whose volumetric mass is much greater than that of the fluid ($\rho_p / \rho_f \geq 10^3$), only viscous drag and gravity forces could be considered and the simplified lagrangian velocity equations write as

$$\begin{cases} \dfrac{dV_i(t)}{dt} = \dfrac{\left[u_i^P(\vec{y}(t),t) - V_i(t)\right]}{\tau_p} f(\Re_{e_p}) - g\delta_{i2} \\ \dfrac{dy_i(t)}{dt} = V_i(t) \end{cases} \tag{1}$$

where V_i represents the particle velocity at position y_i and time t, τ_p the relaxation time of the particle, $f(\Re_{e_p})$ the drag coefficient and $\Re e_p$ is the particle Reynolds number. The driving fluid velocity $u_i^P(\vec{y}(t),t)$ characterizes the velocity of the fluid along the particle trajectory. As a consequence to its inertial effects and its different responses to gravity, the particle will deviate from the fluid element which originally contains it (the crossing trajectory effect), inducing a decorrelation. The main difficulty lies in the determination of such a fluid particle driving velocity along the particle trajectory. This fluid velocity is computed using a one particle, one time scale stochastic equation (Thomson, 1987).

In analogy with fluid element, we suppose that increments in driving fluid velocity evolve as a continuous Markov process which can be represented by a stochastic equation. For a one-dimensional stochastic model, the equation writes as

$$du^p = a(u^p, V, y, t)dt + b(u^p, V, y, t)d\xi \quad (2)$$

Recently, Shao (1995) and Reynolds (2000) have pointed out some contradictions relative to the structure function and suggested that this velocity should be evaluated using a fractional Langevin equation. In fact, Wiener increments necessarily lead to a structure function proportional to dt when, in the limiting case of large drift velocity and negligible inertia, the driving fluid correlation approaches the eulerian space-time correlation which is proportional to $dt^{2/3}$. However, the saltating particles being far from these limiting cases, in a way identical to Reynolds (2000), we will forsake considerations of the structure function for increments in fluid velocity and treat increments $d\xi$ as increments of a Wiener process.

The coefficient $b(u^p, V, y, t)$ will be expressed as

$$b = \sqrt{2/T_L^p} \quad (3)$$

T_L^p being interpreted as a lagrangian decorrelation timescale of the fluid velocity along the particle trajectory. In order to account for the gravity and the inertia effects, we expect the modified timescale to be shorter than the fluid lagrangian timescale T_L. The velocities to which a solid particle is subjected will not be as well correlated than those to which a fluid element is subjected. Moreover, as noted by Rogers and Eaton (1990), a frequency measured in a lagrangian frame is always smaller than a frequency measured in an eulerian one. Different forms have been previously proposed as Sawford and Guess (1991), Zhuang et al. (1989) for example. We propose the following form $T_L^p = T_L/(\alpha_{grav} + \alpha_{inert})$ where α_{grav} and α_{inert} are coefficients relative to the gravity and the inertia effects.

The gravity effect is estimated following the Csanady's approximation (1963). Csanady proposed an interpolation between the lagrangian correlation for vanishing inertia and small terminal velocity $V_{\lim}$ and the eulerian correlation for large $V_{\lim}$. In a direction parallel to gravity, with β an empirical constant, we obtain:

$$\alpha_{grav} = \left[1 + \left(\beta \frac{V_{\lim}}{\sigma_u}\right)^2\right]^{1/2} \quad (4)$$

The inertia effect can be evaluated in the limit of large inertia and vanishing terminal velocity. A turbulent structure (lenghtscale ℓ), passing by the moving particle would have a frequency of

$$\nu_{part} = \frac{u^p - V}{\ell} \approx \frac{u^p - V}{\sigma_u} \nu_L \quad (5)$$

where σ_u represents the root-mean-square fluid velocity and ν_L the lagrangian correlation timescale. This relation could easily be extended and so, in a general case, the inertia coefficient is expressed as

$$\alpha_{inert} = \frac{\left|u^p - (V - V_{\lim})\right|}{\sigma_u} \quad (6)$$

The modified correlation timescale of the driving fluid velocity writes

$$T_L^p = T_L \left\{ \left[1 + \left(\frac{\beta V_{\lim}}{\sigma_u}\right)^2\right]^{1/2} + \frac{\left|u^p - (V - V_{\lim})\right|}{\sigma_u} \right\}^{-1} \quad (7)$$

For the limiting case, when gravity and inertia effects are negligible, the asymptotic behavior is satisfied.

Finally, following Reynolds (2000), the deterministic term $a(u^p, V, y, t)$ is determined with the Fokker-Planck equation relative to the pdf $P(u^p, V, y, t)$ and in coherence with the well-mixed criteria. For the limiting cases of small inertia and small drift velocity or small inertia and large terminal velocity, the one-dimensional stochastic equations for an inhomogeneous Gaussian turbulence are respectively given by:

$$du^p = \left[-\frac{u^p}{T_L^p} + \frac{1}{2}\left(1 + \frac{u^{p2}}{\sigma_u^2}\right)\frac{\partial \sigma_u^2}{\partial y} \right] dt + \sigma_u \sqrt{\frac{2}{T_L^p}} d\xi \quad (8)$$

$$du^p = \left[-\frac{u^p}{T_L^p} + \frac{1}{2}\left(1 + \frac{u^p(u^p + V_{\lim})}{\sigma_u^2}\right)\frac{\partial \sigma_u^2}{\partial y} \right] dt + \sigma_u \sqrt{\frac{2}{T_L^p}} d\xi \quad (9)$$

The stochastic parameterization

The formulation of the recursive model is as follows. Particles are first lifted off by random ejections which stand for the aerodynamic entrainment and the effects of the turbulent structures on particles. Then particles are transported. At every time step, the decorrelation between fluid and particle is computed. When saltating particles impact the soil, rebound and

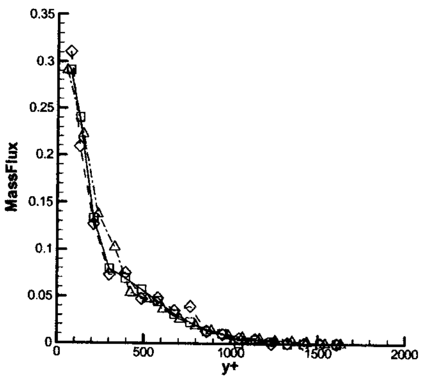

Figure 1. Dispersion of glass particles in a turbulent boundary layer with collision (squares) and without collision (diamonds) between particles. Comparison with the experimental results of Taniere et al. (1997) - triangles.

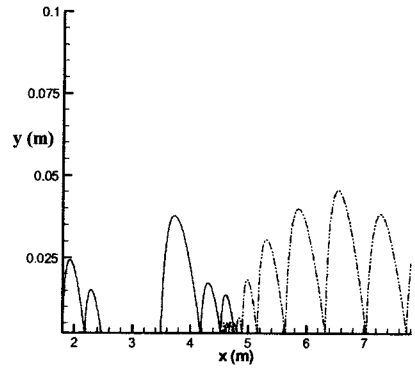

Figure 2. Trajectories of two injected particles (d=188μm).Only one particle is ejected (dashed and dotted trajectory) at the second rebound of second initially injected particle. There is a probability of rebound, of ejection and a Poisson distribution for the number of ejected particles per impact. u_*=0.35m/s

emission are modelised by some probability and random ejection laws, the splash function. Because of the complexity of the grain-bed interaction, we may know the characteristics of the rebounded and ejected grains, in only a statistical sense. Finally, particles are transported by the turbulent flow.

The particles are lifted up in different ways depending on the diameter, density ratio, trajectories of the fluid and particles, kinetic energies and inertia of the particles and the fluid. Many different situations can occur but it appears that in all cases, the turbulent structures are the primary entrainment mechanism and that the ejection depends on the characteristics of the particles (Kaftori et al., 1995). Modelised particles are injected into the flow randomly, the injection angle and vertical velocity are random variables with Gaussian Probability Density Functions. Particles are then transported under the influence of the turbulent flow.

When the particle impacts the soil, the angle of rebound and the norm of the rebounding velocity are random variables with a Gaussian Probability Density Function as well as the velocity and the angle of ejection of the ejected particles. All the statistical laws considered in this model are deduced from experimental data.

Due to the gravitational acceleration, all saltating particles return and impact the soil, so rebounding and ejecting more particles in air. The probabilistic model for the splash process has been taken from a simplified analytic model proposed by M. Sørensen (1991). Based on the experiments done by Anderson and Haff (1991), Sørensen states that grains shot into a loose bed of similar particles at high speeds rebound with a probability of 94%. In their computer simulation Anderson and Haff (1991) find a probability of 95% for a particle to rebound after impact. If $p(R/V_2)$ is the probability of rebound for an impinging grain with vertical impact velocity V_2 and V_{cr} is a critical vertical velocity beneath which grains do not rebound, then the probability has the following simple form:

$$p(R/V_2)=\begin{cases}0.94 & if \quad V_2>V_{cr}\\ 0 & if \quad V_2<V_{cr}\end{cases} \qquad (10)$$

For Sørensen V_{cr} was not far from 0.5 m/s, therefore after a few tests the value of 0.3 m/s was chosen.

As long as the ejection is concerned, the average number $n(\vec{V})$ of grains ejected by a particle hitting the sand bed with a vertical velocity $\vec{V}$ was found by Anderson and Haff (1991) to be proportional to $|\vec{V}|$. Sørensen assumes that all saltating grains hit the bed at an angle of 12°, which is between the boundary values suggested by Nalpanis et al. (1993). Therefore Sørensen establishes that $n(V_2)=0.029V_2$. The assumption for the impacting angle is practically verified by our simulation also, since the mean angle of impact over 638 trajectories is 8.8° (*see table 1*).

Finally, Sørensen establishes a probability distribution of the number k of dislodged grains. He considers that given the vertical component V_2 of the impact velocity, k follows a Poisson distribution with mean value $n(V_2)$:

$$p(k/V_2)=\frac{n(V_2)^k e^{-n(V_2)}}{k!} \qquad (11)$$

Therefore, the probability for ejection to happen at all depends only on the vertical component of the impacting velocity, and is

$$p(ejection)=1-p(0/V_2)=1-e^{-n(V_2)} \qquad (12)$$

NUMERICAL RESULTS

First, the stochastic transport model has been tested against the dispersion of heavy particles released in a turbulent boundary layer, the source being flushed with the floor (Taniere et al., 1997). The glass particles, with a diameter of 60 μm are lifted off by aerodynamic entrainment in a modified suspension motion. Once impacting the bed, particles rebound following an elastic rebound law. The predicted dispersion agrees well with the experimental results (figure 1). This indicates that it is

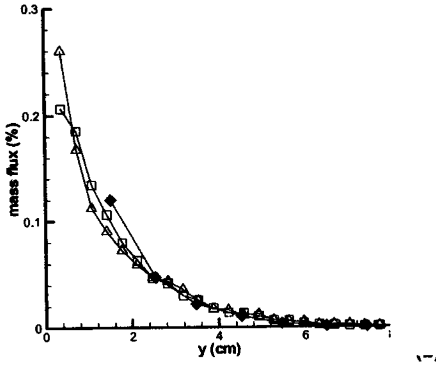

Figure 3. Mass flux profiles for two different splash functions (ejection at every impact –squares, and a splash law with a probability of rebound, of ejection and a Poisson distribution for the number of ejected particles –triangles). Comparison with the wind tunnel results of Nalpanis et. Al. (1993) - diamonds. $u^* = 0.35\, m/s, \delta = 0.14\, m$.

appropriate to use the lagrangian stochastic model with the appropriate modified timescale, to simulate fluid velocities along heavy particle trajectories.

The model has been tested also against heavy particle dispersion data in turbulent boundary layer (Nalpanis et al., 1993). The studied transport mode in this case was pure saltation. Nalpanis et al. studied the dispersion of sand particles covering the bed of the wind tunnel. The initial injection of particles into the flow was done by aerodynamic entrainment. As published by Nalpanis et al. (1993) the mean and the standard deviation for the injected particles are :

$$\begin{cases} < V_2 >= 2u_* \quad \sigma_{V_v} = u_* = 0.35 m/s \\ < \alpha >= 30° \quad \sigma_\alpha = 10° \end{cases} \tag{13}$$

As an example some particle trajectories are shown on figure 2 with the probabilistic splash function described above. With the adjustment of different parameters used in the probabilistic expressions we could simulate different types of soils, grain distributions and packages.

The statistical characteristics of the rebound and ejected particles are:

$$Rebound \begin{cases} < V >= 0.4(norm\ of\ impact\ velocity) \\ \sigma_V = \dfrac{< V >}{2} \\ < \alpha >= 30° \quad \sigma_\alpha = 30° \end{cases} \tag{14}$$

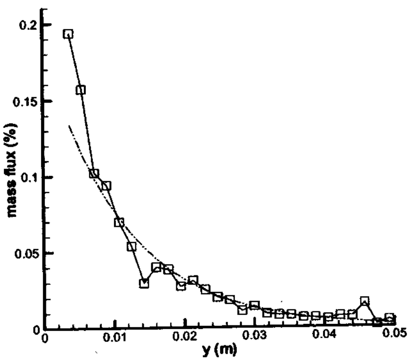

Figure 4. Mass flux profile for 1000 dust particles (squares) and its exponential trend line (dashed and dotted curve). The profile is computed 3m from the downwind edge of the dust bed, d= 2μm and u_* = 0.6 m/s.

$$Ejection \begin{cases} < Ve_1 >= 0.1(norm\ of\ impact\ velocity) \\ < Ve_2 >= 0.1(norm\ of\ impact\ velocity) \\ \sigma_{Ve_1} = \dfrac{< Ve_1 >}{2} \quad \sigma_{Ve_2} = \dfrac{< Ve_2 >}{2} \end{cases} \tag{15}$$

In order to compare with the wind tunnel results of Nalpanis et al. (1993) relative mass flux profiles were computed. Figure 3 shows relative mass flux profiles for two different splash laws as well as the experimental results of Nalpanis et al. (1993). The only difference between the splash functions is that one is taken as described above while the second one is a simpler version where there is one ejection at each rebound. There is no Poisson law for the number of ejected particles per impacting grain in the second case. Our computation results match almost perfectly with the wind tunnel measurements.

In their article Nalpanis et al. (1993) published some of the basic statistical characteristics of the trajectories of particles, such as the mean and standard deviation of the ejection velocity and angle, of the impacting velocity and angle and the height and length of each trajectory between two rebounds. In our simulation, the characteristics of the ejection and injection velocities and angles were fixed by the initial conditions and the splash function, whereas the height, length and impacting characteristics were calculated over a total number of 638 trajectories (100 initially injected particles and 538 ejections). Their measurements and our computed results are all presented in table 1. The computed results are within the experimental boundaries for each parameter.

We also tested our model with the experiments of Shao and Raupach (1993) who studied three configurations of saltation and particle transport by a turbulent flow over a flat bed. In a portion of 3m in a wind tunnel they considered the three following cases : the pure dust configuration - three meters of pure dust (d=2μm), the bombardment configuration – one meter of sand (d=200μm) and the second two meters with pure dust

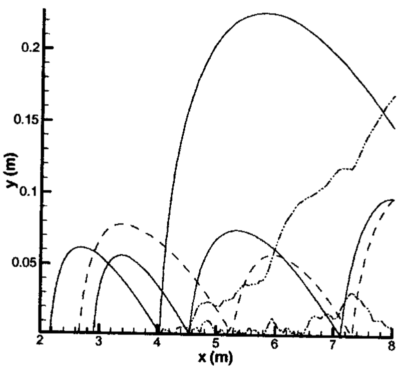

Figure 5. Trajectories of three sand grains (d=200μm) – two plane trajectories and one dashed, that eject dust particles (d=2μm)- dashed and dotted trajectories, each time they impact the bed between 3 and 5m. Sand particles have deterministic trajectories whereas those of dust grains follow the turbulent fluctuations of the flow.

and the mixed configuration – three meters of a sand/dust mixture. As shown on figure 4, the mass flux profile for dust suspended particles as a function of height presents an exponential decrease. These results conform to the field measurements of suspended sediment concentrations made by different field experiments (Nickling, 1978, Gillette, 1977).

Finally, as an illustration of our main objective, we computed a simulation of the bombardment configuration where sand particles eject some dust particles. One can clearly see on figure 5 the different behaviour between particles. Sand particles have deterministic trajectories whereas those of dust grains are random and follow the turbulent fluctuations of the flow.

As when the pure suspension case was studied, mass flux profiles were computed 6m downwind from the upwind edge of the tested bed. The mass flux profile is exponential. In this case two types of particles having different volumes are considered and the mass flux should take the mass of the particle into account. Figure 6 shows the comparison between the usual mass flux and the weighted mass flux. Even though there is almost the same number of particles of dust and sand (725 dust grains ejected by 1000 sand particles), the relative mass flux peak near the bed characterising the presence of near bed particle transport, typical for small ejected particles disappeared. This is of course, because the mass of dust particles is 10^6 times smaller, and their relative mass flux is therefore negligible. Note also that the usual relative mass flux profile is not sensible to the number of ejected particles per impact, because the computed values are all relative. The weighted mass flux profile should change if the number of ejections per impact is increased.

CONCLUSION

We have presented a simulation of the transport of heavy particles in turbulent boundary layers based on a general stochastic approach. A continuous stochastic equation was developed in order to estimate the driving fluid velocity along the particle trajectory, despite the inability of a Wiener process to describe the small time behaviour. With an appropriately modified lagrangian correlation timescale, the continuous lagrangian stochastic model can estimate quite well the transport of heavy particles in inhomogeneous turbulence. Comparisons with wind tunnel experiments agree quite well.

We also tested different splash configurations based on the observations. A probabilistic model for the grain-bed interaction, proposed by M. Sørensen (1991) was introduced. Different forms of the splash process gave results in good agreement with the experiments conducted. The splash function with one ejection and one rebound at each impact was later used for the simulation of dust emission by impacting sand grains because we assumed that saltating grains have always enough energy to eject dust particles. The splash function with one ejection only at the first rebound requires a shorter computation time compared to the other computed splash functions. Finally, the most realistic model for the grain-soil interaction is the probabilistic one. In this model each event (rebound, ejection and number of ejected grains) has a probability distribution. Therefore, only by adjusting the probability coefficients, different soil configurations (moisture, grain packing, cohesive forces, portion of dust and clay) may be simulated.

The model was developed under the assumptions of a steady wind and a flat bed with sand of a single size. No parameters about the soil characteristics (crust, moisture, packing arrangements, elasticity or aggregate content) were taken into account. The model was only studied in two dimensions. Each one of these simplifying assumptions could make the issue of a future study and development of the program.

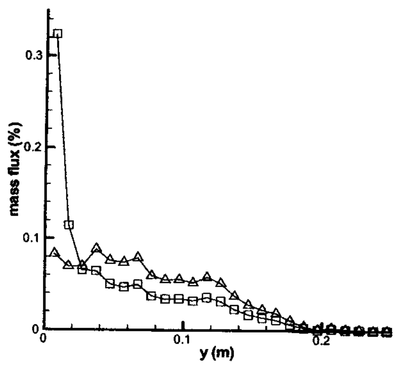

Figure 6. Usual mass flux profile (squares) and weighted mass flux profile (triangles) for 1000 saltating sand grains that eject 725 dust particles. u_*=0.6m/s and δ=0.28m

Table 1. Statistical results of height, length, impacting angle and impacting velocity. Comparison between the measurements of Nalpanis et al. (1993) and the results of the numerical simulation

	Nalpanis, Hunt and Barrett	**Numerical Simulation**
mean height	$1.01cm \leq h \leq 1.39cm$	$h = 1.2cm$
standard deviation of the height	$\sigma_h = mean$	$\sigma_h = 1.1cm$
$\frac{\text{mean length}}{\text{mean height}}$	$11 \leq \frac{l}{h} \leq 14$	$\frac{l}{h} = 12.6$
mean impact angle	$11^\circ \leq \alpha \leq 14^\circ$	$\alpha = 8.8^\circ$
$\frac{\text{impact velocity}}{\text{ejection velocity}}$	$1.6 \leq \frac{V_{impact}}{V_{ejection}} \leq 2$	$\frac{V_{impact}}{V_{ejection}} = 1.6$

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