

# MODELLING STRONG INHOMOGENEITY EFFECTS ON PRESSURE STRAIN IN SECOND MOMENT CLOSURE BY ELLIPTIC RELAXATION

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## ABSTRACT

A new inhomogeneous correction of elliptic relaxation equation(ERE) in Reynolds Averaged Navier Stokes(RANS) modelling is proposed to intermediate between near wall and far from the wall. The quasi-homogeneous pressure strain models using in second moment turbulence closures are usually applied to the source term of ERE. This elliptic model approach is to avoid use of the wall distances and wall normal vectors, so that it makes the model applicable to the flows bounded with complex geometries. The boundary conditions of elliptic relaxation operator affect to the quasi-homogeneous pressure strain model in the near wall region. That is, it gives the correct damping of the redistribution at the wall, and then enables the reproduction of the two-component limit of turbulence. However, the original relaxation operator (Durbin 1993) induces the amplification of redistribution in the logarithmic layer. Accordingly, it is necessary to modify the elliptic operator to reproduce the acceptable results in the logarithmic region. In order to modify the elliptic operator, a strong inhomogeneous correction for the source term of ERE is proposed in the present study.

The present inhomogeneous correction for the elliptic relaxation equation is applied to the inertial and non-inertial channel flows. Results are compared with DNS data for the channel flows. The present model shows good agreements for non-rotating and rotating channel flows.

## INTRODUCTION

In contrast to the conventional second moment modelling for the pressure strain correlation, Durbin(1993) introduced a novel approach. He proposed to model directly the two-point correlation in the integral equation of pressure strain term, which use an isotropic exponential function model. That is, a convolution production is obtained from this model, which can be inverted to give the so-called "elliptic relaxation approach". Thus, the pressure strain term is no longer given by an algebraic relation, but rather by a differential equation. The non-local character is preserved through the elliptic operator and the model can be integrated down to the wall. A notable feature of this approach is that the source term of the elliptic relaxation equation can be given by any quasi-homogeneous model. Even though some intuitive assumptions have been made, Durbin's model is based on a theoretical approach, leading to the hope that it is somewhat

universal, unlike the other models using the wall damping function, wall normal vector and the distance from a wall.

Despite the remarkable success, room for improving the elliptic relaxation model exists. In particular, as pointed out by Wizman et al.(1996), the elliptic operator does not behave correctly in the logarithmic layer. This result shows that the elliptic operator leads to an amplification of the redistribution. Note that the cause of amplification in the logarithmic region does not relate to the quasi-homogeneous model for the source term of the ERE.

Based on the above considerations, Laurance & Durbin (1994) and Wizman et al.(1996) proposed new elliptic formulations using the gradient of turbulent length scale. And Manceau et al.(2001) introduced a new correlation function between the fluctuating velocity and the Laplacian of the pressure gradients. These achievements are obtained by taking into account the influence of strong inhomogeneity and anisotropy on the redistribution term, using a spatially variable length scale and an asymmetric model of the correlation function. The modified elliptic relaxation equations as compared with Durbin's original model for channel flows represent the improved results for the reduction of amplification in the logarithmic layer. However, because the improvements are not satisfactory, especially, in the channel center region, the problems for the amplification of redistribution are still remains.

The present study aims at proposing a new elliptic relaxation equation considering inhomogeneous situations. In this modelling process, we decompose the source term of elliptic relaxation equation into homogeneous and inhomogeneous parts. The present model is applied to the inertial and non-inertial channel flows and the results are compared with DNS data to test the ability of model.

## MODELLING STRONG INHOMOGENEITY EFFECTS ON PRESSURE STRAIN

### Governing Equation

The problem under consideration is that of incompressible and fully developed turbulent channel flow rotating with constant angular velocity about the spanwise direction. The momentum equation and the Reynolds stress equation coupled with pressure strain and dissipation can be written in Cartesian tensor notation as:

$$\frac{DU_i}{Dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_k} \left( \mu \frac{\partial U_i}{\partial x_j} - \rho \overline{u_i u_j} \right) - 2\rho \Omega_j U_k \varepsilon_{ijk} \quad (1)$$

and

$$\frac{D \overline{u_i u_j}}{Dt} = P_{ij} + R_{ij} + D_{ij}^v + D_{ij}^t + F_{ij} - \frac{\overline{u_i u_j}}{k} \varepsilon \quad (2)$$

where

$$F_{ij} = \Phi_{ij} - \varepsilon_{ij} + \frac{\overline{u_i u_j}}{k} \varepsilon \quad (2a)$$

$$\Phi_{ij} = \frac{\rho}{\rho} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2b)$$

$$P_{ij} = - \left( \overline{u_k u_i} \frac{\partial U_j}{\partial x_k} + \overline{u_k u_j} \frac{\partial U_i}{\partial x_k} \right) \quad (2c)$$

$$R_{ij} = -2\Omega_k \left( \overline{u_i u_j} \varepsilon_{ikm} + \overline{u_i u_m} \varepsilon_{jkm} \right) \quad (2d)$$

$$D_{ij}^v = \frac{\partial}{\partial x_k} \left( \nu \frac{\partial \overline{u_i u_j}}{\partial x_k} \right) \quad (2e)$$

$$D_{ij}^t + D_{ij}^b = -\frac{\partial}{\partial x_k} \left( \overline{u_i u_j} u_k \right) - \frac{1}{\rho} \frac{\partial}{\partial x_k} \left( \overline{p u_i} \delta_{jk} + \overline{p u_j} \delta_{ik} \right) \quad (2f)$$

are the production due to mean shear ( $P_{ij}$ ) and system rotation ( $R_{ij}$ ), viscous diffusion ( $D_{ij}^v$ ), turbulent diffusion ( $D_{ij}^t$ ) and pressure diffusion ( $D_{ij}^b$ ), pressure strain ( $\Phi_{ij}$ ), dissipation ( $\varepsilon_{ij}$ ) and relaxed redistribution tensor ( $F_{ij}$ ). The generation terms need no modeling and enter the Reynolds stress equation in their exact form. The viscous diffusion term can also be retained in its exact form whereas the unclosed turbulent diffusion and pressure diffusion are modelled together by gradient diffusions.

The redistribution tensor  $F_{ij}$  is obtained from the solution of elliptic relaxation equation proposed by Durbin(1993), which can be put into coordinate independent form and written as:

$$f_{ij} - L^2 \nabla^2 f_{ij} = \frac{F_{ij}^h}{k} \quad (3)$$

$$F_{ij} = k f_{ij} \quad (4)$$

$$F_{ij}^h = \frac{\Phi_{ij}^h}{k} + \frac{2b_{ij}}{T} \quad (5)$$

where,  $\Phi_{ij}^h$  denotes a quasi-homogeneous form of  $\Phi_{ij}$ ,  $k$  is the turbulent kinetic energy and  $b_{ij}$  is the Reynolds stress anisotropy tensor defined as  $b_{ij} = \overline{u_i u_j} / 2k - \delta_{ij} / 3$ . Also, Length scale  $L$  is prevented from going to zero at the wall by using the Kolmogorov scale and time scale  $T$  also introduces the Kolmogorov scale in the viscous layer.  $L$  and  $T$  are defined as:

$$L = C_L \max \left( \frac{k^{3/2}}{\varepsilon}, C_\tau \left( \frac{\nu^3}{\varepsilon} \right)^{1/4} \right) \quad (6)$$

$$T = \max \left( \frac{k}{\varepsilon}, C_T \left( \frac{\nu}{\varepsilon} \right)^{1/2} \right) \quad (7)$$

The model equations for Reynolds stress  $\overline{u_i u_j}$  are finally closed with the transport equation for dissipation rate  $\varepsilon$  of turbulent kinetic energy.

$$\frac{D\varepsilon}{Dt} = \frac{C_{\varepsilon 1} P - C_{\varepsilon 2} \varepsilon}{T} + \nu \frac{\partial^2 \varepsilon}{\partial x_k \partial x_k} + \frac{\partial}{\partial x_i} \left( \frac{C_\mu}{\sigma_\varepsilon} \overline{u_i u_j} T \frac{\partial \varepsilon}{\partial x_j} \right) \quad (8)$$

where  $C_{\varepsilon 1} = 1.35(1 + 0.1P/\varepsilon)$ ,  $C_{\varepsilon 2} = 1.83$ ,  $\sigma_\varepsilon = 1.4$  and  $C_\mu = 0.26$ , and  $P$  is the production rate of turbulent kinetic energy.

Concerning the derivation of the above relations, see Durbin(1993) for further details.

### Inhomogeneity Correction by Length Scale Anisotropy

The major feature of elliptic relaxation approach is the damping effect of the pressure strain correlation at the near wall region. However, the problem of elliptic relaxation equation (3) as emphasized by Wizman et al.(1996) and Manceau et al.(2001) is that the redistribution is amplified in the logarithmic region.

In order to avoid this problem, Laurence & Durbin(1994) and Wizman et. al(1996) proposed the empirical length scale gradient models, respectively, transformed as

$$f_{ij} - L \nabla^2 (L f_{ij}) = \frac{F_{ij}^h}{k} \quad (9)$$

$$f_{ij} - \nabla^2 (L^2 f_{ij}) = \frac{F_{ij}^h}{k} \quad (10)$$

On the other hand, Manceau et al.(2001) showed that the behaviour of amplification is a consequence of the fact that the anisotropy of the correlation function  $f(x, x')$  which is defined to obtain the integral solution of the Poisson equation and, in particular, its asymmetry in the wall normal direction due to the strong inhomogeneity in the vicinity of the wall, is not accounted for by the simple model as  $f(x, x') = \exp(-r/L)$  introduced by Durbin(1993). Thus, they proposed to use the gradient of the length scale to identify the main direction of inhomogeneity, in the following manner:

$$f(x, x') = \exp \left( \frac{-r}{L + \beta(x' - x) \cdot \nabla L} \right) \quad (11)$$

where,  $r = |x - x'|$  and  $L$  is the correlation length scale.

Considering Eq. (11) and using a Taylor series expansion, the Manceau et al.(2001) suggested a new neutral formulations with  $\beta = 1/12$  as follows.

$$(1 + 16\beta(\nabla L)^2) f_{ij} - L^2 \nabla^2 f_{ij} - 8\beta L \nabla L \cdot \nabla f_{ij} = F_{ij}^h / k \quad (12)$$

To investigate the quantity of amplification, the present work introduced an elliptic effect  $\Gamma_{ij}$  raised from the elliptic operator instead of the amplification factor  $\Gamma$  originated by Manceau et al.(2001). That is, the "elliptic effect" is induced by the elliptic operator  $L^2 \nabla^2$  and defined as,

$$\Gamma_{ij} = \frac{k f_{ij}}{F_{ij}^h} \quad (13)$$

where  $F_{ij}^h$  is the source term of the elliptic relaxation equation in homogeneity situation.  $f_{ij}$  is solved by elliptic relaxation equation (3) and is affected by boundary condition. The ratio of  $k f_{ij}$  to  $F_{ij}^h$  means the deviation of redistribution calculated by elliptic operator from the quasi-homogeneous redistribution model.

In order to test the "elliptic effect", an *a priori* test is performed with the DNS data for a channel flow at  $Re_\tau = 590$ . The *a priori* test consists in solving the Durbin's elliptic relaxation equation(1993) with the terms  $F_{ij}^h$  and  $L$  taken from the DNS data. The exact boundary condition is also applied at the wall. The equation is solved in only one-half of the channel using a symmetry boundary condition at the center of the channel.

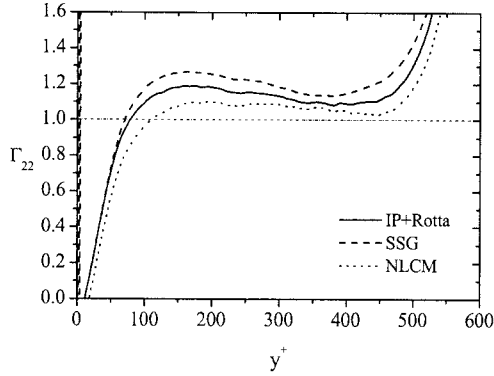


Fig. 1 The comparison of elliptic effect for the wall normal direction  $\Gamma_{22}$  with three source terms based on Durbin's elliptic relaxation equation(1993).

For the source term  $F_{ij}^h$ , any quasi-homogeneous model can be used. Here, for linear model, we adopt the simple model recommended by Launder et al.(1975), which consists of a sum of Rotta's return to isotropy and the isotropization of production model (hereafter, IP+Rotta). Also, for quasi-linear model and non-linear model, we adopt Speziale, Sarkar & Gatski(1991, hereafter SSG) model and modified Launder & Teselepidakis (1991, hereafter NLCM; non-linear cubic model) model, respectively.

According to the above mentioned source term models, the elliptic effect for wall normal direction  $\Gamma_{22}$  is depicted in Fig. 1. From the *a priori* test, although the discrepancies are existed in the adopted all models, we can see the similar trends in the whole region. The amplified distributions of  $\Gamma_{22}$  are shown in the logarithmic region and in the channel center region. The amplification is gradually decreasing in the logarithmic region and rapidly increasing in the channel center. Also,  $\Gamma_{22}$  brings about damping effect in viscous sublayer and buffer layer region inner  $y^+ \approx 50$ . The same characteristics as profiles for  $\Gamma_{22}$  are obtained from the  $\Gamma_{11}$  and  $\Gamma_{33}$  components. Since this amplification effect is caused by the elliptic operator, as mentioned the above, the modified elliptic operators are suggested by Laurence & Durbin(1994), Wizman et al.(1996) and Manceau et al.(2001).

#### Strong Inhomogeneous Correction by Quasi-Homogeneous Pressure Strain Gradient

In the source terms modelling process of Durbin's ERE and the other ERE, generally, the homogeneous situation is assumed. Thus, we think that it is sufficient to adopt the quasi-homogeneous models for the source term of ERE. However, in case of considering the inhomogeneous situation, it is natural to introduce an inhomogeneous pressure strain model in the source term of ERE.

According to Manceau et al.(2001), the elliptic relaxation equation in the primitive form is written as,

$$\Phi_{ij} - L^2 \nabla^2 \Phi_{ij} = -\frac{L^2}{\rho} \Psi_{ij} \quad (14)$$

$$\Psi_{ij} = \left( u_i \nabla^2 \frac{\partial p}{\partial x_j} + u_j \nabla^2 \frac{\partial p}{\partial x_i} \right) \quad (15)$$

In homogeneous situation, the second term of the LHS

on Eq. (14),  $L^2 \nabla^2 \Phi_{ij}$ , vanishes and the RHS of Eq. (14) can be replaced by any quasi-homogeneous model  $\Phi_{ij}^h$ . However, through imposing boundary conditions on Eq. (14), non-homogeneous effects induced by the presence of a wall can be represented. In order to consider the strong inhomogeneity situation in the near wall region, we introduce an inhomogeneous redistribution model in the source term. For the inhomogeneous situation,  $\Phi_{ij}$  and  $\Psi_{ij}$  on Eq. (14) can be decomposed into homogeneous and inhomogeneous parts as follows.

$$\Phi_{ij} = \Phi_{ij}^h + \Phi_{ij}^{inh}, \quad \Psi_{ij} = \Psi_{ij}^h + \Psi_{ij}^{inh} \quad (16)$$

The substitution of Eq. (16) into Eq. (14) yields the following source term equations.

$$-\frac{L^2}{\rho} \Psi_{ij}^h = \Phi_{ij}^h - L^2 \nabla^2 \Phi_{ij}^h \quad (17)$$

$$-\frac{L^2}{\rho} \Psi_{ij}^{inh} = \Phi_{ij}^{inh} - L^2 \nabla^2 \Phi_{ij}^{inh} \quad (18)$$

First of all, in homogeneous part Eq. (17), it is assumed that the second term of the RHS,  $L^2 \nabla^2 \Phi_{ij}^h$ , vanishes and the first term of the RHS can be replaced by any quasi-homogeneous model. However, in inhomogeneous part Eq. (18), it is necessary to model the first and second term of the RHS respectively. In the second term  $L^2 \nabla^2 \Phi_{ij}^{inh}$ ,  $\nabla^2 \Phi_{ij}^{inh}$  is modelled by using a parameter  $f_a$  as follows. The parameter connects the far from the wall situation and near wall effect.

$$\nabla^2 \Phi_{ij}^{inh} = (1-f_a) [\nabla^2 \Phi_{ij}^{inh}]_{near\ wall} + f_a [\nabla^2 \Phi_{ij}^{inh}]_{far\ wall} \quad (19)$$

In Eq. (19), in order to model the term  $[\nabla^2 \Phi_{ij}^{inh}]_{near\ wall}$ , a special approximation is introduced by subtracting  $L^2 \nabla^2 \Phi_{ij}$  from  $\nabla^2 L^2 \Phi_{ij}$  as,

$$[\nabla^2 \Phi_{ij}^{inh}]_{near\ wall} = \frac{1}{L^2} (\nabla^2 L^2 \Phi_{ij} - L^2 \nabla^2 \Phi_{ij}) \quad (20)$$

The first term of parenthesis of the RHS on Eq. (20) is the Wizman et al.'s elliptic operator(1996) and the second term is Durbin's original one(1993). Thus, the difference of the two operators is assumed to model  $[\nabla^2 \Phi_{ij}^{inh}]_{near\ wall}$  in this study.

On the other hand, the term  $[\nabla^2 \Phi_{ij}^{inh}]_{far\ wall}$  appeared from the second term of the RHS on Eq. (19) can be reconstructed by composition of homogeneous and in-homogeneous parts, and then considering  $[\nabla^2 \Phi_{ij}^h]_{far\ wall} \approx 0$  in the far from the wall, the term arrives at following station.

$$[\nabla^2 \Phi_{ij}^{inh}]_{far\ wall} \approx [\nabla^2 \Phi_{ij}^h]_{far\ wall} + [\nabla^2 \Phi_{ij}^{inh}]_{far\ wall} = \nabla^2 \Phi_{ij} \quad (21)$$

Inserting Eqs. (19)-(21) into Eq. (18) and then substituting of Eq. (17) and (18) into Eq. (14), ultimately, a new formulation for elliptic relaxation operator can be derived as

$$\Phi_{ij} - (1-f_a) \nabla^2 L^2 \Phi_{ij} = \Phi_{ij}^h + \Phi_{ij}^{inh} \quad (22)$$

where,  $f_a = A$  and  $A$  is Lumley's stress flatness invariant.

In order to model inhomogeneous pressure strain  $\Phi_{ij}^{inh}$ ,

we adopted the wall-echo correction proposed by Launder & Li(1994). Thus, the effective homogeneous pressure-strain gradient is defined as

$$\Phi_{ij}^{inh} = C^{inh} L^2 \frac{\partial f(A)}{\partial x_k} \frac{\partial \Phi_{ij}^h}{\partial x_k} \quad (23)$$

where,  $f(A) = A^{0.3}(1 + 2.5A^3)$  suggested by Launder and Li(1994) and the coefficient  $C^{inh}$  is calibrated from the calculations for channel flow as 0.08.

To test the prediction ability for the amplification by the present formulation Eq. (22), we perform the *a priori* test for the "elliptic effect" with the channel flow DNS data at  $Re_\tau = 590$  and the result is compared with the other formulations proposed by Durbin's original model Eq. (3), Wizman's empirical model Eq. (10) and Manceau et al.'s neutral model Eq. (12).

Fig. 2 shows the distributions of damping effect in the near wall and the amplification effect in logarithmic region. These results are obtained by using IP+Rotta and SSG pressure-strain models as the source term. In order to compare the "elliptic effect" of elliptic operators, the length scale coefficient  $C_L$  appeared in Eq. (6) is fixed to 0.2. However, the model of Manceau et al.(2001) is computed by both 0.2 and 0.28. Wizman et al.'s ERE and Manceau et al.'s ERE which are known to the neutral models show the very similar trends with the small amplification quantity relatively in the logarithmic region as compared with Durbin's original model. The present formulation, also, gives good profiles for the reduction of amplification in the logarithmic layer and, especially, shows the superior results to those of Wizman et al.'s ERE and Manceau et al.'s ERE in the core region.

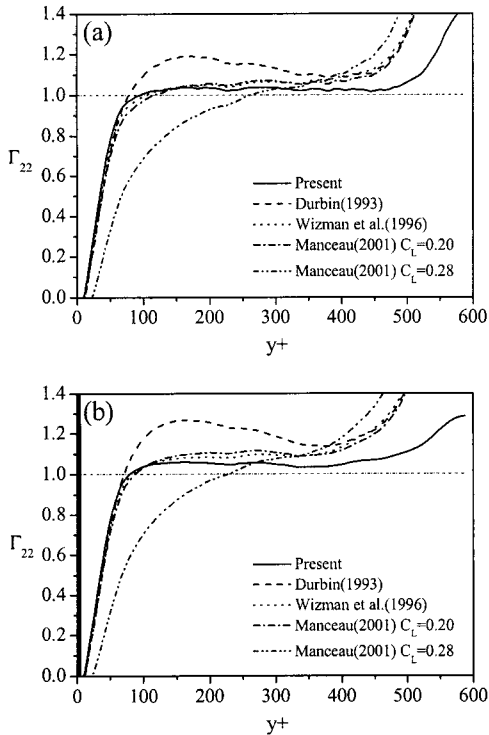


Fig 2. The effects of elliptic relaxation operator on  $\Gamma_{22}$  for (a) Rotta+IP model (b) SSG model.

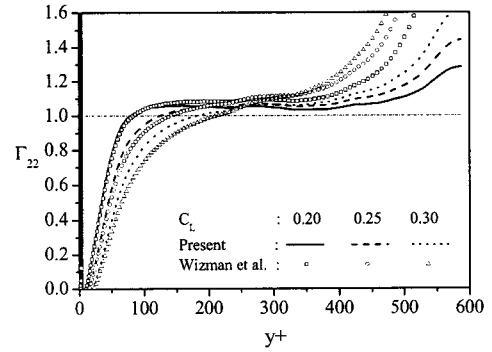


Fig. 3 Distributions of  $\Gamma_{22}$  due to various turbulent length scale coefficients with SSG model.

That is, the amplification effect can be reduced by modification of the original Durbin's model, which is achieved by additional term obtaining from the asymmetry correlation such as Wizman et al. and Manceau et al.'s ERE or by introducing the inhomogeneous effect in the source term as the present model. In case of adopting  $C_L = 0.28$  in Manceau et al.'s ERE, so enlarged damping effects are reproduced in near wall region.

On the other hand, when compared with the source term models of ERE, the elliptic effect for wall normal direction  $\Gamma_{22}$  is generally amplified by more SSG than IP+Rotta.

The effect of length scale coefficient  $C_L$  on Eq. (13) is examined for Wizman et al.'s ERE and the present ERE with SSG model as shown in Fig. 3. Three coefficients are adopted in  $C_L = 0.2, 0.25$  and  $0.3$ . It can be seen that the increasing of length scale coefficient leads to over-damping effect in the near wall region and to over-amplifying in the core region.

## RESULTS AND DISCUSSION

In this section, simulations are performed with Reynolds stress equation involving the elliptic relaxation equation, in both cases of non-rotating and rotating channel flows as shown in Fig. 4.

First of all, in order to investigate the elliptic effects in the non-rotating channel flow, as can be seen in Fig. 5, the mean velocity and Reynolds stress profiles are compared with DNS data ( $Re_\tau = 194$ , Kristoffersen & Andersson 1993) for four elliptic relaxation equations (Durbin's original model 1993; Wizman et al. 1996; Manceau et al. 2001 and the present model Eq. (22)). For all models, the coefficients appeared in the model equations are adopted on the same values except elliptic operator only.

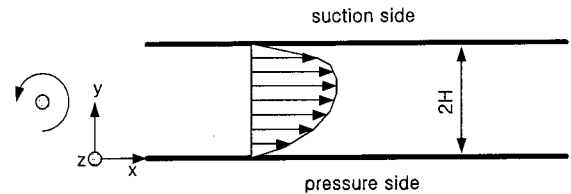


Fig. 4 Schematic diagram of flow configuration and coordinate system.

The coefficient of the length scale is, in particular, fixed to 0.2, because the elliptic effect is strongly influenced by the length scale coefficient. And, the SSG model is used as the source term of the elliptic relaxation equations.

The profiles of the mean velocity show very similar results for all models in the logarithmic region, but a discrepancy is existed in the core region by Durbin's model. In Fig. 5(b) and (c), the distributions of streamwise and wall normal turbulence intensity are predicted well by the present model than any other models, especially, in the core region. This distinct behaviors in the core region by the present model are related to the distributions of the "elliptic effect" in the same region. That is, it is already mentioned, through the *a priori* test, that the present model gives the weaker amplification effect than other models in the core region.

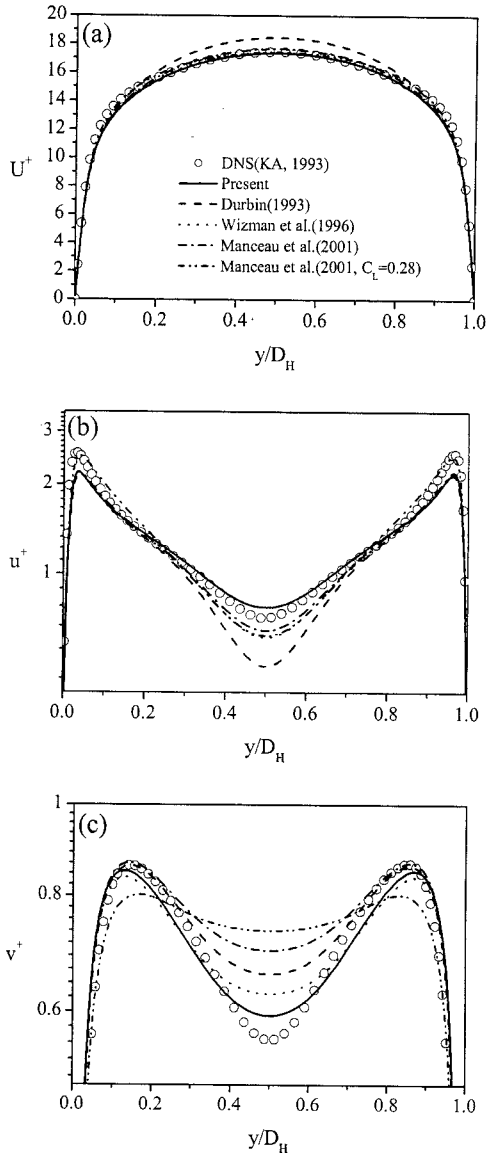


Fig. 5 A comparison of (a) normalized mean velocity profiles; (b) streamwise and (c) wall-normal intensity profiles with  $Re_\tau=194$  DNS(symbols, Kristoffersen & Andersson, 1993).

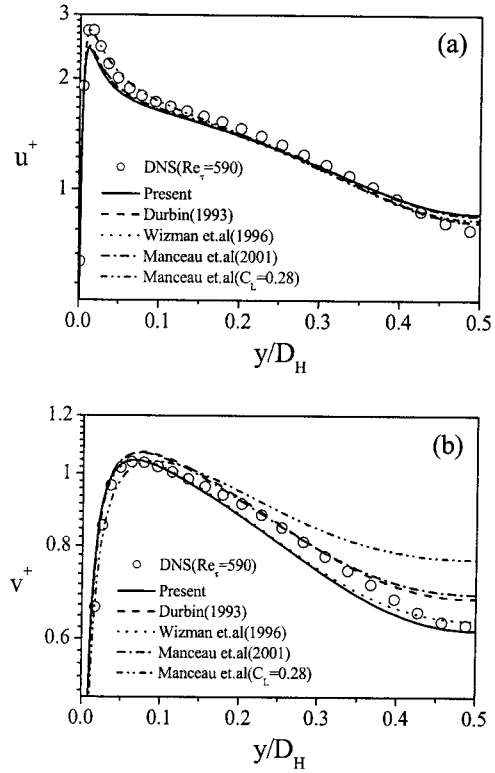


Fig. 6 A comparison of (a) streamwise intensity, (b) wall-normal intensity profiles with  $Re_\tau=590$  DNS data (symbols, Moser et al. 1999).

Also, it can be observed in Fig. 6 that the present model, for the test case of  $Re_\tau=590$ (Moser et al. 1999), gives overall correct normal stress profiles in the whole region. This result shows that the prediction sensitives due to the variation of Reynolds number can be appreciably reflected by the present model. Generally, in non-rotating channel flow calculation, Wizman et al.(1996) and Manceau et al.(2001) models show good results in comparison with the DNS data. As will be seen later, however, elliptic operator of Wizman et al. and Manceau et al. offers the unstable solution on the imposed system rotation.

In the simulations for rotating channel flow using elliptic relaxation model, Wizman et al.'s model with SSG and Pettersson & Andersson(1997) model with non-linear pressure strain proposed by Ristorcelli et al.(1995) were successfully applied to the rotating flow. However, in order to reproduce the inclination of relaminarization in the suction side, an additional rotating source term( $C_{\Omega} k e_{ijk} \Omega_j U_{k,i}$ ) in the dissipation equation is adopted by both models.

On the examination for the dissipation rate of isotropic turbulence, Speziale et al.(1991) demonstrated that the additional term in dissipation equation is not only theoretically unfounded but also is limited at local rotation number. Therefore, it is noted that, in the present calculations, the additional source term in the dissipation equation is not used. Since the additional term is purely empirical, we think that the use of the term can induce the unphysical phenomena on the industrial applications. In case of excluding the

additional rotating source term in the dissipation equation, the above both models have failed to reproduce the DNS data at high rotation rate due to a numerical instability. Thus, the prediction for rotating channel flows is limited to the present model. And, the rotational effect is induced only from the production due to rotation and the absolute vorticity of the pressure-strain term in the full Reynolds stress equation.

Fig. 7 shows the associated mean velocity distributions across the channel with increasing rotation number ( $Ro=0.1, 0.2$  and  $0.5$ ). The position of maximum mean velocity is shifted towards the suction side of the channel and the profiles become approximately linear with a slope  $dU/dy$  of  $2\Omega$  and the width of linear slope region increases with the rotation number. The model predictions are compared well with DNS data. Fig. 8 represents the profiles of Reynolds stresses. The streamwise intensity  $u^+$  significantly reduced close to the suction side with increasing rotation.

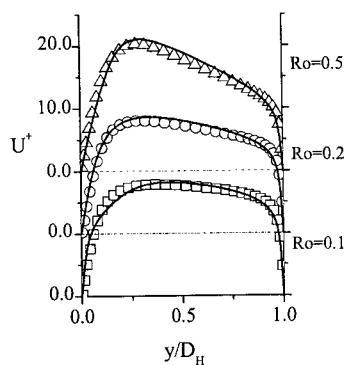


Fig. 7 Streamwise mean velocity profiles for the present model with increasing rotating rates; DNS data (Kristoffersen & Andersson, 1993)-symbols; present model-lines.

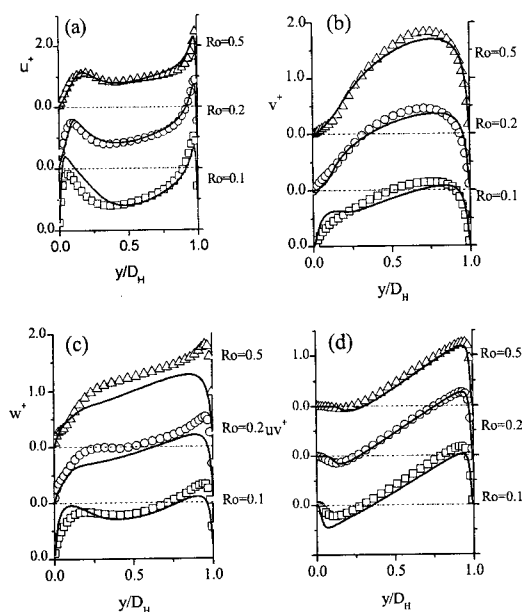


Fig. 8 Reynolds stresses profiles for the present model with increasing rotating rates: DNS, symbols; present model, lines.

The wall normal intensity  $v^+$  increases gradually in between pressure side and the channel center as shown in Fig. 8(b). In turbulent shear stress  $uv^+$ , the positions of zero shear stress are shifted towards the suction side with increasing rotation. The distributions of mean velocity and intensity components except  $w^+$  are in excellent agreement in comparison with DNS data. On the other hand, the spanwise intensity is slightly under-estimated with increasing rotation. Overall, the present predictions by the imposed system rotation show good agreement with DNS data at higher rotation number than low rotation number.

## CONCLUSION

A new inhomogeneous correction of elliptic relaxation equation in RANS modelling is proposed to intermediate between near wall and far from the wall. Especially, in considering the inhomogeneous situation, the source term of ERE is modeled by decomposing into homogeneous and inhomogeneous parts. From this modelling process, we can more reduce the amplification of redistribution by the present model than the other models.

In the non-rotating channel flow, the profiles of the mean velocity show very similar results for all models in the logarithmic region. The normal turbulence intensity distributions are predicted well by the present model than any other models, especially, in the core region. This distinct behaviors in the core region by the present model are related to the distributions of the "elliptic effect" in the same region.

The other models except the presents model have failed to reproduce the DNS data at high rotation rate due to a numerical instability. Thus, the prediction for rotating channel flows is limited to the present model. The present predictions by the imposed system rotation show good agreement with DNS data in the wide range of the rotating number.

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