A GENERALIZED EARSM BASED ON A NONLINEAR PRESSURE STRAIN RATE MODEL

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ABSTRACT

The use of a pressure strain rate model including terms nonlinear in the strain and rotation rate tensors in an explicit algebraic Reynolds stress model (EARSM) is considered. For 2D mean flows the nonlinear contributions can be fully accounted for in the EARSM formulation. This is not the case for 3D mean flows and a suggestion of how to modify the nonlinear terms to make the EARSM formulations for 2D and 3D mean flows consistent is given. The corresponding EARSM is derived. This is all done in conjunction with the use of streamline curvature corrections emanating from the advection of the Reynolds stress anisotropy. The proposed model is tested for rotating homogeneous shear flow, rotating channel flow and rotating pipe flow. The nonlinear contributions are shown to have a significant effect on the flow characteristics. Finally, model predictions are investigated using a $K-\omega$ platform with a change in the dissipation rate production, $\mathcal{P}_\varepsilon$. It is found that even for a small change of $\mathcal{P}_\varepsilon$ the effect is considerable and it is argued that some of the shortcomings of the model can be attributed to the modelling of $\mathcal{P}_\varepsilon$ (or corresponding $\mathcal{P}_\omega$).

INTRODUCTION

Effects of strong curvature and rotation represent cornerstone problems in turbulence modelling. An exploration of what can be done in terms of realizability and prediction improvements of rotating flows using a Differential Reynolds stress model (DRSM) was undertaken by Sjögren and Johansson (2000). In the Sjögren and Johansson DRSM (SJ-DRSM) a pressure strain rate model nonlinear in the Reynolds stress anisotropy, $a_{ij} = \overline{u_i u_j}/K - 2\delta_{ij}/3$, was used. Also, tensor groups quadratic in the mean strain and rotation rate tensors were included and were found to improve predictions in rotating flows. Here, the effect of such a nonlinear pressure strain rate model on an EARSM is investigated.

An EARSM can be derived from the transport equation for the Reynolds stress anisotropy

$$K\frac{Da_{ij}}{Dt} - \left(\frac{\partial T_{ijl}}{\partial x_l} - \frac{\overline{u_i u_j}}{K}\frac{\partial T_l^{(K)}}{\partial x_l}\right) = -\frac{\overline{u_i u_j}}{K}(\mathcal{P} - \varepsilon) + \mathcal{P}_{ij} - \varepsilon_{ij} + \Pi_{ij} \quad (1)$$

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in which the dissipation rate tensor ε_{ij} and the pressure strain rate tensor Π_{ij} need to be modelled while the Reynolds stress production $\mathcal{P}_{ij}$ and the turbulence kinetic energy production $\mathcal{P}$ can be expressed explicitly in a_{ij}, K and the mean strain and rotation rate tensors. When normalized with the turbulence time scale ($\tau = K/\varepsilon$) the latter read

$$S_{ij} \equiv \frac{\tau}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \quad \Omega_{ij} \equiv \frac{\tau}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \quad (2)$$

Assuming weak equilibrium, Rodi (1976), which amounts to neglecting the advection and diffusion yields a purely algebraic relation

$$\left(\mathbf{a} + \frac{2}{3}\mathbf{I}\right)\left(\frac{\mathcal{P}}{\varepsilon} - 1\right) = \frac{1}{\varepsilon}(\boldsymbol{\mathcal{P}} - \boldsymbol{\varepsilon} + \mathbf{\Pi}) \quad (3)$$

in boldface matrix notation. The normalized Reynolds stress production can be expressed as

$$\frac{\boldsymbol{\mathcal{P}}}{\varepsilon} = -\frac{4}{5}\mathbf{S} - (\mathbf{aS} + \mathbf{Sa}) + \mathbf{a\Omega} - \mathbf{\Omega a} \quad (4)$$

The pressure strain rate and dissipation rate anisotropy ($e_{ij} = \varepsilon_{ij}/\varepsilon - 2\delta_{ij}/3$) tensors can be lumped together and modelled, with the nonlinear contributions in addition to general quasilinear model, as

$$\frac{\mathbf{\Pi}}{\varepsilon} - \mathbf{e} = -\frac{1}{2}\left(C_1^0 + C_1^1\frac{\mathcal{P}}{\varepsilon}\right)\mathbf{a} + C_2\mathbf{S} + \frac{C_3}{2}(\mathbf{aS} + \mathbf{Sa} - \frac{2}{3}\{\mathbf{aS}\}\mathbf{I}) - \frac{C_4}{2}(\mathbf{a\Omega} - \mathbf{\Omega a}) + C_\Omega(\mathbf{N}^\Omega + \mathbf{N}^S) \quad (5)$$

in which $\mathcal{P}/\varepsilon \equiv -\{\mathbf{aS}\}$, where $\{\}$ denotes the trace, and the nonlinear terms are

$$\mathbf{N}^\Omega = \frac{1}{\sqrt{-II_\Omega}}\left(\mathbf{a\Omega}^2 + \mathbf{\Omega}^2\mathbf{a} - \frac{2}{3}\{\mathbf{a\Omega}^2\}\mathbf{I}\right) \quad (6)$$

$$\mathbf{N}^S = \frac{1}{\sqrt{II_S}}\left(\mathbf{aS}^2 + \mathbf{S}^2\mathbf{a} - \frac{2}{3}\{\mathbf{aS}^2\}\mathbf{I}\right) \quad (7)$$

$\mathbf{N}^\Omega$ and $\mathbf{N}^S$ were used by Sjögren and Johansson (2000) and were shown to improve the prediction of effects of rotation. By using the same model coefficient, C_Ω, for both $\mathbf{N}^\Omega$ and $\mathbf{N}^S$ their total contribution vanishes in flows where $II_S = -II_\Omega$, e.g. channel flow and homogeneous shear flow.

The modelled transport equation of the Reynolds stress anisotropy can now be written

$$\tau\left(\frac{D\mathbf{a}}{Dt}-\mathcal{D}^{(a)}\right)=A_0\Big((A_3+A_4\frac{\mathcal{P}}{\varepsilon})\mathbf{a}+A_1\mathbf{S}-(\mathbf{a\Omega}-\mathbf{\Omega a})+A_2(\mathbf{aS}+\mathbf{Sa}-\frac{2}{3}\{\mathbf{aS}\}\mathbf{I})-c(\mathbf{N}^{\Omega}+\mathbf{N}^{S})\Big) \quad (8)$$

The relations between the A and C-coefficients are

$$A_0=\frac{C_4}{2}-1 \quad A_1=\frac{3C_2-4}{3A_0} \quad A_2=\frac{C_3-2}{2A_0}$$
$$A_3=\frac{2-C_1^0}{2A_0} \quad A_4=\frac{-C_1^1-2}{2A_0} \quad c=-\frac{C_\Omega}{A_0} \quad (9)$$

After assuming weak equilibrium and hence neglecting the advection and diffusion (8) can, in analogy with (3), be written as an algebraic relation

$$N\mathbf{a} = -A_1\mathbf{S}+(\mathbf{a\Omega}-\mathbf{\Omega a})-A_2(\mathbf{aS}+\mathbf{Sa}-\frac{2}{3}\{\mathbf{aS}\}\mathbf{I}) +c(\mathbf{N}^{\Omega}+\mathbf{N}^{S}) \quad (10)$$

where $N=A_3+A_4\frac{\mathcal{P}}{\varepsilon}$.

NONLINEAR MODELLING IN CONJUNCTION WITH STREAMLINE CURVATURE CORRECTIONS

By imposing the weak equilibrium assumption in the streamline based coordinate system instead of the computational system, the advection gives rise to an additional algebraic term, see Girimaji (1997), Sjögren (1997) and Wallin and Johansson (2002). This contribution can be formulated in terms of an antisymmetric tensor $\mathbf{\Omega}^{(r)}$ as

$$-(\mathbf{a\Omega}^{(r)}-\mathbf{\Omega}^{(r)}\mathbf{a}) \quad (11)$$

From (8) it is clear that for an EARSM using a linear pressure strain rate model ($c=0$) the streamline curvature correction (11) can be fully accounted for by replacing the rotation rate tensor $\mathbf{\Omega}$ with

$$\mathbf{\Omega}^*=\mathbf{\Omega}-\frac{\tau}{A_0}\mathbf{\Omega}^{(r)} \quad (12)$$

Problems arise when $c\neq 0$ since the tensor $\mathbf{N}^{\Omega}$ which is nonlinear in the absolute rotation rate tensor must be taken into account while still keeping the two and three dimensional formulations consistent. For 2D mean flows this can be overcome and a mathematically correct formulation can be reached. For 3D mean flows, on the other hand, the nonlinearity of $\mathbf{N}^{\Omega}$ would demand the use of a tensor basis based on three tensors $\mathbf{S}$, $\mathbf{\Omega}$ and $\mathbf{\Omega}^{(r)}$. This is an awkward solution to the problem and a more tractable procedure is needed.

By reformulating $\mathbf{N}^{\Omega}$ as

$$\mathbf{N}^{\Omega}=\frac{\sqrt{-II_\Omega}}{-II_{\Omega^*}}\left(\mathbf{a\Omega}^{*2}+\mathbf{\Omega}^{*2}\mathbf{a}-\frac{2}{3}\{\mathbf{a\Omega}^{*2}\}\mathbf{I}\right) \quad (13)$$

(10) is fully solvable, in terms of N, for 2D and 3D mean flows when using the appropriate approximation of $\mathbf{N}^S$ for 3D mean flows (see below). Furthermore, the EARSM solution for 2D mean flows will be the same as the exact curvature corrected solution and the 3D EARSM formulation will in a consistent way reduce to the 2D formulation in case of 2D mean flows. Also, when the curvature correction is switched off ($\frac{\tau}{A_0}\to 0$), (13) reduces to its original form, (6).

EARSM FORMULATION

To reach an EARSM formulation, (10) is solved by expanding the Reynolds stress anisotropy in terms of a tensor basis derived from the mean velocity gradients by using the Caley-Hamilton theorem. The algebraically least complex way to do this is in terms of a ten element tensor basis, $\mathbf{T}^{(i)}$. The ten basis tensors can, in fact, be reduced to five tensors, but the resulting coefficients will be extremely lengthy, see Taulbee et al. (1994). Thus, the Reynolds stress anisotropy is most conviniently expanded as

$$\mathbf{a}=\sum_{i=1}^{10}\beta_i\mathbf{T}^{(i)} \quad (14)$$

(14) is then inserted into (10) and solved for the β-coefficients which can depend on the invariants of $\mathbf{S}$ and $\mathbf{\Omega}^*$

$$II_S=\{\mathbf{S}^2\} \quad II_{\Omega^*}=\{\mathbf{\Omega}^{*2}\} \quad III_S=\{\mathbf{S}^3\}$$
$$IV=\{\mathbf{S\Omega}^{*2}\} \quad V=\{\mathbf{S}^2\mathbf{\Omega}^{*2}\} \quad (15)$$

The scalar left hand side nonlinearity of (10) in the unknowns (β-coefficients) is adressed by solving a polynomial equation for N after the β-coefficients have been determined.

It has been shown, see e.g. Taulbee (1992), that for $A_2=0$ and $c=0$, (10) maps to five basis tensors for 3D mean flows:

$$\mathbf{T}^{(1)}=\mathbf{S} \qquad \mathbf{T}^{(3)}=\mathbf{\Omega}^{*2}-\tfrac{1}{3}II_{\Omega^*}\mathbf{I}$$
$$\mathbf{T}^{(4)}=\mathbf{S\Omega}^*-\mathbf{\Omega}^*\mathbf{S} \quad \mathbf{T}^{(6)}=\mathbf{S\Omega}^{*2}+\mathbf{\Omega}^{*2}\mathbf{S}-\tfrac{2}{3}IV\mathbf{I}$$
$$\mathbf{T}^{(9)}=\mathbf{\Omega}^*\mathbf{S\Omega}^{*2}-\mathbf{\Omega}^{*2}\mathbf{S\Omega}^* \quad (16)$$

This is convenient since the model based on a full ten tensor basis is much more complex, and the behaviour in terms of singularities is presently not known.

Since we are interested in the curvature corrected model formulation, $\mathbf{\Omega}^*$ rather than $\mathbf{\Omega}$ will be used in the basis tensors (16) and the invariants (15).

EARSM solution for 2D mean flows

For 2D mean flows with $A_2=0$ and $c\neq 0$, the EARSM solution maps to $\mathbf{T}^{(1)}$ and $\mathbf{T}^{(4)}$ and the corresponding β-coefficients are

$$\beta_1=-\frac{A_1N^*}{N^{*2}-2II_{\Omega^*}} \quad \beta_4=-\frac{A_1}{N^{*2}-2II_{\Omega^*}} \quad (17)$$

N^* is given by the solution of the third order equation

$$N^{*3}-A_3^*N^{*2}-(A_1A_4+2II_{\Omega^*})N^*+2A_3^*II_{\Omega^*}=0 \quad (18)$$

which is the same equation as for the WJ-EARSM but with $N\to N^*=N-c(\sqrt{II_S}-\sqrt{-II_\Omega})$ and $A_3\to A_3^*=A_3-c(\sqrt{II_S}-\sqrt{-II_\Omega})$. Note that it is the inertial II_Ω that is used for the A_3^* relation. The physical solution of (18) is given by

$$N^* = \begin{cases} \frac{A_3^*}{3} + (P_1 + \sqrt{P_2})^{1/3} + \\ \mathrm{sign}(P_1 - \sqrt{P_2})|P_1 - \sqrt{P_2}|^{1/3} & P_2 \geq 0 \\ \\ \frac{A_3^*}{3} + 2(P_1^2 - P_2)^{1/6} \\ \cos\left(\frac{1}{3}\arccos\left(\frac{P_1}{\sqrt{P_1^2 - P_2}}\right)\right) & P_2 < 0 \end{cases} \quad (19)$$

where

$$P_1 = \left(\frac{A_3^{*2}}{27} + \frac{A_1 A_4}{6} II_S - \frac{2}{3} II_{\Omega^*}\right) A_3^* \quad (20)$$

$$P_2 = P_1^2 - \left(\frac{A_3^{*2}}{9} + \frac{A_1 A_4}{3} II_S + \frac{2}{3} II_{\Omega^*}\right)^3 \quad (21)$$

It is worth pointing out that the transformation $A_3 \to A_3^*$ is the only difference between the proposed model and the WJ-EARSM for 2D mean flows.

EARSM solution for 3D mean flows

For 3D mean flows the nonlinear tensor $\mathbf{N}^S$ can be shown to map outside the five tensor basis (16). In order to eliminate this problem $\mathbf{N}^S$ can be approximated with its exact expression in 2D mean flows, $\mathbf{N}^S = \sqrt{II_S}\mathbf{a}$.

With $A_2 = 0$, $\mathbf{N}^S = \sqrt{II_S}\mathbf{a}$ and using the nonlinear contribution $\mathbf{N}^\Omega$ in (13) the corresponding EARSM coefficients become

$$\begin{aligned} \beta_1 &= A_1(N^* c^2 II_\Omega + 8c II_{\Omega^*}\sqrt{-II_\Omega} - 14N^* II_{\Omega^*} \\ &\quad +4N^{*3})/(Q_1 Q_2) \\ \beta_3 &= 4IV A_1(2c^2 II_\Omega N^{*2} + c^3 N^*(-II_\Omega)^{3/2} - 18 II_{\Omega^*}^2 \\ &\quad +18cN^*\sqrt{-II_\Omega} II_{\Omega^*} + 7c^2 II_{\Omega^*} II_\Omega) \\ &\quad /(II_{\Omega^*}^2 (2c\sqrt{-II_\Omega} - 3N^*) Q_1 Q_2) \\ \beta_4 &= 4A_1/Q_1 \\ \beta_6 &= -2A_1(N^* c^2 II_\Omega + 4c II_{\Omega^*}\sqrt{-II_\Omega} + 2N^{*2} c\sqrt{-II_\Omega} \\ &\quad -6N^* II_{\Omega^*})/(II_{\Omega^*} Q_1 Q_2) \\ \beta_9 &= 2A_1(4cN^*\sqrt{-II_\Omega} + c^2 II_\Omega - 6II_{\Omega^*}) \\ &\quad /(II_{\Omega^*} Q_1 Q_2) \end{aligned} \quad (22)$$

where

$$\begin{aligned} Q_1 &= -(2N^* - c\sqrt{-II_\Omega})^2 + 2II_{\Omega^*} \\ Q_2 &= N^{*2} - 2II_{\Omega^*} \end{aligned} \quad (23)$$

The equation for N for 3D mean flows is a very lengthy sixth order equation that will not be given here. This equation is of little practical use. Here we instead us the cubic equation (18) for both 2D and 3D mean flows.

The inclusion of $\mathbf{G} = \mathbf{a}^2\mathbf{\Omega} - \mathbf{\Omega}\mathbf{a}^2$

We also made an attempt of including a term nonlinear in the Reynolds stress anisotropy tensor $\mathbf{G} = \mathbf{a}^2\mathbf{\Omega} - \mathbf{\Omega}\mathbf{a}^2$ in the pressure strain rate model (5). $\mathbf{G}$ is one of terms in the most general model linear in $\mathbf{S}$ and $\mathbf{\Omega}$ (see e.g. Johansson and Hallbäck, 1994).

For 2D mean flows the inclusion of $\mathbf{G}$ in the pressure strain rate model will not change the EARSM solution if $A_2 = 0$. This can be explained by the fact that for the particular choice of $A_2 = 0$, the contribution of $\mathbf{G}$ vanishes for 2D mean flows.

For 3D mean flows $\mathbf{G}$ can be shown to map outside of the five tensor basis (16). There is no direct way of approximating $\mathbf{G}$ in such a way that the contribution outside (16) is zero while its characteristics of being nonlinear in the Reynolds stress anisotropy are preserved. In fact, if the β-coefficients are chosen such that the contribution outside (16) is zero, then the trivial (zero) solution will be the only solution.

Recalibration of A_0 .

In the basic curvature corrected model by Wallin and Johansson (2002), the A_0 coefficient was calibrated considering homogeneous rotating shear. By arguing that the model should be neutrally stable for rotation number $Ro = 1/2$, where $Ro \equiv \omega_z^{(r)}/(dU/dy)$, the value $A_0 = -0.72$ was obtained (see Wallin & Johansson (2002) for details). Since we are adding additional terms, $\mathbf{N}^\Omega$ and $\mathbf{N}^S$, that contribute in rotating flows, the value of A_0 needs to be recalibrated in order to preserve neutral stability for $Ro = 1/2$.

For the present model, an analytical relation for the model coefficients cannot be derived, as was done by Wallin & Johansson for the basic model without the nonlinear terms. Instead, the A_0 coefficient was adjusted such that the computed turbulent kinetic energy initially will be almost constant for $Ro = 1/2$. That resulted in $A_0 = -0.9$ which is used in the following.

The inclusion of the $\mathbf{N}^{\Omega,S}$ terms reduces A_3^* for Ro between 0 and 1. Under certain conditions, A_3^* may become negative, *e.g.* when $R_0 \approx 0.5$ and the turbulence is decaying. This must be avoided, since the solution of the cubic N^* equation may change character for negative A_3^* and nonphysical roots may be obtained. This is avoided by a lower limit of $A_3^* = \max(A_3 - c(\sqrt{II_S} - \sqrt{-II_\Omega}), 0)$ which should not have a major impact on the solution.

TEST CASES

The proposed model has been tested for three generic flows: rotating homogeneous shear flow and rotating channel flow for 2D mean flows and rotating pipe flow for 3D mean flows. Comparisons have been made with LES by Bardina et al. (1983) for rotating homogenous shear flow, DNS by Alvelius (1999) for channel flow and experimental data by Imao et al. (1996) for rotating pipe flow.

The curvature correction was used in all computations implying the appropriate use of $\mathbf{\Omega}^*$, as defined in (12). The model parameters were $A_0 = -0.9$ and $C_\Omega = 0.5$ as proposed by Sjögren and Johansson (2000), which yields $c = -C_\Omega/A_0 = 0.56$. Furthermore $A_1 = 1.20$, $A_2 = 0$, $A_3 = 1.80$ and $A_4 = 2.25$, see Wallin and Johansson (2002).

2D mean flow: Homogeneous rotating shear flow

Homogenous rotating shear was computed for three different rotation numbers $Ro \equiv \omega_z^{(r)}/(dU/dy)$ of 1/4, 1/2 and −1/2 using the $K - \varepsilon$ platform together with the proposed model, the WJ-EARSM and an eddy viscosity model.

Figure 1a shows the development of the turbulent kinetic energy for the most energetic case, $Ro = 1/4$. As can

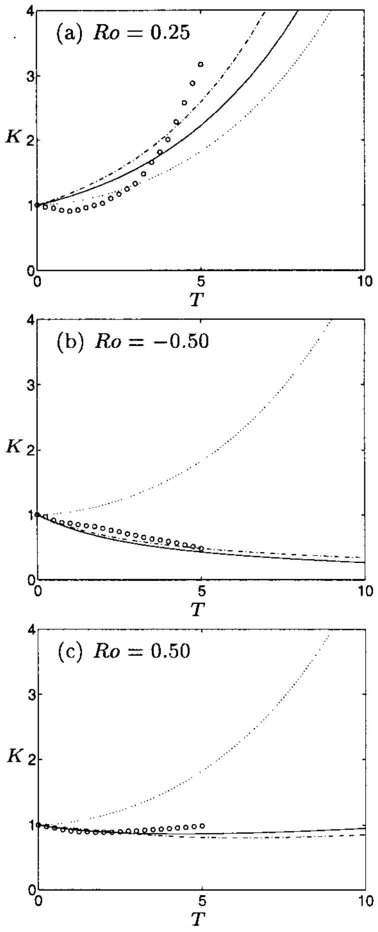

Figure 1: Rotating homogeneous shear flow, $dU/dy = 3.4$, proposed model (–·), WJ-EARSM (–), eddy-visc (···), Large eddy simulation (LES) by Bardina et al. (1983) (○)

be seen the inclusion of the nonlinear terms increases the growth rate. K is, however, still somewhat underpredicted compared to the long time behaviour of the Bardina LES .

The stable case with $Ro = -1/2$ is shown in figure 1b. Predictions of both the WJ-EARSM and the proposed model agree well with the Bardina LES data, the propsed model does, however, show a small improvement over the WJ-EARSM.

The neutrally stable case of $Ro = 1/2$ is shown in figure 1c. As expected, the change in the turbulent kinetic energy is small and the agreement with the Bardina LES good.

2D mean flows: Fully developed rotating channel flow

The second 2D mean flow test case is rotating channel flow. The channel coordinate system $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ is rotating with the rate $\omega_z^{(r)}$ in the $\mathbf{e}_z$ direction. The Reynolds number for the computation was $Re_\tau \equiv u_\tau \delta/\nu = 180$ where δ is the half channel width and $u_\tau^2 = 1/2((u_\tau^s)^2 + (u_\tau^u)^2)$ where u_τ^s and u_τ^u are the stable and unstable side friction velocities, respectively. Comparisons were made with the DNS data by Alvelius and the WJ-EARSM. The $K-\omega$ platform was used for both models and the computations were made such that u_τ assumed the same value as in the DNS data with $Ro \equiv 2\omega_z^{(r)}\delta/U_m = 0.43$. As a result, Ro differs depending on the model used, $Ro = 0.41$ for the proposed model and $Ro = 0.40$ for the WJ-EARSM. The Reynolds numbers based on the mean velocity are ($Re_m = U_m\delta/\nu$) are $Re_m^{(gen)} = 3201$ and $Re_m^{(WJ)} = 3265$ for the propsed model and the WJ-EARSM, respectively.

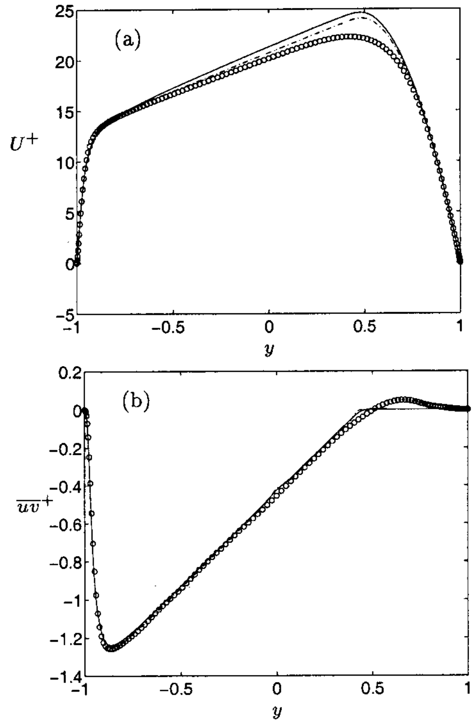

Figure 2: Computed rotating channel flow for $Re_\tau = 180$. Proposed EARSM (–·),WJ-EARSM (–), DNS data by Alvelius (○).

The predictions of U^+ with the proposed model are somewhat improved, but still overestimated, figure 2a. The predictions of $\overline{uv}^+$ are almost unaffected, figure 2b. Both models lack the ability to capture the small amount of positive shear stress on the stable side of the $\overline{uv}^+$ profile.

3D mean flows: Rotating pipe flow

Fully developed flow in a circular pipe rotating around its symmetry axis is a suitable test case since it represents a three dimensional flow that is dependent on only one spatial coordinate, r. For a laminar rotating flow the tangential velocity, U_θ, varies linearly with the radius, r, while in case of turbulence U_θ has a parabola-like profile, which is indicated by the form of the integrated Reynolds equation in the tangential direction

$$U_\theta(r) = U_\theta(R)\frac{r}{R} - \frac{r}{\nu}\int_r^R \frac{K a_{r\theta}}{r'} dr' \qquad (24)$$

An eddy-viscosity model is unable to describe this phenomenon and a fully three-dimensional EARSM is needed, i.e. quadratic EARSMs are not sufficient.

The proposed model has been used to compute rotating pipe flow at Reynolds number 20000, based on the axial bulk velocity and the pipe diameter. Two different rotation rates

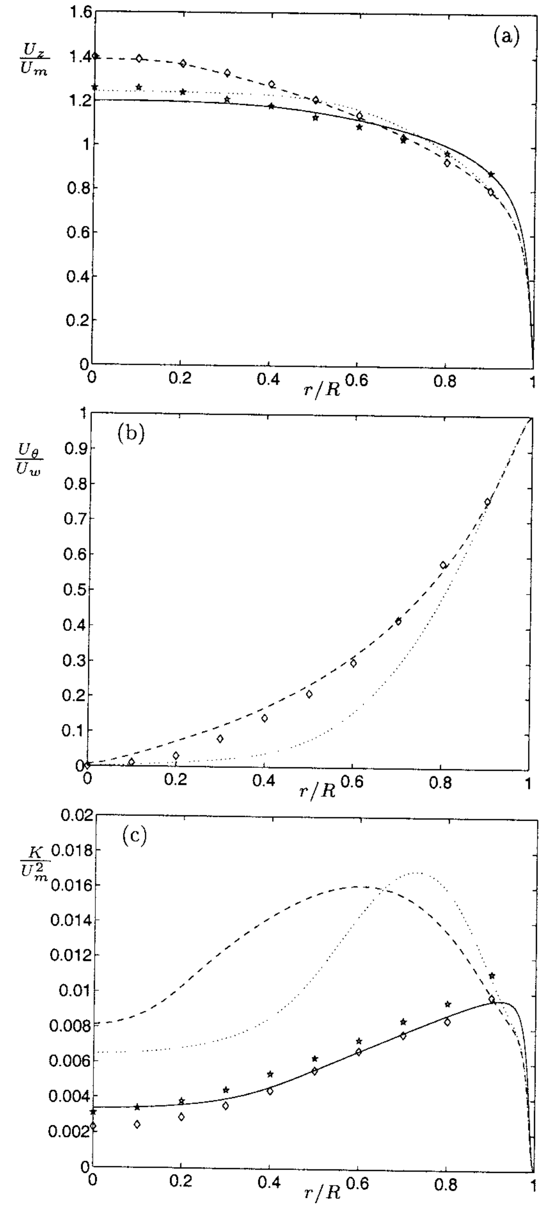

Figure 3: Computed rotating pipe flow for $Z = 0$ and 0.5, Proposed EARSM for $Z = 0.5(--)$ and $Z = 0$ $(-)$, WJ-EARSM for $Z = 0.5$ $(\cdots)$ and $Z = 0$ $(-\cdot)$ (coincides with proposed model for $Z = 0$), Experimental data by Imao et al. for $Z = 0$ $(\star)$ and $Z = 0.5$ $(\diamond)$.

were used, $Z = 0$ and 0.5 where $Z = U_\theta(R)/U_m$, the wall tangential velocity divided by the axial bulk velocity. The results are compared with experimental data by Imao et al. (1996). Comparisons are also made with the WJ-EARSM. The $K - \omega$ model was used as model platform. The model behaviour at larger rates of rotation (e.g. $Z = 1$) is presently investigated.

Figure 3a shows the computed axial velocities compared with experimental data. For $Z = 0$ both models predict a U_z that is somewhat too flat in the centre of the pipe compared to the experimental data. For $Z = 0.5$ the proposed model performs very well compared to the WJ-EARSM and the agreement with experimental data is excellent.

The tangential velocity profiles are shown in figure 3b. The proposed model clearly shows an improvement over the WJ-EARSM and the predicted U_θ agrees very well with the experiment.

The computed turbulence kinetic energy, on the other hand, is not in good agreement with the experiment for either model. Here, instead of decreasing with increasing Z, as in the experiments, K increases drastically, see figure 3c.

The importance of $\mathcal{P}_\varepsilon$

The difficulties of consistently predicting the characteristics of a rotating flow is a well known problem for both the Reynolds stress modelling and the underlying two equation platform, see Speziale et al. (1998). Especially, the modelling of the dissipation rate production is critical.

As a demonstration of this, one could study rotating pipe flow using a dissipation rate model which is a blend of the eddy viscosity assumption and an EARSM. Such a model could read $\mathcal{P}_\varepsilon = C_{\varepsilon 1}\varepsilon/K(2C_\alpha \nu_T II_S/\tau^2 + (1 - C_\alpha)P_K)$ where C_α is the parameter that determines the relative weight of the eddy viscosity assumption. Since the prodution of ω is given by $\mathcal{P}_\omega = \frac{\mathcal{P}_\varepsilon}{C_\mu K} - \frac{\omega}{K}\mathcal{P}_K$, $\mathcal{P}_\omega$ becomes

$$\mathcal{P}_\omega = 2C_\alpha(\alpha + 1)\frac{II_S}{\tau^2} + (\alpha - C_\alpha - C_\alpha \alpha)\frac{\omega}{K}\mathcal{P}_K \qquad (25)$$

where $\alpha = C_{\varepsilon 1} - 1$ and $\mathcal{P}_K$ denotes the turbulent kinetic energy production.

By using (25) in the $K - \omega$ platform the characteristics of the flow are radically changed even for a small value of C_α. Figure 4 displays the results for $C_\alpha = 0.1$ and $C_\alpha = 0$. As can be seen, the profiles are almost unaffected for $Z = 0$. For $Z = 0.5$ the relative changes of the axial and tangential velocity profiles are small compared to the change in turbulence kinetic energy, K, which is reduced considerably and is comparable to the level in the experiment. By increasing C_α further the turbulence level is reduced even more without affecting the velocity profiles significantly.

It is not the intention here to investigate the advantages and disadvantages of (25) as a model. The purpose is merely to demonstrate that the modelling of $\mathcal{P}_\omega$ is important and that the delicate nature of the problem implies that a small change in the model can have a significant effect on the flow characteristics. It is, on the other hand, promising to see that a change in a quantity not appearing explicitly in the EARSM formulation has a positive effect on the overall predictions made with the complete model.

CONCLUDING REMARKS

The contribution of the nonlinear extensions ($\mathbf{N}^{\Omega,S}$) of the quasilinear pressure strain rate model can be included in the EARSM formulation and a consistent solution can be reached. The nonlinear contributions have a significant effect on the model predictions. It is also evident that the modelling of the production of the length scale determining quantity is critical and that further development in this area is needed.

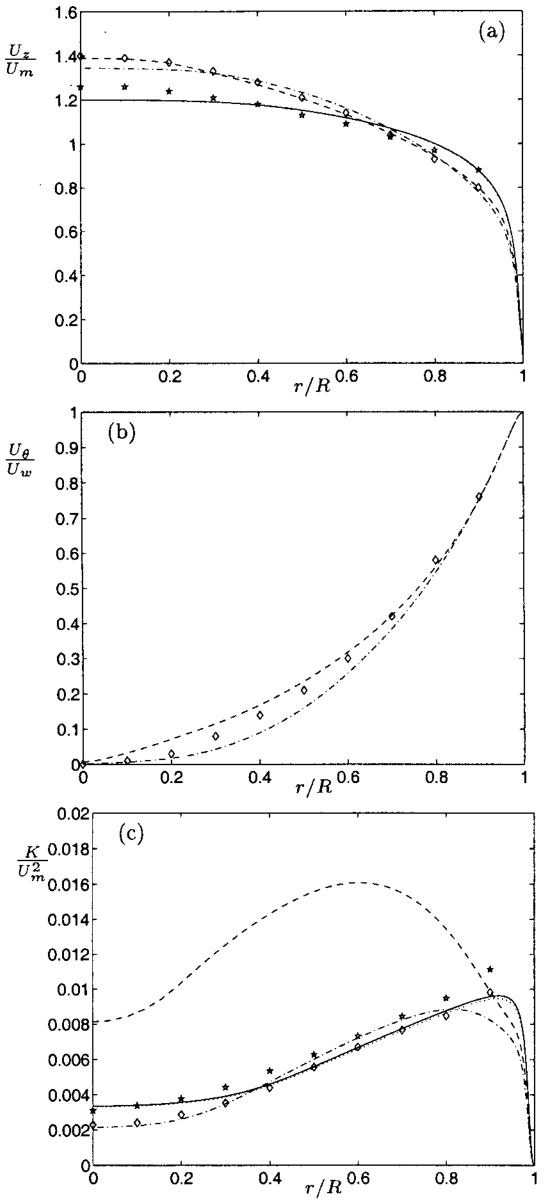

Figure 4: Rotating pipe flow for $Z = 0$ and $Z = 0.5$. Proposed model with $C_\alpha = 0.1$ for $Z = 0.5$ (-·) and $Z = 0$ (···) and with $C_\alpha = 0$ for $Z = 0.5$ (−−) and $Z = 0$ (−). Experimental data by Imao et al. for $Z = 0$ ($\star$) and $Z = 0.5$ ($\diamond$).

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