UNSTEADY REYNOLDS-AVERAGED NAVIER-STOKES COMPUTATIONS OF SHOCK INDUCED OSCILLATIONS OVER TWO-DIMENSIONAL RIGID AIRFOILS

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ABSTRACT

This paper deals with results from on-going research conducted at ONERA, regarding validation of turbulence models for unsteady transonic flows, for which shock wave/boundary layer interaction develops. The goal is to predict Shock Induced Oscillations that appear over airfoils. Two-dimensional unsteady Navier-Stokes computations have been applied to two test cases, using two complementary strategies: the "wall layer" one involving a coarse grid and a more "classic" one applied to a refined grid. Turbulence closure has been achieved using transport equation models.

INTRODUCTION

The present study is devoted to the resolution of the Unsteady Reynolds-Averaged Navier-Stokes equations, with the aim of predicting the onset and extent of Shock Induced Oscillations (SIO) over two-dimensional (2D) rigid airfoils. For transonic flows applications, such oscillations are mainly caused by shock wave/boundary layer interaction producing separated regions. The response of the wing structure to these aerodynamic instabilities corresponds to the well-known buffeting phenomenon; this latter has to be clearly identified since it limits the flight envelope of a given aircraft. Thus, the numerical prediction of SIO onset and extent requires to evaluate the ability of turbulence modelling to capture separated flows and to reproduce correctly the various time scales of the unsteady viscous flows.

Turbulence models, either simple (eddy viscosity type) or complex (Reynolds stress transport type), employed in RANS methods are very adequate for computed steady flows without any region of reverse flow (Spalart, 1999). Nevertheless, such models cannot handle the prediction of flows characterised by massive separation, for which Detached Eddy Simulations (DES) or Large Eddy Simulations (LES) should be more appropriate to capture 3D turbulent eddies and their burst (Squires et al., 2002). However, several authors dealt with unsteady computations with "standard" RANS models, taking then into account the time scales separation between SIO and turbulence. Such modelling is referred to as URANS-type models and has been applied in the present paper for the prediction of SIO onset and extent.

In former studies developed at ONERA, the evaluation of the ability of turbulence modelling to capture separated flows had been carried out at conditions prior to the onset of SIO, for which a steady approach could still be valid (Furlano, 2001, Furlano et al., 2001). Then, first unsteady results were obtained with the ONERA RANS solver, using the two transport-equation model k-l from Smith (1994) and a very fine mesh, but were strongly affected by the threshold level of the maximum value of the turbulent viscosity necessary to get any SIO (Furlano et al., 2001). On the other hand, a two-layer closure with wall functions in the inner layer and the same k-l turbulence model in the outer layer had provided a damping of SIO for the same flow conditions without any threshold level (Goncalves et al., 2001a, 2001b). Work has been pursued using that same solver but with other transport-equation turbulence models: Spalart-Allmaras (1994) and Jones-Launder (1972) with a SST-type corrector on the eddy viscosityfor the latter. SIO was then recorded without any threshold level on the turbulent viscosity. The dependency of the results upon the above-cited threshold value was then suppressed (Furlano et al., 2002). Thus, these studies showed up the capability to predict periodic self-excited shock oscillations. Moreover, new numerical developments have allowed to emphasise investigations of unsteady flows and, thus, characterise precisely enough the extent of the SIO.

Research has been consistently continuing at ONERA, using the brand new "elsA" software (Cambier et al., 2002, Gazaix et al., 2002). Turbulence validation has been pursued using the Spalart-Allmaras model (Spalart et al., 1994), the model from Menter (1994) and two ONERA models (Aupoix et al., 2000, Daris, 2002, Daris et al., 2002). Thus, this paper deals with recent numerical results obtained with these four models, when applied to the two following test cases:

- a 2D OALT25 Airfoil, tested at the ONERA T2 wind tunnel (Caruana et al., 1996) - Free-stream Mach number, M=0.78 - Reynolds number based on the chord of the airfoil, c, Re_c=5.8 & 20.15 millions.
- a 2D RA16SC1 Airfoil, tested at ONERA S3MA wind tunnel (Benoit et al., 1987) - M=0.732 - Re_c=4.2 millions.

Comparisons between computations and experiments are detailed, with special emphasis on global aerodynamic coefficients, pressure distributions, pressure fluctuations and SIO frequency. The existence of side wall effects in the test cases has been taken into account in the computations by correcting the values of M and α, from estimates provided by an inviscid / viscous coupling method.

SOLVER AND TURBULENCE MODELS

Solver and basic numerical method

The object-oriented "elsA" software solves the three-dimensional compressible Reynolds Average Navier-Stokes (RANS) system for multi-domain structured meshes, using finite volume method with cell-centered discretisation (Cambier et al., 2002, Gazaix et al., 2002). For computations involving transport equations turbulence models, the system of equations for the mean and turbulent fields is uncoupled in order to keep the modular feature of the solver. Time explicit integration is done with a four step Runge-Kutta algorithm (2^{nd} order accurate), the fluxes being computed with two 2^{nd} order schemes: Jameson's one for the mean flow and Roe's one for the turbulent transport equations.

For unsteady computations (URANS), the initial conditions are provided from either a converged or a non-converged steady solution, for which existing acceleration techniques have been applied, such as local-time stepping, implicit residual smoothing (IRS) or implicit LU factorization and multi-grid method. Then, two complementary strategies for SIO prediction have been developed:

- a Wall Layer approach involving a coarse grid (the height of the first cell close to the wall is about 70 wall units) and a wall law treatment imposing diffusive flux densities needed for integration process (Goncalves et al., 2001a, 2001b). It will be referred to as "WL" (Wall Layer) strategy.

- a more standard approach, using "classic" refined grid (height of the first cell around 0.5 wall units), referred to as "RG" (Refined Grid) strategy.

Dual Time Stepping method: "DTS" method

The dual time stepping method is time implicit and allows to choose the time step value only relatively to the frequencies of the investigated phenomenon. Having defined the fictitious dual time, sub-iterations are performed at each physical time step to solve the equations. A tolerance criterion based on the L2-norm of the residual variations for the density variable has been fixed to 10^{-3}; if it is not fulfilled, a maximum of 2000 sub-iterations is imposed. The sub-iterations process is equivalent to a steady-state process with respect to the dual time. Thus, convergence acceleration techniques previously mentioned can be used. Description of the "DTS" method implementation in the ONERA solver can be found in Rouzaud et al. (2000a and 2000b).

At last, this "DTS" method is applied to both "WL" and "RG" strategies.

Turbulence modelling

Turbulence closure is achieved using Boussinesq assumption; the turbulent viscosity, μ_t, being then expressed using the turbulence length and velocity scales obtained by solving transport equations. Several models have been considered for the present applications:

- the one-transport equation model from Spalart et Allmaras (1994);

- the two transport equations k-ω/k-ε model from Menter (1994), including SST corrector; that latter limits the ratio $-u'v'/k$ allowing the turbulent boundary layer to relax downstream of the shock and to delay turbulence production in that area.

- the two-transport equation model k-φ, where $\varphi=\varepsilon/k^{1/2}$ (Aupoix et al., 2000).

- the two-transport equation model k-kl (Daris, 2002, Daris et al., 2002). It can presently be applied only with "WL" strategy.

These four models are referred to as [SA], [SST], [kPHI] and [kkl], respectively. The [SA] and [SST] models have been recommended by NASA Langley for evaluating transonic flows over different test cases (Marvin et al., 1996), while the [kPHI] et [kkl] models have been rather recently developed at ONERA and are successfully applied to several types of flow conditions especially for adverse pressure gradient.

OALT25 AIRFOIL

The ability of the above-cited models to capture separated flows and SIO has been checked with applications to the case of turbulent transonic flows developing over two airfoils: OALT25 and RA16SC1. For these relatively thick airfoils, the pressure gradient ahead of the shock is negative for the former and almost zero for the latter; such a behaviour affects the sensitivity to SIO amplitude.

Experimental data base

Tests were conducted in the transonic, cryogenic ONERA T2 wind tunnel, with self-adaptive upper and lower walls of the test section. The model is a 2D OALT25 airfoil (c=0.25m, relative thickness t/c=12.18%) equipped with 47 pressure taps in a transverse section. Transition is tripped on both sides of the model at x/c=5%. Detailed measurements have been obtained for several angles of attack of the model (from 0.5^o to 2.5^o) and at a given free stream Mach number M=0.78, but for two values of the Reynolds number based on the chord length, Re_c: 5.8 10^6 and 20.15 10^6 (Caruana et al., 1996). Experimental distributions of r.m.s. values of pressure fluctuations had reported gloomy appearance of buffeting onset for $\alpha=2.5^o$ at Re_c=20.15 10^6. Spectral analysis of Kulite transducers pointed out an energy peak for frequencies close to 80-100Hz. This peak could be attributed to important separation areas from shock footprint to trailing edge since shock oscillation remains small due to strong pressure gradient.

Mesh - Conditions for computations

Mesh convergence has been considered using C-type grid topology. For the "RG" strategy, the mesh contains 321x107 nodes, with 241 points describing the airfoil surface. The grid extents at about 50 chord lengths apart from the airfoil, in each direction, with a grid refinement on the suction side at the average shock location and in the trailing edge vicinity. On the suction side, the y^+ value of the first cell adjacent to the wall is varying from about 0.45 to 0.65 ahead of the shock, with lower values downstream of it; on the pressure side, the y^+ value can reach 1.3 on the suction side, but remains less than unity over about 95% chord length. For the "WL" strategy, the mesh has been obtained from the "RG" one, by removing 16 lines near the wall. The y^+ value ahead of the shock is then about sixty times greater.

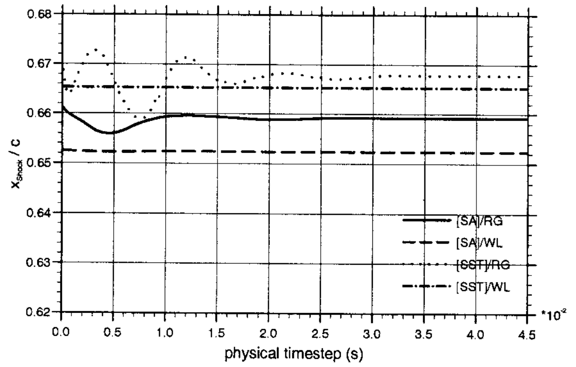

Figure 1: OALT25 Airfoil - Shock oscillations (M=0.78, Re_c=20.15 10^6, $\alpha = 2.5^o$).

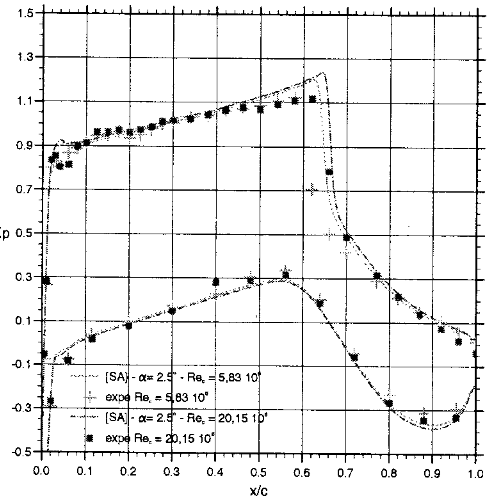

Figure 2: OALT25 Airfoil - Pressure distributions (M=0.78, α=2.5^o, [SA] model, "RG" strategy - Re_c=5.83 and 20.15 10^6).

Taking into account the above-cited side walls effects leads to adapt the values of M and α, such that ΔM=-0.01 and $\Delta\alpha$=-0.3^o; these estimates are kept constant whatever the values of M, Re_c or α.

Unsteady results

Unsteady computations have been performed for conditions close to flight applications, i.e. at Re_c=20.15 10^6 and α=2.5^o, using the [SA] and [SST] models and considering both "RG" and "WL" strategies. Results pointed out the damping of the shock oscillations (Fig. 1); the lift coefficient reaches the steady solutions, with slight variations attributed to turbulence model or strategy. The resulting steady computed pressure distribution clearly reveals a rather strong pressure gradient in front of the shock, which is then fixed on the suction side. At this angle of attack, the effect of Reynolds number on the experimental as well as numerical pressure distributions is rather weak with a small upward motion of the shock for lower Re_c (Fig. 2); the observed differences upstream of the shock are attributed to 3D effects due to model size compared to the dimensions of the test section (Furlano et al., 2001). Moreover, at this Re_c value, using [SA] model and "RG" strategy, no shock oscillation could be obtained when increasing α from 2.5^o to 4.5^o.

Thus, varying either α or Re_c for M=0.78 did not allow to find SIO with [SA] model. The computed pressure gradient, ahead of the shock, being even stronger than the measured one, could explain the absence of numerical SIO.

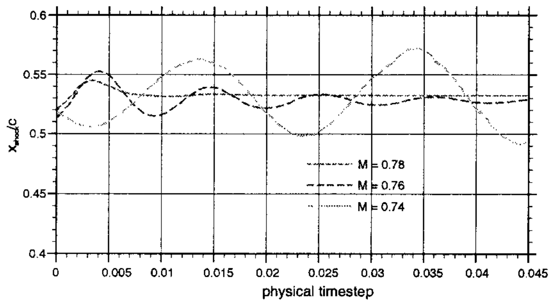

Figure 3: OALT25 Airfoil - Shock oscillations (α=4.5^o, Re_c=20.15 10^6, [SA] model, "RG" strategy).

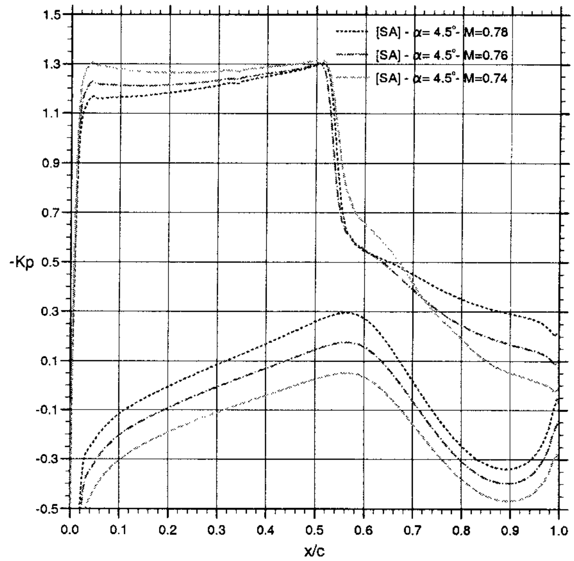

Figure 4: OALT25 Airfoil - Pressure distributions (Re_c=20.15 10^6, α=4.5^o, [SA] model, "RG" strategy).

This has been confirmed from further computations made at fixed α (4.5^o) and Re_c (20.15 10^6), when varying M. Indeed, a decrease in Mach number from 0.78 to 0.74 allows to get self-sustained oscillations using the [SA] model (Fig. 3). This should be attributed to the weakening of the pressure gradient ahead of the shock, with a slight decrease of the local Mach number (Fig. 4); at M=0.76, damping has also been obtained. Thus, the (α, M) band of buffeting extent is very narrow for this airfoil.

For these same aerodynamic conditions (M=0.74, Re_c=20.15 10^6 and α=4.5^o), more computations were performed using the three other turbulence models. Considering refined grids towards the wall, well established SIO at the same frequency (49Hz) could be recorded for both [SA] and [kPHI] models (Fig. 5) although the relative amplitude of the lift coefficient C_L being greater for the [kPHI] model (21% vs. 15%) in agreement with a larger extent of SIO (13% vs. 11%). However, no SIO has been obtained with the [SST] model, whatever "RG" or "WL" strategy has been applied (Fig. 5 & 6).

When applying the "WL" strategy, the [SA] and [kkl] models were the only ones to provide self-sustained SIO, though at different level orders: $\Delta C_L/C_L$=39% for the [kkl] model vs. 11% for the [SA] one. The [kkl] model has pro-

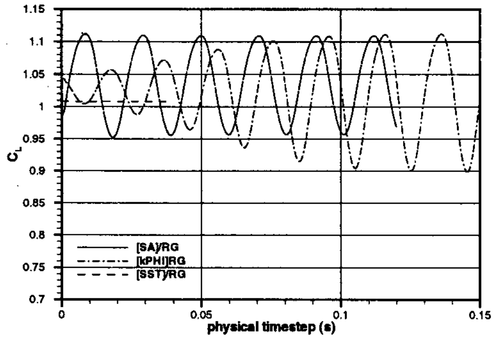

Figure 5: OALT25 Airfoil - Lift oscillations (Re_c=20.15 10^6, α=4.5^o, M=0.78, "RG" strategy).

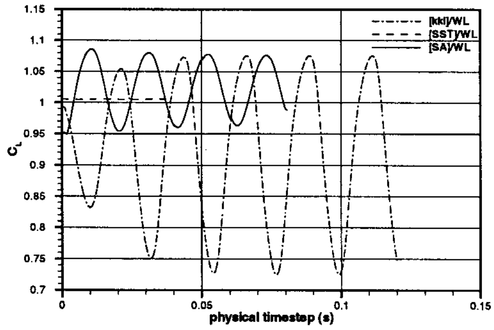

Figure 6: OALT25 Airfoil - Lift oscillations (Re_c=20.15 10^6, α=4.5^o, M=0.78, "WL" strategy).

vided the greater extent of SIO associated to either a strong separation area or a deep recovery downstream of the shock footprint; this can be clearly noticeable on the skin friction distribution over a period of SIO (Fig. 7).

On the other hand, the [SA] model is the one which has presented less variations of both the local Mach number upstream of the shock, the pressure fluctuations and the separation importance or extent (Fig. 8).

Moreover, the [kPHI] model did not converge when applying the "WL" strategy, due to stability concern on turbulent quantities within the boundary layer.

At last, it should be pointed out that the turbulence models which have provided SIO are the ones for which no converged steady solution could be obtained. The [SST] model always provided a steady convergence, at comparable levels (C_L or shock location) for the "RG" and "WL" strategies. Using that solution (or one slightly different) as initial condition for the unsteady computations did not allow to observe any SIO.

RA16SC1 AIRFOIL

Experimental data base

Tests were conducted in the ONERA S3 wind tunnel with a 2D RA16SC1 airfoil (c=180mm, t/c=16%), at M=0.732, Re_c=4.2 10^6 and different angles of attack between 0^o and 4.5^o (Benoit et al., 1987). Transition was tripped on both sides of the airfoil at x/c=7.5%. Strong shock oscillations were recorded above 3^o, over about 17% to 32% chord length depending upon α. SIO frequency data was recorded and was slightly dependent upon α (88Hz - 108Hz). At last,

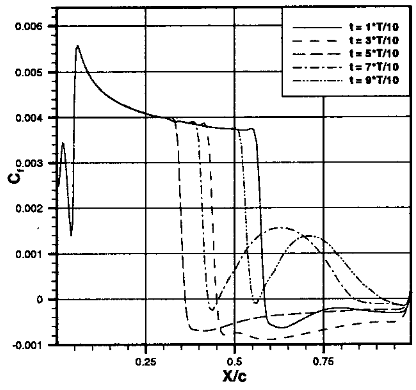

Figure 7: OALT25 Airfoil - Skin friction distribution over a period of SIO (α=4.5^o, Re_c=20.15 10^6, M=0.78, [kkL] model, "WL" strategy).

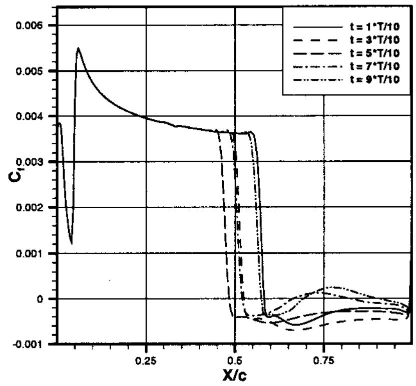

Figure 8: OALT25 Airfoil - Skin friction distribution over a period of SIO (α=4.5^o, Re_c=20.15 10^6, M=0.78, [SA] model, "RG" strategy).

Kulite sensors provided r.m.s. values of the pressure.

Mesh convergence - Computations conditions

As for the preceding test case, care about the mesh definition was brought, leading to 337x105 and 337x89 nodes, for the "RG" and "WL" strategies, respectively. The computational domain extents equally over 50 chord lengthes. Compared to the OALT25 application, the y^+ of the first cell of the domain is slightly greater than 1.1 either ahead of the shock location or on the suction side. For the "WL" strategy, the y^+ value ahead of the shock is about fifty times greater. The mesh should be slightly refined for further turbulence validation.

The estimates for taking into account the side walls effects led to: ΔM=-0.009 and $\Delta\alpha$=-1.0^o, supposed constant whatever α is.

Unsteady results

Unsteady computations were at first made at M=0.732 and $\alpha = 4^o$ with the [SA] model, starting from initial conditions provided by non-converged results for steady state. Rather strong SIO were observed when using the "RG" strategy, over almost 19% chord length; a smaller extent (~6%) was recorded when applying the "WL" strategy (Fig.

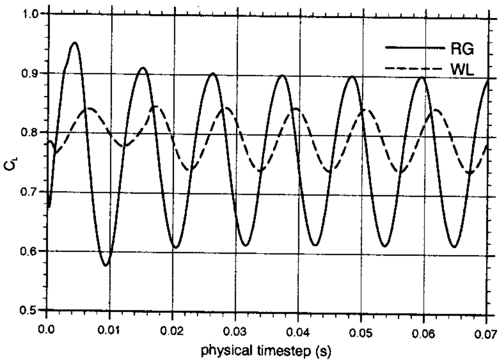

Figure 9: RA16SC1 Airfoil - Lift oscillations (α=4^o, Re$_c$=4.2 10^6, M=0.732, [SA] model, "RG" & "WL" strategies).

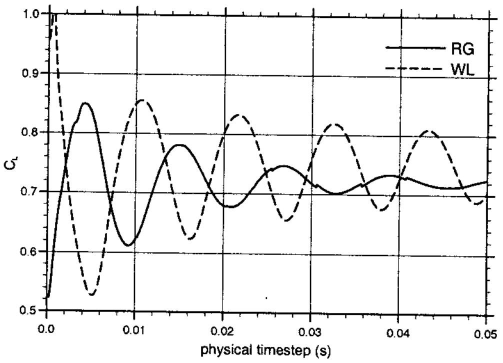

Figure 10: RA16SC1 Airfoil - Lift oscillations (α=4^o, Re$_c$=4.2 10^6, M=0.732, [SST] model, "RG" & "WL" strategies).

9). The relative variations of C_L are then less important for the "WL" strategy compared to the "RG" one: 0.105 vs. 0.285. This is not consistent with the afore-mentioned results obtained with the OALT25 airfoil; the difference in amplitude variations between the two strategies is greater for the RA16SC1 airfoil compared to the OALT25 one. The two strategies led to the same SIO frequency: 89Hz, yet.

Using the [SST] model, variations of C_L versus time are plotted in Fig. 10 for the two strategies. The use of a refined grid towards the wall has led to the damping of the oscillations while applying the "WL" strategy should provide opposite behaviour; further timesteps could confirm that statement. Indeed, self sustained shock wave oscillations at a frequency of 90.5Hz were recorded over an extent comparable to that of the [SA] model with the "WL" strategy, i.e. 6.5%. Thus, with the [SST] model, great sensitivity to the considered strategy has been evidenced, while SIO had always been recorded with the [SA] model.

When plotting the r.m.s. value of pressure fluctuations along both sides of the airfoil, one can observe that levels obtained with the "WL" strategy are really weaker compared to the measured ones (Fig. 11). However, the [SA] model used on a refined grid has been able to reproduce the correct level on both the pressure and suctions sides of the airfoil.

The idea was then to perform computations at the same value of M, using the [SA] model and the "RG" strategy, but for higher angles of attack in order to define the extent of SIO domain. Firstly, steady computations were performed for α varying between 3^o and 5.5^o; they are used as initial conditions and are providing some information on the average flow behaviour. No SIO was recorded for α = 3.0^o and 5.5^o, while the maximum of oscillations has been reached at α = 4.5^o, value very close to the measured one (4.0^o). The non-dimensional SIO frequency (Strouhal number) grows with α, but is slightly weaker (6-8%) than the measured one for all values of α (Fig. 12), while the computed oscillations amplitudes appeared to be much higher than the experimental ones (Fig. 13). These results are very important since they confirmed the opportunity to numerically predict the extent of SIO on this airfoil. Further investigations with the [kPHI] and [kkl] models are in progress at ONERA.

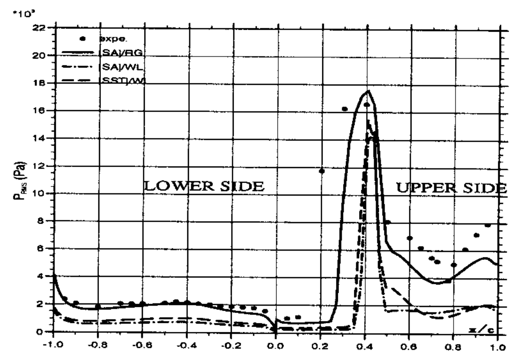

Figure 11: RA16SC1 Airfoil - r.m.s. pressure fluctuations (α=4^o, Re$_c$=4.2 10^6, M=0.732, [SA] & [SST] models, "RG" & "WL" strategies).

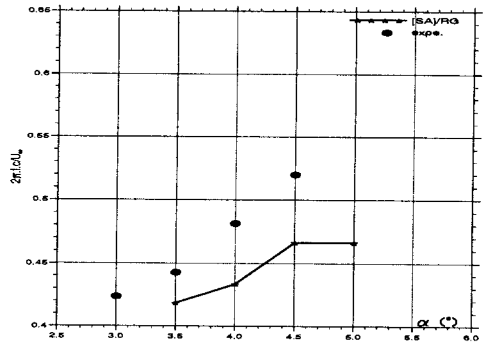

Figure 12: RA16SC1 Airfoil - Effects of α (Re$_c$=4.2 10^6, M=0.732, [SA] model, "RG" strategy).

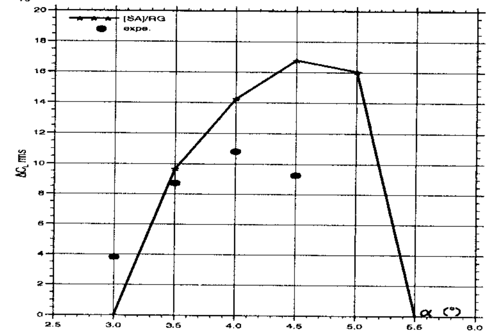

Figure 13: RA16SC1 Airfoil - Effects of α (Re$_c$=4.2 10^6, M=0.732, [SA] model, "RG" strategy).

CONCLUDING REMARKS

Results presented in this paper dealt with unsteady computations of turbulent flows that develop on 2D rigid airfoils (OALT25 & RA16SC1 test cases) at conditions for which strong Shock Induced Oscillations (SIO) as well as important separated areas could be present. Using the ONERA solver, URANS turbulence models have been investigated from transport equation models and two different strategies involving either a fine grid ("RG") or a coarse grid with the wall layer concept ("WL").

The sensitivity of results to turbulence models has been clearly showed up, especially for the two test cases for which the pressure gradient ahead of the shock is completely different, then generating either strong or weak SIO. A compromise will have to be settled between "RG" and "WL" strategies, since the latter might not be purely appropriate for turbulence validation but might be interesting for computations of 3D flows. From the four tested models ([SA], [SST], [kPHI] and [kkl]) on the OALT25 airfoil, the first one appears as the most robust one, whatever "RG" and "WL" strategy is. The [SST] model does not indicate any SIO for the OALT25 airfoil and show up a great sensitivity to grid refinement when applied to the RA16SC1 airfoil. The [kPHI] model provides results in good agreement with the [SA] model but did not converge when using the "WL" strategy. At last, the [kkl] model seems to over-estimate the SIO amplitude and can presently be applied only with "WL"-type grids. Further investigations with these two last models are in progress at ONERA, especially by applying them to the second test case.

A parametric investigation, using URANS formulation, has been conducted in the framework of the RA16SC1 test case, by varying the angle of attack at fixed Mach and Reynolds numbers. Results obtained are very important since they have confirmed the opportunity to numerically predict the onset and extent of SIO for a 2D rigid airfoil, using turbulence model schemes expressed as for steady-type computations.

All the reported unsteady computations are 2D with the use of URANS-type models. The needs to perform 3D calculations, especially for the first investigated test case, has been clearly shown in former studies for steady conditions (Furlano et al., 2001). 3D unsteady computations should then lead to a weaker pressure gradient ahead of the shock, allowing then to obtain SIO which has been observed in experiments but not with the present 2D unsteady computations.

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