

# EVOLUTION OF DISTURBANCES IN BOUNDARY LAYERS WITH WALL SUCTION

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## ABSTRACT

An experimental investigation of free stream turbulence (FST) induced transition in asymptotic suction boundary layers (ASBL) has been performed. It is known that FST induces elongated disturbances consisting of high and low velocity regions, denoted streaky structure, into the boundary layer. Emphasis is here placed on the disturbance growth and the modification of the streaky structure when subjected to various uniform wall suction rates. The most salient feature of the ASBL is its constant boundary layer thickness along the plate which experimentally is reached after some entry length when uniform suction is applied over a large area. This makes it possible to change the Reynolds number and the boundary layer thickness independently, which is unique for the ASBL case. Three different FST levels ( $Tu = 1.6-2.3\%$ ) have been investigated with an active turbulence generating grid. This grid offers the possibilities to vary the  $Tu$ -level while keeping the FST characteristic scales.

Wall suction is shown to suppress the disturbance growth and delay or inhibit the breakdown to turbulence depending on the strength of suction and the level of  $Tu$ . Two-point correlation measurements in the spanwise direction reveal that the averaged streak spacing is affected by the boundary layer thickness (with  $Re = \text{const.}$ ) and the  $Tu$ -level, but is independent of the Reynolds number (with boundary layer thickness /  $Re = \text{const.}$ ). The streaky structure in a Blasius boundary layer is known to take the aspect ratio of around unity and in this investigation we are able to show how this structure is flattened with increasing suction. Furthermore, by keeping the Reynolds number constant while varying the boundary layer thickness we provide the result of a linear relationship between the streak spacing and the boundary layer thickness.

## INTRODUCTION

Boundary layers under high free stream turbulence (FST, hereafter) is of great significance in terms of practical application. Reliable methods for the prediction of bypass transition is at present limited (Westin and Henkes, 1997), and is a serious problem in the design process for certain applications. For the control of boundary layer transition to delay transition to turbulence, which is termed LFC (Laminar Flow Control), many methods have been proposed (e.g. Gad-el-Hak, 2000). The present study aims at extending this LFC technique to boundary layers affected by FST. FST induces streamwise elongated disturbances consisting of high and low velocity regions (streaky structures) into the boundary layer (Kendall, 1985, 1998, Westin et al., 1994, Alfredsson and Matsubara, 2000). It has been hypothesised that the disturbances are so called transiently growing disturbances, see e.g. Andersson et al. (1999), Luchini (2000)

and Fransson and Corbett (2003). For boundary layers subjected to FST such disturbances grow algebraically to fairly high amplitude before breaking down to turbulence (Westin et al. 1994, Matsubara and Alfredsson, 2001). This breakdown may be caused by the secondary instability on the streaky structures (Andersson et al., 2001). Westin et al. (1998) pointed out that the continuous forcing by FST along the boundary layer edge is necessary for the breakdown. This transition process is one type of so called bypass transition.

Distributed suction at the wall has long been known to be a possibility to stabilize laminar boundary layers with respect to instability waves (TS-waves) in order to avoid or postpone transition. Fransson (2001) and Fransson and Alfredsson (2003) showed that also FST induced disturbances could be stabilized by suction at the wall, indicating the suction is also effective to delay bypass transition. In the present study these results are extended to other Reynolds numbers and a parametric study is made in order to understand both the receptivity to free stream turbulence of the boundary layer as well as the subsequent disturbance development.

Recent studies indicate that the spanwise scale of the streaky structures is of the order of the local boundary layer thickness (Matsubara and Alfredsson, 2001). In the present experimental situation it is possible to change the boundary layer thickness and Reynolds number independently. Furthermore in the asymptotic region the boundary layer thickness is independent of downstream position. This makes it possible to do a parametric study of the scaling of the streaky structures.

When wall suction,  $-V_0$ , is applied to the laminar boundary layer, the streamwise velocity distribution in wall normal direction  $U(y)$  can be analytically obtained and expressed as follows (see e.g. Schlichting, 1979):

$$U(y) = U_\infty \left( 1 - e^{-\frac{V_0 y}{\nu}} \right), \quad (1)$$

where  $U_\infty$  and  $\nu$  denote free stream velocity and kinematic viscosity, respectively. It should be noted that the velocity profile is independent of streamwise position  $x$ , and thus the boundary layer thickness becomes constant. The displacement thickness  $\delta_1$ , momentum thickness  $\delta_2$ , and shape factor  $H_{12}$  are:

$$\delta_1 = \frac{\nu}{V_0}, \quad \delta_2 = \frac{1}{2} \frac{\nu}{V_0} \quad \text{and} \quad H_{12} = 2. \quad (2)$$

From Eqs. (1) and (2), the relationship between boundary layer thickness  $\delta_{99}$  and displacement thickness  $\delta_1$  is:

$$\delta_{99} = \ln(100)\delta_1. \quad (3)$$

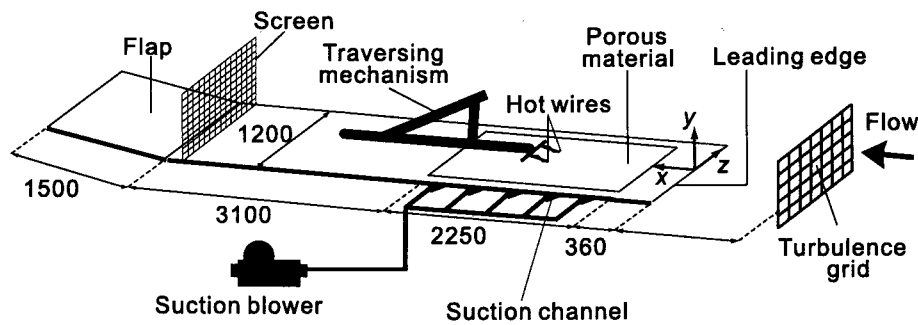


Figure 1: Schematic of experimental facility. Dimensions are in mm.

The Reynolds number based on displacement thickness  $\delta_1$  is written as:

$$Re = \frac{U_\infty \delta_1}{\nu} = \frac{U_\infty}{V_0}. \quad (4)$$

As indicated by Eq. (2) and Eq. (4), boundary layer thickness and Reynolds number can be independently adjusted by changing  $U_\infty$  and  $V_0$ .

#### EXPERIMENTAL SET UP

The experiments were conducted in the MTL-wind tunnel at KTH. A schematic of the experimental set up is shown in figure 1. The test section is 7 m long, 0.8 m high and 1.2 m wide. A horizontal test plate, which spans the whole width of the test section was fixed here. A porous material, 3.2 mm in thickness, covers 2.25 m (length)  $\times$  1.0 m (width) of the upper surface of the test plate. Through this porous surface, uniform suction was applied starting from  $x=0.36$  m, see figure 1. The porous plate consists of a sintered plastic material with an average pore size of  $16 \mu\text{m}$  and is even enough to be considered smooth. There is a plenum chamber below the porous plate which is connected to an electric blower through 18 holes. This ensures that the pressure distribution in the plenum chamber is uniform. An extra aluminum plate section of 3.1 m was added downstream of the test plate and was followed by a trailing-edge flap. The trailing-edge flap as well as the fine-meshed screen placed just upstream the flap make it possible to adjust the blockage ratio between the upper and lower sides of the test plate. This makes it possible to adjust the position of the stagnation line at the leading edge on the upper side of the plate to avoid separation. For the leading edge, an asymmetric shape was employed to minimize the pressure gradient. The pressure gradient along the plate beyond the leading edge was set to  $\Delta C_p = \pm 0.01$  by adjusting the test section ceiling. For details of the plate and leading edge, see Fransson (2001).

For the velocity measurements, hot-wire anemometry with single wire sensors was used. The sensors were made by platinum and their diameter and length were  $2.5 \mu\text{m}$  and 0.5 mm. Johansson and Alfredsson (1982), where an extra term is added to King's law for compensation of natural convection which makes it suitable for sensing low speed flow, was employed. The hot-wire probe was mounted on the traversing mechanism as shown in figure 1. For the evaluation of the spanwise scale of the streaky structures, two sensors were used for the simultaneous measurement of the instantaneous streamwise velocity. As illustrated in figure 1 the origin of the coordinate system was fixed at the centre of the leading edge of the plate. The  $x$ -,  $y$ - and  $z$ - axes were directed downstream, wall normal and spanwise directions.

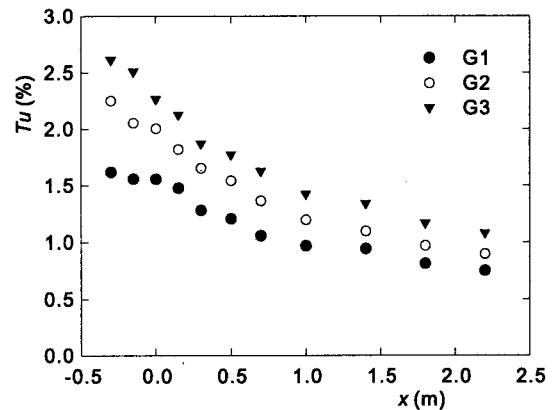


Figure 2: Variation of free-stream turbulence level,  $Tu$ , for different injection rates at  $U_\infty=5.0$  m/s.

The free stream turbulence was generated by an active turbulence generating grid mounted upstream of the leading edge. This grid has a mesh width of 50 mm and is constructed by 33 cylindrical pipes (5 mm diameter), 20 vertically and 13 horizontally. A total of 254 jet orifices (diameter 1 mm) have been drilled in the pipes and point towards the upstream direction. From these jet orifices, secondary air is ejected into the freestream. The distance from the grid to the plate is 1.4 m, which ensures a fairly homogeneous turbulence at the plate. The turbulence intensity depends on the injection rate and in the present study three cases are studied. These are denoted by G1 (without injection), G2 and G3, hereafter. In figure 2, the downstream variation of the FST level,  $Tu = u_{rms}/U_\infty$  is shown. In all three cases, the FST level gradually decreases in the downstream direction. The FST level at the leading edge,  $x = 0$ , is 1.6%, 2.0% and 2.3% for the G1, G2 and G3 cases respectively. An advantage with using the active grid to create different levels of turbulence is that the turbulence scales are fairly independent of the injection rate as shown by Fransson (2001). For the present grid he showed that the Taylor scale was approximately 8 mm.

The measurements were made at a free stream velocity of 5 m/s if not otherwise stated. The suction velocity  $V_0$  was for this case varied between 0–0.4% of  $U_\infty$ . However one advantage of the asymptotic suction boundary layer is that the Reynolds number and the boundary layer thickness can be changed independently. Some measurements were therefore carried out with free stream velocities  $U_\infty$  in the range 2.0–6.0 m/s and the suction velocity was changed such that the Reynolds number was constant. In that way the boundary layer thickness, but not the Reynolds number, changes.

In the transitional region where both laminar and turbulent flows exist a method was developed to distinguish

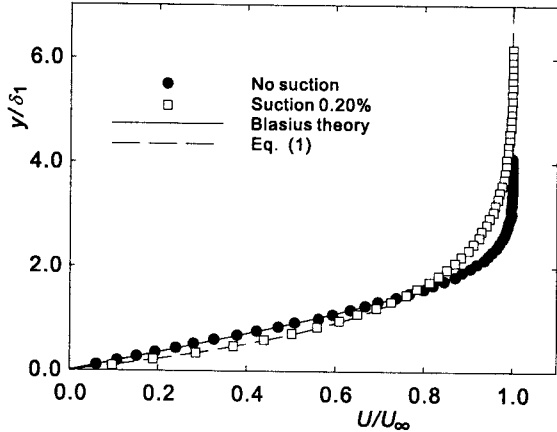


Figure 3: Mean velocity profiles for boundary layers with and without wall suction, without FST.

Table 1: Reynolds numbers for test cases

$V_0/U_\infty$	0.10%	0.20%	0.29%	0.40%
$Re$	1000	500	350	250

between these different flow regimes. First the signal was Fourier transformed whereafter the contribution below 100 Hz was set to zero. By an inverse transform the high frequency part of the signal was extracted and a threshold was applied to the square of the signal to determine the turbulent sequences of the signal. It was shown by a trial and error procedure that this method was rather insensitive to small changes of the evaluation parameters involved. This methodology was used both to determine the intermittency factor (which was determined at the  $y$ -position where  $u_{rms}$  had its maximum) as well as the spanwise spatial scale of the streaky structures in the transitional region.

## RESULTS AND DISCUSSION

### Disturbance Growth in a Boundary Layer

Prior to discussion on the boundary layer affected by FST, velocity distributions without FST were evaluated at a fixed free stream velocity of  $U_\infty=5$  m/s. Figure 3 shows mean streamwise velocity distributions,  $U(y)$ , at  $x=2.2$  m with ( $V_0/U_\infty = 0.20\%$ ) and without suction. Note that the distance from the wall is normalized by the measured displacement thickness ( $\delta_1$ ). The theoretical velocity profiles obtained from the Blasius solution and Eq. (1) are also shown in the figure for comparison. It is seen that the measured velocity distributions are in good agreement with theory.

The downstream evolution of FST induced disturbance amplitude in the boundary layer is investigated next. The random motion of the streaky structures gives rise to low frequency fluctuations in the streamwise velocity as sensed by a stationary hot wire. The amplitude distribution of such disturbances has a maximum in the central region of the boundary layer and this maximum gives a good indication of the disturbance development. In this case the free stream velocity was fixed to  $U_\infty=5$  m/s, and the suction velocity was changed to vary the Reynolds number. Four Reynolds numbers were selected for the test case and are summarized in Table 1.

In figure 4 the variation of the maximum values of  $u_{rms}$  measured at each  $x$  position is shown. Figure 4(a) shows the G1 case. When suction is not applied,  $u_{rms}$  increases parabolically until  $x=2.0$  m, and thereafter  $u_{rms}$  increases faster due to the start of the transition process, i.e. formation of turbulent spots. This is indicated by the variation of the shape factor,  $H_{12}$ , and the intermittency factor,  $\gamma$ , shown in figure 5(a). Even at the most upstream position,  $x=0.5$  m, the shape factor is  $H_{12} = 2.5$ , which is less than that value of the Blasius profile,  $H_{12} = 2.6$ . This is due to the presence of the streaky structures inside the boundary layer. The shape factor is almost constant up to  $x=1.5$  m but decreases to  $H_{12} = 2.3$  at  $x=2.2$  m. Corresponding to this decrease, the intermittency factor  $\gamma$  rises from 0.0 to 0.2.

From figure 4(a) it is obvious that the development of the disturbance amplitude is less dramatic when suction is applied. In fact the growth of  $u_{rms}$  with downstream distance seems eliminated for a suction rate  $V_0/U_\infty$  below 0.2%, and  $u_{rms}$  is decreasing in the downstream direction, for even higher suction rate. The shape factor  $H_{12}$  and intermittency factor  $\gamma$  for all the suction cases presented in figure 5(a) is close to  $H_{12} = 2.0$  and  $\gamma = 0.0$  which means that the boundary layer state is laminar and transition does not occur.

In figure 4(b) the results obtained from the case G2, i.e. a higher FST level than that of G1, are shown. Without suction,  $u_{rms}$  first increases with downstream distance as for the G1 case, whereas the fast increase due to the start of the transition occurs around  $x=1.4$  m, which is further upstream as compared to the G1 case. It reaches a peak level around  $x=1.8$  m of  $u_{rms} = 0.19\%$ , and thereafter decreases. The latter decrease is caused by changing the boundary layer state from an intermittent state to a fully turbulent state. This phenomenon is also clearly demonstrated by the downstream variation of  $H_{12}$  and  $\gamma$ , shown in figure 5(b).

In the no suction case, at the most upstream position,  $x=0.5$  m, the shape factor is  $H_{12} = 2.4$ . The shape factor decreases from around  $x=1.0$  m and reaches a typical turbulent value of  $H_{12} = 1.4$  at  $x=2.2$  m. The intermittency factor  $\gamma$  is zero that showing the boundary layer state is fully laminar at the most upstream position, whereas it gradually increases from  $x=1.0$  m, where the shape factor  $H_{12}$  starts decreasing, and finally it reaches  $\gamma = 1.0$  at  $x=2.2$  m, showing the boundary layer state is fully turbulent. The  $x$ -position of the peak value of  $u_{rms}$  corresponds to the position where the intermittency is approximately 0.5.

For the suction rates  $V_0/U_\infty = 0.1\%$  and  $0.2\%$ , the growth of  $u_{rms}$  is suppressed as compared with the no-suction case, see figure 4(b). However at the end of the test region the start of transition is seen both as an increase in  $u_{rms}$  and as an increase in  $\gamma$  (see figure 5(b)). A corresponding decrease in the shape factor is also seen. For the two highest suction rates (0.29%, 0.40%) there is hardly any sign of transition, neither in the amplitude nor in the intermittency result. For the highest suction case  $V_0/U_\infty = 0.40\%$  in particular,  $u_{rms}$  decreases in the downstream direction in the same way as the G1 case.

For the highest FST level (G3) all cases except the highest suction rate become transitional in the measurement region (see figures 4(c) and 5(c)). It is also clear, especially from the intermittency data, that the start of transition moves upstream as compared with that of the G2 case.

The results above clearly show that wall suction inhibits the growth of disturbance induced by FST. The next question is how the suction modifies the distribution of turbulence in the boundary layer. This is shown in figure 6 for the

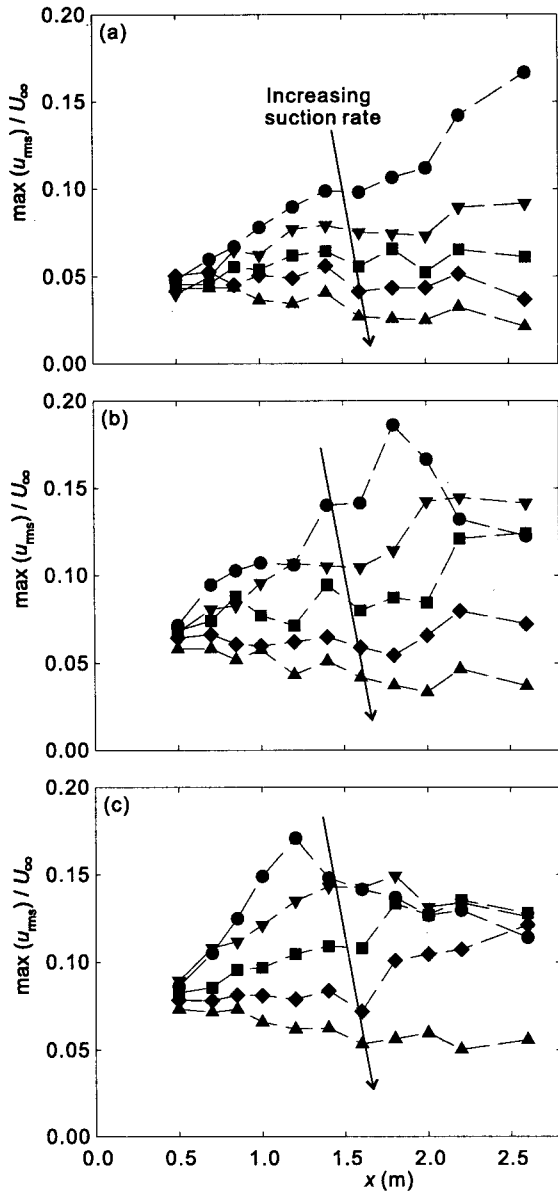


Figure 4: Downstream evolution of  $u_{rms}$  -maximum. (a) G1 (b) G2 (c) G3.  $\bullet$ , No suction;  $\blacktriangledown$ ,  $V_0/U_\infty=0.1\%$ ;  $\blacksquare$ , 0.2%;  $\blacklozenge$ , 0.29%;  $\blacktriangle$ , 0.40%. Lines are visual aid only.

case G2 at  $x=2.2$  m, where, depending on the suction rate, the flow is either fully turbulent, transitional or laminar (see figure 5(b)). In the no suction case, the distribution is that of a typical turbulent boundary layer with a maximum very close to the wall. Also the amplitude is in the range which one would expect from a turbulent boundary layer.

In the suction case  $V_0/U_\infty = 0.10\%$ , shown by open triangles, the turbulence intensity is larger than that of no-suction case especially in the region close to the wall. This is due to the transitional state of the boundary layer. It is obvious that increasing suction provides a reduction of  $u_{rms}$  across the whole height of the boundary layer. It is seen that the peak position in the  $y$ -direction moves away from the wall and becomes broader with increasing suction rate.

#### Modification of Streaky Structures by Wall Suction

A well known hypothesis is that in bypass transition the breakdown to turbulence is caused through a secondary

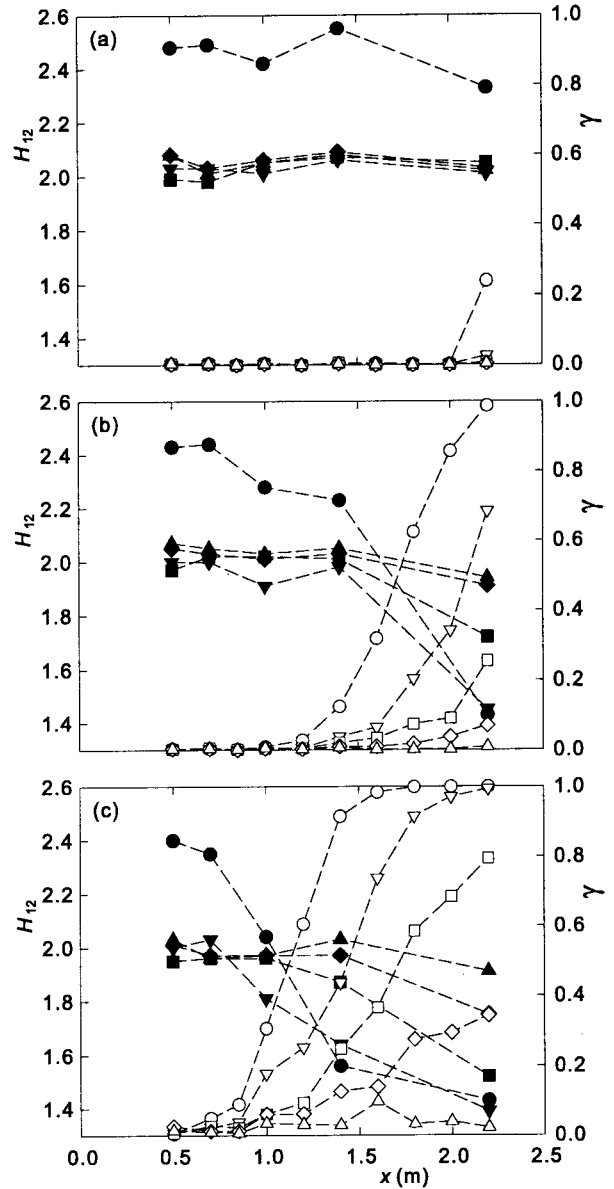


Figure 5: Downstream variation of shape factor and intermittency factor. (a) G2 (b) G3. Symbols as in figure 4. Filled symbols,  $H_{12}$ ; open symbols,  $\gamma$ . Lines are visual aid only.

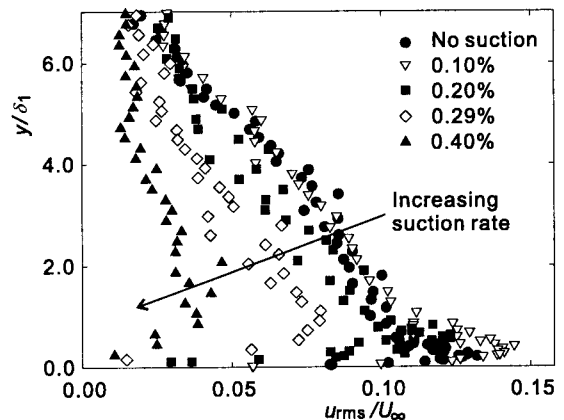


Figure 6: Effect of wall suction on distributions of turbulent intensity at G2,  $x=2.2$  m

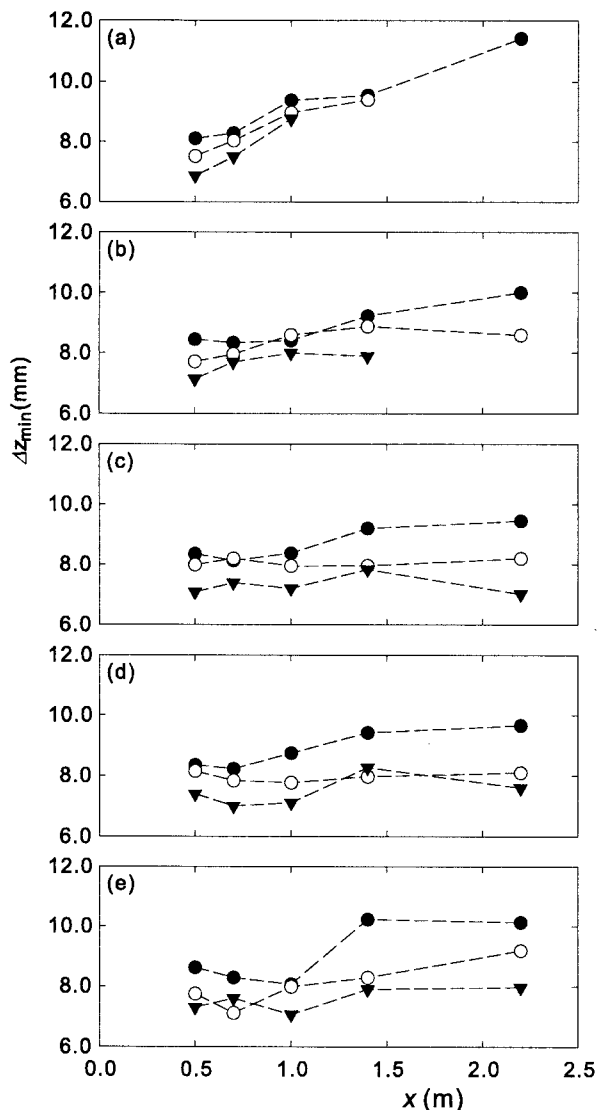


Figure 7: Streamwise variation of streak spacing.  $V_0/U_\infty =$  (a) 0, (b) 0.10%, (c) 0.20% (d) 0.29% (e) 0.40%. ●, G1; ○, G2; ▼, G3. Lines are visual aid only.

instability developing on the spanwise inflectional velocity distribution formed by the streaky structures. Results obtained from the above section reveals that the wall suction suppresses the breakdown. It is thus important to discover how the streaky structure is affected by the wall suction in the transitional stage. This paper hereafter investigates the modification of the streaky structure in terms of the spanwise scale of the streaks. The spanwise scale is determined from the two point correlation of the fluctuating streamwise velocity simultaneously obtained at two different spanwise positions but at the same streamwise and wall normal positions. The spanwise distance,  $\Delta z_{\min}$ , where the two point correlation takes a negative local minimum is considered to correspond to half of the spacing of the streaky structures. In the transitional region where turbulent spots are intermittently formed, laminar periods are extracted to determine  $\Delta z_{\min}$  as was described previously.

In figure 7 the streamwise variation of the streak spacing  $\Delta z_{\min}$  is shown. In the case without suction (figure 7(a))  $\Delta z_{\min}$  increases in the downstream direction for all three FST cases (G1, G2 and G3). This indicates that the streak spacing becomes wider as the boundary layer grows. In the

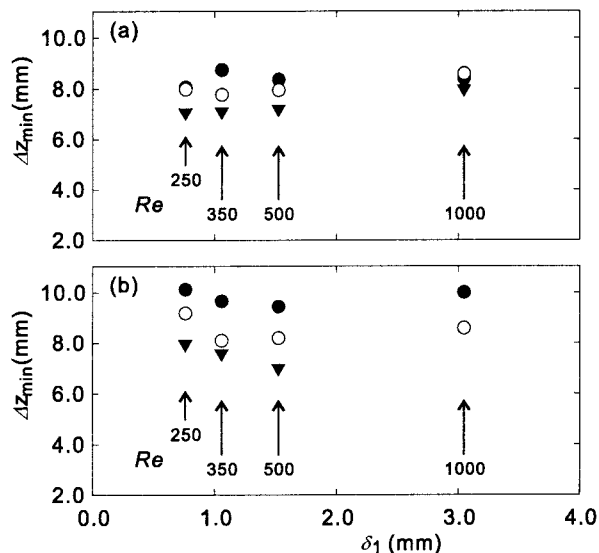


Figure 8: Variation of streak spacing as a function of displacement thickness at (a)  $x=1.0$  m, (b)  $x=2.2$  m. Symbols as in figure 7.

most downstream position the distance is close to the boundary layer thickness which is in agreement with the results of Matsubara and Alfredsson (2001). Fransson (2001) observed that higher levels of FST gives smaller  $\Delta z_{\min}$ . The same effect can be noted here. In the cases of G2 and G3 some downstream positions are not shown because in this region the transition is completed and the boundary layer state is fully turbulent. When suction is applied, see figures 7(b)-(e), the increase of  $\Delta z_{\min}$  is suppressed and stays fairly constant. The trend that higher FST gives smaller  $\Delta z_{\min}$  is still observed in these suction cases. According to above observation the streak spacing is affected by both the local boundary layer thickness and the FST level. On the other hand, it could also be noted that the absolute value of  $\Delta z_{\min}$  is around 7–10 mm in all suction cases. This suggests that  $\Delta z_{\min}$  is not strongly affected by the change of the boundary layer thickness due to the different rates of suction. Figure 8 shows  $\Delta z_{\min}$  as a function of displacement thickness  $\delta_1$  obtained from Eq. (2) at  $x=1.0$  m and 2.2 m. In this figure both the boundary layer thickness and the Reynolds number are changed simultaneously by changing the suction rate while keeping  $U_\infty = \text{const.}$  (resulting in  $\delta_1/Re = \nu/U_\infty \approx \text{const.}$ ). Note that the streak spacing seems to be constant. This result implies that the streaky structure is flattened (compared with the no suction case) with decreasing  $Re$  or  $\delta_1$ .

In the above experiments the Reynolds number changes with the suction rate. One experiment was made where the Reynolds number was kept constant by simultaneously changing the free stream velocity and the suction velocity such that their ratio was constant. In this way the thickness of the boundary layer was changed and gives us the possibility to investigate the relationship between the boundary layer thickness and the streak spacing without influence of the Reynolds number. Note that in this case the  $Re$  is kept constant but the ratio  $\delta_1/Re = \nu/U_\infty$  changes.

In figure 9  $U_\infty$  was varied from 2 to 6 m/s and  $V_0$  was adjusted to keep the Reynolds number at  $Re=350$ , see Eq. (4). The injection rates of secondary airflow from the active turbulence grid for G1, G2 and G3 cases were tuned to the same  $Tu$ -levels at the leading edge to those of 5 m/s cases. However, the correct  $Tu$ -level could not be obtained in the G1

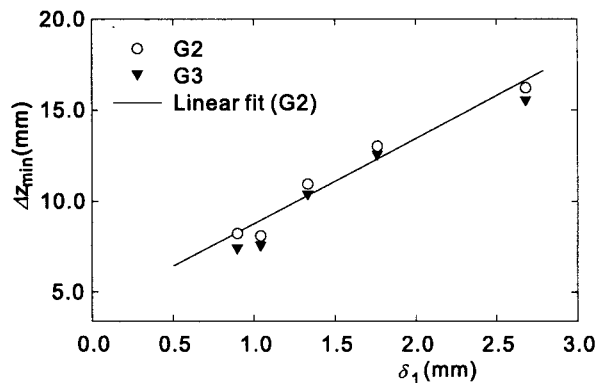


Figure 9: Streak spacing against displacement thickness at constant  $Re$ .

case when  $U_\infty$  is higher than 5 m/s and therefore G1 is excluded in figure 9. It seems to be a linear relationship between  $\Delta z_{\min}$  and  $\delta_1$ . It is also noted that the streak spacing approaches the thickness of the boundary layer with increasing  $\delta_1$ . This is hypothesized by recent research but the present results are the first demonstration in an experiment.

## SUMMARY

The primary motive for the present investigation is to enhance the knowledge of the boundary layer transition scenario caused by moderate to high levels of free stream turbulence (FST). It is believed that the level of FST and the characteristic length scales in the FST are important parameters for the receptivity process taking place at the leading edge while the boundary layer thickness acts as an upper limit for the evolution of the streaky structure. The present experimental set-up is unique in the sense that it allows a change of the boundary layer thickness while keeping the Reynolds number constant. Furthermore, the active turbulence generating grid used in this work is able to maintain (roughly) the same Taylor scale while varying the intensity.

In this investigation four important conclusions have been made regarding the disturbance evolution: **1.** The disturbance growth is suppressed with wall suction, and with a suction rate of  $V_0/U_\infty=0.4\%$ , which corresponds to  $Re = 250$ , the disturbance amplitude decays for all  $Tu$ -levels investigated here. **2.** Increasing FST-level reduces the averaged streak spacing. **3.** When both the boundary layer thickness and the Reynolds number are changed simultaneously, keeping  $U_\infty=const.$ , the spanwise scale seems to stay constant. This result implies that the streaky structure is flattened (compared to the no-suction case) with decreasing  $Re$ . **4.** By keeping the Reynolds number constant at  $Re=350$  changing  $\delta_1$  it was shown that the spanwise scale increases linearly with the boundary layer thickness. This results in an aspect ratio of the structure approaches unity.

The second point above is a verification of the result by Fransson and Alfredsson 2003 while the first and third is an extension of that work by considering different Reynolds numbers. The last point is the first experimental demonstration that the streaky structure approaches an aspect ratio of unity with increasing boundary layer thickness, something that has been observed previously in developing (i.e.  $Re \neq const.$ ) Blasius boundary layers but not quantified (see e.g. Matsubara and Alfredsson, 2001). The third and fourth points are not contradictory. However, they confirm the difficulty in the streak scaling and indicates that the ratio of the boundary layer thickness to the Reynolds number is es-

sential for the evolution of spanwise structures. Note that this ratio is directly proportional to the viscous length scale ( $l^* = \delta_1 Re^{-1/2}$ ). A final remark worth mentioning is that the spanwise scale is around 8 mm close to the leading edge, which in turn is the Taylor length scale of the active grid.

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