A NON-DISSIPATIVE DYNAMIC SUBGRID SCALE SIMILARITY MODEL

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ABSTRACT

Two new subgrid scale turbulence models are proposed. The first one is an extension of the classical structure function model. It is shown, that the modified structure function model can be used in Germano's dynamic process (Germano 1992). The second model utilizes a given scale similarity ansatz, which models the subgrid scale stresses in terms of resolved stresses and determines the scaling parameter assuming assuming scale invariance of an eddy-viscosity model. The performance of the derived subgrid scale models has been tested in fully developed turbulent channel flow at low Reynolds number (based on the channel height H and friction velocity u_τ) of $Re = 360$. The same flow was computed by means of underresolved direct numerical simulation (DNS) without model and LES with a dynamic Smagorinsky model. First and second order statistics of the statistically averaged flow field generated in LES using the scale similarity model agree well with DNS data by Kim et al. (Kim 1987).

INTRODUCTION

Principally, there are three different approaches to predict turbulent flows numerically. Due to the dramatically increased efficiency of the super computers and new numerical methods it is possible to solve the time-dependent Navier-Stokes equations (1) and (2)

$$\nabla \cdot \vec{u} = 0 \qquad (1)$$

$$\partial \vec{u}/\partial t + \nabla \cdot (\vec{u}\vec{u}) = -\nabla p + \nabla^2 \vec{u}/Re_\tau \qquad (2)$$

without any turbulence model in a Direct Numerical Simulations (DNS). In eq. (1) and (2) the incompressible dimensionless Navier-Stokes equations are presented. Furthermore, the notation of eq. (1) and (2) is that of the equations used to simulate the turbulent channel flow. In this respect the velocity vector $\vec{u} = (u_x, u_y, u_z)$ and the nabla operator ∇ are non-dimensionalized with the friction velocity u_τ and the channel height H. p and t denote dimensionless values of pressure and time and by $Re_\tau = Hu_\tau/\nu$ the Reynolds number, which contains the molecular kinematic viscosity ν, is defined.

The huge computational resources, which are required to conduct a DNS restricts their application to turbulent flows at lower Reynolds numbers with a in most cases academic objective. However, the obtained turbulent flows solutions are characterized by random three-dimensional fluctuations with a continuous spectrum of length scales ranging down to flow structures which dissipate the excessive energy, the Kolmogorov scales.

Turbulent flow calculation with a more applied objective were and are still performed solving the Reynolds averaged Navier-Stokes equations (RANS) to obtain the statistically averaged mean flow field $\langle \vec{u} \rangle$ and $\langle p \rangle$. In many cases the obtained solution is stationary and, depending on the number of homogeneous directions involved, one- or two-dimensional. The statistical approach is associated with the highest loss of information and with a closure problem which is not satisfactory solved. Spectral information is completely lost, since any statistical quantity is an average over all turbulent scales. The obtained flow field describes the mean flow, which is enough for many applied problems, while the turbulent information is described with the Reynolds stress tensor $\langle u_i'' u_j'' \rangle$, which has to be modelled with empirical or semi-empirical models.

A first reduction of the number of unknowns can be accomplished applying the eddy viscosity principal, which was first introduced by Boussinesq (Boussinesq 1925),

$$-\langle u_i'' u_j'' \rangle = \nu_t 2 \langle S_{ij} \rangle - 2/3 \delta_{ij} k \qquad (3)$$

where $u_i'' = u_i - \langle u_i \rangle$ represents the statistical velocity fluctuation in tensor notation, $k = 1/2 \langle u_i''^2 \rangle$ the turbulent kinetic energy, $S_{ij} = 1/2(\partial \langle u_i \rangle/\partial x_j + \partial \langle u_j \rangle/\partial x_i)$ the mean strain rate tensor and δ_{ij} the identity tensor. Furthermore, eq. (3) contains the eddy viscosity ν_t, which has to be determined from known flow variables. This fundamental assumption defines the wide class of eddy viscosity models. There are a huge number of different turbulence modelsi, most of which use the eddy viscosity ansatz, but so far there is no generally valid statistical turbulence model.

In the last decade the interest in time-dependent, three-dimensional analysis of turbulent flows increased, since many technical problems are associated with large scale motions. To obtain these time-dependent flow structures researchers conduct instationary RANS simulation on three dimensional meshes with increasing number of grid points approaching the third technique called Large-Eddy Simulation (LES). This method basically resembles a compromise between RANS and DNS since it allows to predict the dynamics of the large turbulent scales while the effect of the fine scales are modeled with a subgrid-scale model.

The governing equations, which have to be solved in a LES are derived applying a filter function on the Navier-Stokes equations. Usually the filter function is chosen according to the applied discretization technique, i.e. a spectral cut-off filter is used for spectral discretization and a box-filter in finite-difference or finite-volume discretization. The kernel used to filter the flow field on a finite volume ΔV is presented in eq. 4.

$$G^V(x - x') = \begin{cases} 1/\Delta V & \text{for all } |x' - x| \in \Delta V \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

Filtering the velocity vector u_i for example, one obtains the low-pass filtered velocity vector $\overline{u_i}$, which differs from u_i by the local velocity fluctuation or subgrid scale u_i', as pointed out in eq. (5).

$$u_i = \overline{u}_i + u_i' \quad \text{with} \quad \overline{u}_i = \int_{\Delta V} G^V u_i \, dV \qquad (5)$$

Low-pass filtering of the Navier-Stokes equations (1) and (2) with eq. (4) formally removes scales with a wavelength smaller than the grid mesh. The obtains filtered Navier-Stokes equations (6) and (7) read:

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \qquad (6)$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{2}{Re_\tau} \frac{\partial \overline{S}_{ij}}{\partial x_j}. \qquad (7)$$

Eq. (7) contains the unknown subgrid scale stress tensor $\tau_{ij} = \overline{u_i^{'} u_j^{'}}$, which is the analog to the Reynolds stress tensor, and the filtered strain rate tensor $\overline{S}_{ij} = 1/2(\partial\overline{u}_i/\partial x_j + \partial\overline{u}_j/\partial x_i)$. To use the eddy viscosity concept eq. (3) to approximate τ_{ij} for LES was first proposed by the meteorologist Smagorinsky, who simulated large scale atmospheric motions. In Smagorinsky's model (Smagorinsky 1963) the eddy viscosity is proportional to a mean grid length scale Δ and the local strain rate $|\overline{S}| = \sqrt{1/2\overline{S}_{ij}\overline{S}_{ij}}$.

$$\tau_{ij} = \overline{u_i^{'} u_j^{'}} = -(C_{SM}\Delta)^2 \cdot \sqrt{\frac{1}{2}\overline{S}_{ij}\overline{S}_{ij}} \cdot \overline{S}_{ij} + 1/3\overline{u_k^{'} u_k^{'}}\delta_{ij} \qquad (8)$$

where $\nu_{SM} = (C_{SM}\Delta)^2\sqrt{\frac{1}{2}\overline{S}_{ij}\overline{S}_{ij}}$ defines the eddy viscosity of the Smagorinsky model. An approximate value for C_{SM} was derived by Lilly (Lilly 1965), assuming that the cut-off wavenumber in Fourier space lies within the $k^{-5/3}$ decay of an Kolmogorov cascade (k denotes the wavenumber). In this range, the energy spectrum $E(k) = C_K\varepsilon^{2/3}k^{-5/3}$ is characterized by C_K, the Kolmogorov constant. Assuming $C_K = 1.4$ a Smagorinsky constant $C_{SM} \approx 0.18$ is obtained.

Applying the Smagorinsky model for engineering applications though, the value $C_{SM} = 0.18$ leads to excessive dissipation. That's why a value of $C_{SM} = 0.1$ was used for channel simulations by Moin and Kim (Moin 1982) and for LES of a backstep flow by Arnal and Friedrich (Arnal 1992).

For wall-bounded flows it is necessary to adjust the value of C_s as it is done in the dynamic model by Germano (Germano 1992). The latter provides an expression for the constant C_{SM} in terms of the resolved scales which is a function of time and space.

Another class of widely used eddy-viscosity models are the so-called structure-function models. Assuming three-dimensional isotropic turbulence in Fourier space, where all wavenumbers k greater than the cutoff wavenumber k_c are suppressed and a Kolmogorov cascade exists, Kraichnan (Kraichnan 1976) derived the so-called spectral eddy viscosity in spectral space. Presuming a subgrid-scale kinetic energy dissipation, which equals the overall dissipation ε, the spectral eddy-viscosity was transformed into the physical space by Leslie and Quarini (Leslie 1979).

$$\nu_{ST}(\vec{x},\Delta x) = \frac{2}{3}C_k^{-\frac{3}{2}}\left[\frac{E(k_c)}{k_c}\right]^{1/2}, \quad \text{with} \quad k_c = \frac{\pi}{\Delta x} \qquad (9)$$

For a given local kinetic-energy spectrum $E_x(k_c)$ and a uniform mesh width Δ, the local spectrum can be calculated in terms of the local second-order structure function of $\overline{u}$ using Batchelors (Batchelor 1953) relation

$$F(\vec{x},\Delta) = \langle|\,\vec{\overline{u}}(\vec{x}) - \vec{\overline{u}}(\vec{x}+\vec{r})\,|^2\rangle = 4\int_0^{k_c} E(k)\left[1 - \frac{sin(k\Delta)}{k\Delta}\right]dk \qquad (10)$$

to obtain the eddy-viscosity of the structure function model by Métais and Lesieur (Métais 1980)

$$\nu_{ST}(x_i,\Delta) = 0.105C_k^{-\frac{3}{2}}\,\Delta[F(x_i,\Delta)]^{1/2}. \qquad (11)$$

Both, the Smagorinsky model and the structure function model were derived under similar assumptions. The first who addressed the question, how these two models relate was Comte (Comte 1994). He pointed out, that the structure function model can be replaced by

$$\nu_{ST} \approx 0.777(C_{SM}\Delta)^2\sqrt{2\overline{S}_{ij}\overline{S}_{ij} + \overline{\omega}_i\overline{\omega}_i} \quad , \qquad (12)$$

where ω_i denotes the vorticity vector, in a first-order approximation in the limit of vanishing grid spacing.

Any eddy viscosity model is purely dissipative. Thus, these models are unable to account for backscatter effects, i.e. the transfer of energy from small to larger scales. Furthermore, in analysis of experimental and numerical data (Liu et al. (Liu 1994) and Clark et al. (Clark 1979)) it was shown that the exact subgrid scale stress tensor correlates very poorly with the strain rate tensor. However, between τ_{ij} and the Leonard stress tensor $L_{ij} = \widetilde{\overline{u}_i\overline{u}_j} - \tilde{\overline{u}}_i\tilde{\overline{u}}_i$, for the two top-hat filter widths $\overline{\Delta}$ and $\tilde{\overline{\Delta}} > \overline{\Delta}$, they observed high corellations.

Assuming, that the subgrid scale stress tensor can be estimated directly from the resolved velocity field, Bardina et al. (Bardina 1980) suggested the first scale similarity model, for which filtering is applied twice with the same filter width $\overline{\Delta}$.

$$\tau_{ij} = \overline{\overline{u}_i\,\overline{u}_j} - \overline{\overline{u}}_i\overline{\overline{u}}_j \qquad (13)$$

Filtering with two different filter widths is performed for the scale similarity model by Liu et al. (Liu 1994). But this model contains an unknown constant C_L, which are positive with values lower than 1 according to measurements by Liu et al. (Liu 1994).

$$\tau_{ij} = C_L L_{ij} \qquad (14)$$

However, when these scale similarity models (eq. (13) and (14)) were used in LES both models hardly dissipate any energy, although the correlation between τ_{ij} and the Leonard tensor L_{ij} turned out to be comparably high, if top-hat filtering or Gauss filtering was applied. Faced with this difficulty Bardina et al. (Bardina 1980) already suggested to add a dissipative eddy-viscosity term in his mixed model

$$\tau_{ij} = \overline{\overline{u}_i}\,\overline{\overline{u}_j}i - \overline{\overline{u}_i\,\overline{u}_j} - 2(C_{SM}\Delta)^2\,|\,\overline{S}\,|\,\overline{S}_{ij} \qquad (15)$$

Although adding the eddy-viscosity term in the mixed model was performed in a rather ad hoc manner, this cured most of the problems of Bardina's model. Later many variations of mixed models were proposed. One of them is the one by Liu et al. (Liu 1994), who calculated both constants C_L and C_{SM} in their mixed model

$$\tau_{ij} = C_L L_{ij} - 2(C_{SM}\Delta)^2\,|\,\overline{S}\,|\,\overline{S}_{ij} \qquad (16)$$

with Germano's dynamic process solving a 2×2-system of equation in order to minimize the error.

It is the aim of this work to investigate, how closure of Liu et al.'s scale similarity model can be obtained utilized the dynamic process by Germano (Germano 1992). Furthermore, the structure function model is modified to use it in the dynamic process. The performance of the newly derived models will be tested in LES of turbulent channel flows for low Reynolds number.

NUMERICAL METHOD

The incompressible Navier-Stokes equations (1) - (2) are integrated applying Schumann's volume balance procedure (Schumann 1973) in cartesian coordinates. This leads to a set of spatially discrete equations on staggered grids, which represent the discrete counterpart of the filtered incompressible Navier-Stokes equations (6) - (7). Utilizing second order central interpolation and differentiation schemes leads to a method which is suitable for LES. Time integration is performed with a explicite second order accurate Leapfrog time step, which is restricted by a linear stability criterion.

A fractional step approach provides the oscillation-free coupling between pressure and velocity fields and leads to a three dimensional Poisson equation for the pressure correction, which has to be solved at each time step. The direct solutions of these Poisson problems are obtained using FFT's in x- and y-directions and a tridiagonal matrix algorithm for the remaining 1D-Helmholtz problems.

OUTLINE OF THE COMPUTATIONS

Large-Eddy simulation of turbulent channel flow are conducted to test the performance of subgrid scale turbulence models. The domain of computation extending $2\pi \times \pi \times 1$ in cartesian $x \times y \times z$-directions is bounded by two horizontally extending plates. The channel height H denotes vertical separation of these plates. Periodic boundary conditions are applied in the horizontal streamwise and spanwise directions, x and y. No-slip conditions for the tangential velocity components u_x and u_y are used at the wall. The impermeability condition prescribes a vanishing wall-normal velocity component $u_z = 0$ for $z/H = 0$ and $z/H = 1$, the z-coordinates of the plates.

The flow is driven by a constant pressure gradient $\partial\overline{p}/\partial x = -2$. The Reynolds number, based on the friction velocity u_τ, the channel height H and the kinematic viscosity ν, is $Re_\tau = (u_\tau H)/\nu = 360$. The simulations are performed on meshes with 64 and 32 equidistantly distributed grid points in x-direction and y-direction, and 95 grid points, which are refined in wall normal z-direction using a tangent hyperbolic law. Finally, at time $t = 0$ the simulations were started with random and small wave number velocity perturbations, which were superimposed on a analytical solution of laminar channel flow.

A MODIFIED STRUCTURE FUNCTION MODEL

As already noted by Comte (Comte 1994), the Smagorinsky model and the structure function model are closely related. This is not surprising, since both models were derived under the same assumption. However, the structure function model of Métais and Lesieur (Métais 1980) is not compatible with the dynamic process by Germano (Germano 1992). Therefore, it is worthwhile to investigate the structure function model in more detail.

Discretizing the structure function model eq. (11) leads to the following expression for the function $F(x_i,\Delta)$:

$$\begin{aligned} F(x_i,\Delta) &= \langle |\, \overline{u}_x(x_i) - \overline{u}_x(x_i+\Delta)\,|^2 \\ &+ |\, \overline{u}_y(x_i) - \overline{u}_y(x_i+\Delta)\,|^2 \\ &+ |\, \overline{u}_z(x_i) - \overline{u}_z(x_i+\Delta)\,|^2 \rangle \end{aligned} \tag{17}$$

Assuming isotropic meshes with equidistant grid spacing Δ, the finite differences in eq. 18, for example $\overline{u}_x(x_i) - \overline{u}_x(x_i+\Delta)$, are equivalent to a second order accurate central differences, if eq. (refdisStr) is divided by Δ. Therefore, eq. (18) can be expressed as follows

$$\begin{aligned} F(x,y,z,\Delta) &= \Delta^2 \langle \left(\frac{\partial\overline{u}_x}{\partial x}\right)^2 + \left(\frac{\partial\overline{u}_x}{\partial y}\right)^2 + \left(\frac{\partial\overline{u}_x}{\partial z}\right)^2 \\ &+ \left(\frac{\partial\overline{u}_y}{\partial x}\right)^2 + \left(\frac{\partial\overline{u}_y}{\partial y}\right)^2 + \left(\frac{\partial\overline{u}_y}{\partial z}\right)^2 \\ &+ \left(\frac{\partial\overline{u}_z}{\partial x}\right)^2 + \left(\frac{\partial\overline{u}_z}{\partial y}\right)^2 + \left(\frac{\partial\overline{u}_z}{\partial z}\right)^2 \rangle \end{aligned} \tag{18}$$

Introducing the velocity gradient tensor $\overline{A}_{ij} = \partial\overline{u}_i/\partial x_j$ in eq. (19), eq. (11) can be expressed by

$$\nu_{ST}(x_i,\Delta) = (C_{ST}\Delta)^2\sqrt{\overline{A}_{ij}\overline{A}_{ij}} \quad \text{with} \quad C_{ST}^2 = 0.105 C_k^{-3/2} \tag{19}$$

The main advantage of eq. (19) is its applicability in the dynamic process, because the velocity gradient tensor can be determined on the grid filter level $\overline{\Delta}$ and on test filter level $\tilde{\overline{\Delta}}$. For the latter the flow field is explicitly filtered over the volume element $\Delta\tilde{V} = 2\Delta x 2\Delta y \Delta r$ applying the kernel of the top-hat filter

$$G^{\tilde{V}}(x-x') = \begin{cases} 1/\Delta\tilde{V} & \text{for all } |\, x' - x \,| \in \Delta\tilde{V} \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

The deviatoric part of the subgrid scale tensor t_{ij} reads

$$t_{ij} = \tau_{ij} - 1/3\tau_{kk}\delta_{ij} \approx -C_{ST}^2\overline{\Delta}^2 \,|\, \overline{A} \,|\, \overline{S}_{ij} \tag{21}$$

where $\overline{A} = \sqrt{\overline{A}_{ij}\overline{A}_{ij}}$ represents a norm of the local velocity gradient tensor.

The deviatoric part of the subgrid scale stress tensor T_{ij} is evaluated from the velocity field of the test filter.

$$T_{ij} = \widetilde{\overline{u_i u_j}} - \tilde{\overline{u}}_i\tilde{\overline{u}}_j - 1/3(\widetilde{\overline{u_k u_k}} - \tilde{\overline{u}}_k\tilde{\overline{u}}_k)\delta_{ij} \approx -2C_{ST}{}^2\tilde{\overline{\Delta}}^2 \,|\, \tilde{\overline{A}} \,|\, \tilde{\overline{S}}_{ij} \tag{22}$$

Furthermore, following Germano (Germano 1992) the deviatoric part of the Leonard tensor L_{ij} reads

$$L_{ij} - 1/3 L_{kk}\delta_{ij} = T_{ij} - \tilde{t}_{ij} \tag{23}$$

If T_{ij} and t_{ij} in eq. (23) are substituted by eq. (22) and eq. (21) a equation is obtained, which can be solved for C_{ST} applying the least square formulation by Lilly (Lilly 1992).

$$C_{ST}^2 = \frac{L_{ij}(\tilde{\overline{\Delta}}^2 \,|\, \tilde{\overline{S}} \,|\, \tilde{\overline{S}}_{ij} - \widetilde{[\overline{\Delta}^2 \,|\, \overline{S} \,|\, \overline{S}_{ij}]})}{(\tilde{\overline{\Delta}} \,|\, \tilde{\overline{S}}\tilde{\overline{S}}_{ij} - \widetilde{[\overline{\Delta} \,|\, \overline{S} \,|\, \overline{S}_{ij}]})^2} \tag{24}$$

Negative values for C_{ST}^2 are obtained frequently in a region close to the wall, if eq. (24) is computed in a LES of turbulent channel flow for a Reynolds number $Re_\tau = 360$. Therefore, as proposed by Lilly (Lilly 1994), C_{ST}^2 is statistically averaged in streamwise and spanwise directions.

Results

In Fig. 1 and Fig. 2 - 4 the mean velocity profiles and profiles of rms velocity fluctuations obtained in two LES, one utilizing the dynamic Smagorinsky model and the other one applying the above derived dynamic modified structure function model, are compared with DNS results by Kim et al. (Kim 1987) who performed spectral simulations. Additionally, results of underresolved DNS using the above described finite-volume method are shown. The mean velocity profiles of the two LES are similar but high compared to Kim et al.'s results (Fig. 1). This is typical for LES of low Reynolds number flows with eddy viscosity models. On the other hand the underresolved DNS leads to mean axial velocities which are low, indicating the need for an effective turbulence model to reliably predict this flow. That the two subgrid scale model lead to similar results is also indicated in Fig 2 - 4, where the rms velocity fluctuations of both LES reflect almost similar results. But u_x, rms is too high and u_y, rms and u_z, rms are low compared to the spectral DNS results, again a typical feature of eddy viscosity modeling.

DYNAMIC SCALE SIMILARITY MODEL

To model the subgrid scale tensor based on scale similarity with the Leonard stress tensor was first proposed by Liu et al. (Liu 1994).

$$\tau_{ij} = C_L L_{ij} = C_L(\widetilde{\overline{u}_i\overline{u}_j} - \tilde{\overline{u}}_i\tilde{\overline{u}}_j) \tag{25}$$

In measurements of turbulent jet flow he observed high correlations between the subgrid scale tensor and Leonard stress tensor and constants C_L ranging from 0.6 to 0.8, but no applicable method to determine C_L was proposed.

In this work we propose to determine C_L utilizing the dynamic process (Germano 1992). To achieve this the deviatoric parts of the subgrid scale stress tensors t_{ij} and T_{ij} are approximated with any eddy viscosity subgrid scale model. Below we apply the Smagorinsky model, but the modified structure function model could be used as well.

$$t_{ij} \approx -2C_s{}^2\overline{\Delta}^2 \,|\, \overline{S} \,|\, \overline{S}_{ij} \quad \text{and} \quad T_{ij} \approx -2C_s{}^2\tilde{\overline{\Delta}}^2 \,|\, \tilde{\overline{S}} \,|\, \tilde{\overline{S}}_{ij} \tag{26}$$

The scale similarity model eq. (25) is rewritten in terms of the deviatoric parts of the subgrid scale stress tensor and the Leonard stress tensor (see eq. (23)),

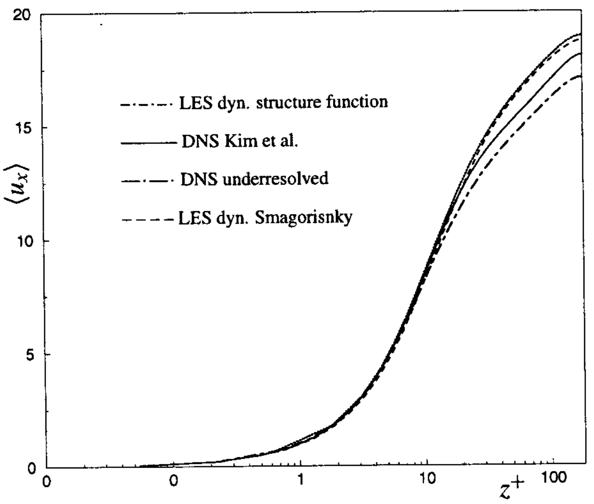

Figure 1: Mean velocity profiles calculated in LES, using the dynamic Smagorinsky model and the new dynamic modified structure function model compared to underresolved DNS results and Kim et al.'s (Kim 1987) spectral DNS data.

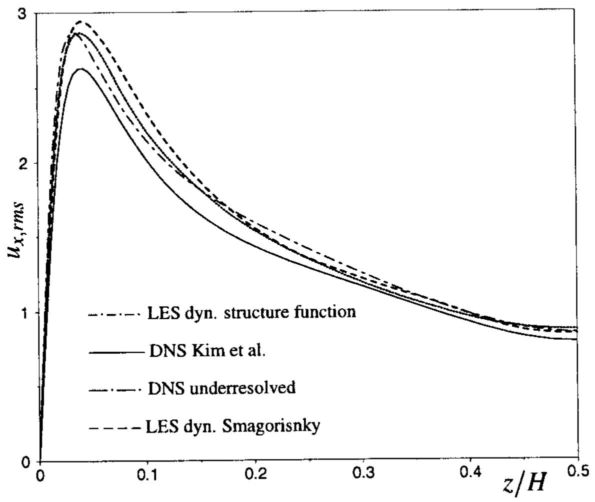

Figure 2: Profiles of streamwise rms velocity fluctuations calculated in LES, using the dynamic Smagorinsky model and the new dynamic modified structure function model compared with an underresolved DNS results and Kim et al.'s DNS data.

$$t_{ij} = C_L(L_{ij} - 1/3L_{kk}\delta_{ij}) = C_L(T_{ij} - \tilde{t}_{ij} - 1/3\left[T_{kk} - \tilde{t}_{kk}\delta_{ij}\right]) \quad (27)$$

Substituting t_{ij} and T_{ij} in eq. (27) by eq. (26) leads to the following relation for the unknown factor C_L.

$$\overline{\Delta}^2 \mid \overline{S} \mid \overline{S}_{ij} = C_L(\tilde{\overline{\Delta}}^2 \mid \tilde{\overline{S}} \mid \tilde{\overline{S}}_{ij} - \widetilde{[\overline{\Delta}^2 \mid \overline{S} \mid \overline{S}_{ij}]}) \quad (28)$$

Eq. 28 can be solved with the least square formulation by Lilly (Lilly 1994). Principally, C_L is a function of space and time. However, the high correlation between τ_{ij} and L_{ij} (eq. (25)) and between their statistically averaged counterparts $\langle\tau_{ij}\rangle$ and $\langle L_{ij}\rangle$ must be conserved. Thus, it is necessary that C_L behaves smoothly in all homogeneous directions and in time. To enforce this, C_L is statistically averaged as given by

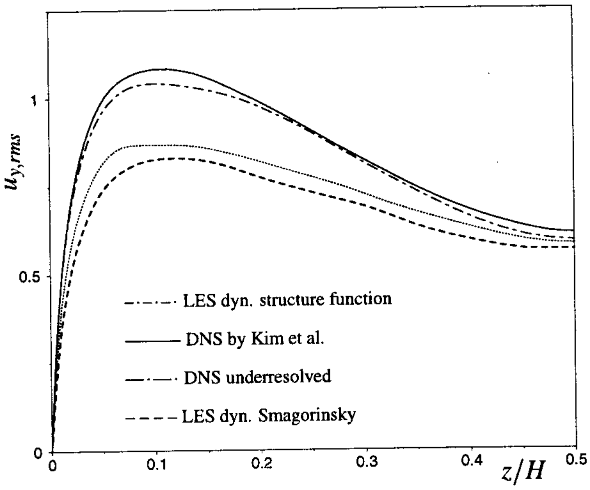

Figure 3: Profiles of spanwise rms velocity fluctuations calculated in LES, using the dynamic Smagorinsky model and the new dynamic modified structure function model compared with an underresolved DNS results and Kim et al.'s DNS data.

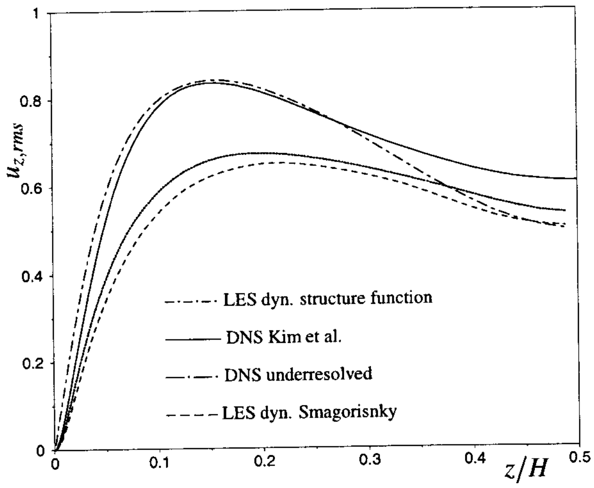

Figure 4: Profiles of wall-normal rms velocity fluctuations calculated in LES, using the dynamic Smagorinsky model and the new dynamic modified structure function model compared with an underresolved DNS results and Kim et al.'s DNS data.

$$\langle C_L\rangle = \frac{\langle\overline{\Delta}^2 \mid \overline{S} \mid \overline{S}_{ij}(\tilde{\overline{\Delta}}^2 \mid \tilde{\overline{S}} \mid \tilde{\overline{S}}_{ij} - \widetilde{[\overline{\Delta}^2 \mid \overline{S} \mid \overline{S}_{ij}]})\rangle}{\langle(\tilde{\overline{\Delta}}^2 \mid \tilde{\overline{S}} \mid \tilde{\overline{S}}_{ij} - \widetilde{[\overline{\Delta}^2 \mid \overline{S} \mid \overline{S}_{ij}]})^2\rangle} \quad (29)$$

With eq. (29) closure for the scale similarity model in eq. (27) is achieved.

Results

To test the scale similarity model a LES of turbulent channel flow is performed. The dynamically computed distribution of $\langle C_L\rangle$ for a certain time is depicted in Fig. 5. The value of $\langle C_L\rangle$ ranges from ≈ 0.3 at the wall to ≈ 0.7 for $y/H \approx 0.15$. The values agree remarkably well with those, Liu et al. (Liu 1994) obtained in their measurements of turbulent jet flow. Additionally, C_L turned out to be positive everywhere in the flow field, therefore, no stability problems were observed during the computation.

In Fig. 6 the streamwise mean velocity profiles computed

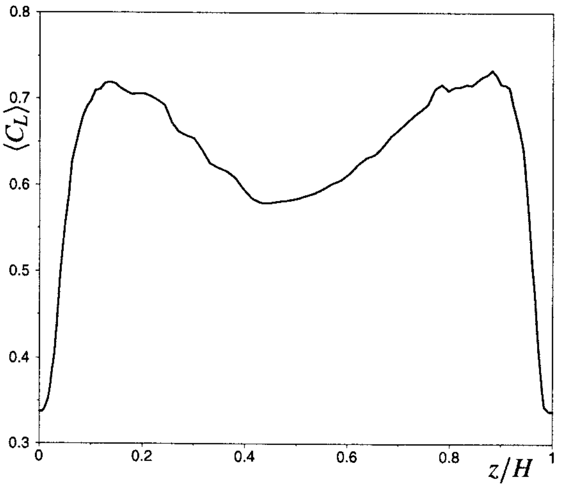

Figure 5: Wall-normal distribution of the scale similarity constant $\langle C_L \rangle$.

with underresolved direct numerical simulation (DNS) (LES without model), with LES using the dynamic Smagorinsky model and with LES utilizing the new scale similarity model are presented in logarithmic scaling over wall distance in wall units, i.e $z^+ = zRe_\tau$. They are compared to DNS data by Kim et al. (Kim 1980). While the underresolved DNS leads too low mean velocity profile, the use of dynamic Smagorinsky model generates velocity profiles which are too high. The mean velocity generated with the new model agrees remarkably well with the data of Kim et al. (Kim 1980). In Fig. 7 the resulting deviatoric part of the Reynolds stress tensor

$$R_{ij} = \langle u_i u_j \rangle - \langle u_i \rangle \langle u_j \rangle - 1/3(\langle u_k u_k \rangle - \langle u_k \rangle \langle u_k \rangle)\delta_{ij} \qquad (30)$$

is compared to the according data of Kim et al. (Kim 1980). It is demonstrated that the used scale similarity model produces reliable results in LES of turbulent wall-bounded flows.

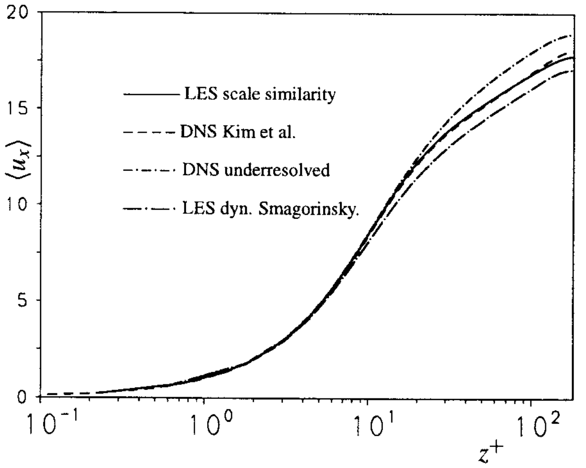

Figure 6: Mean velocity profiles calculated in LES compared to to fully resolved DNS data by Kim et al. (Kim 1987) and to underresolved DNS data.

SUBGRID SCALE ENERGY DISSIPATION

Dissipation of subgrid scale energy is the amount of energy which is transferred from resolved to unresolved scales. It is well known that

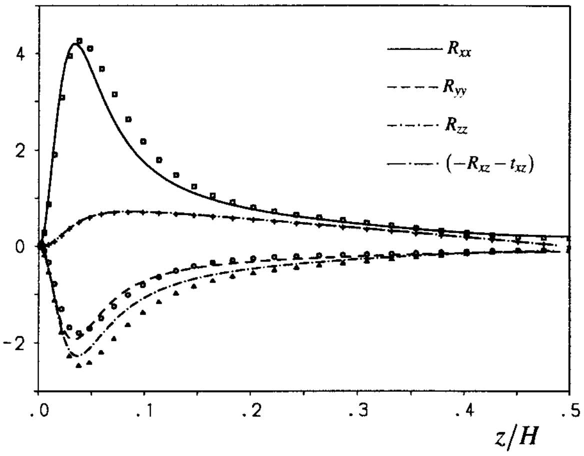

Figure 7: Profiles of the deviatoric part of the Reynolds stress tensor calculated in LES with the new scale similarity model compared to fully resolved DNS data by Kim et al. (Kim 1987) denoted by symbols.

the mean dissipation of subgrid scale energy is negative everywhere in the flow field, but backscatter of energy form small to large scales can be observed locally in space and time. In order to investigate this energy transfer Härtel (Härtel 1994) proposed to split the production term of the transport equations for kinetic subgrid scale energy into a mean and a fluctuating term, as presented in eq. (31).

$$\langle \tau_{ij}\overline{s_{ij}} \rangle = \langle \tau_{ij} \rangle \langle \overline{S_{ij}} \rangle + \langle \tau''_{ij} \overline{S_{ij}}'' \rangle . \qquad (31)$$

The term on the left hand side of eq. 31 represents the total production of subgrid scale energy in the subgrid scale energy equation (not shown). It is split into the first term on the right hand side of eq. (31), which expresses the production due to the mean strain rate and the second term, which is the production due to the fluctuating strain rates. These production terms of the subgrid scale energy equation are equivalent to the dissipation terms of the resolved scale energy (turbulent kinetic energy) equation.

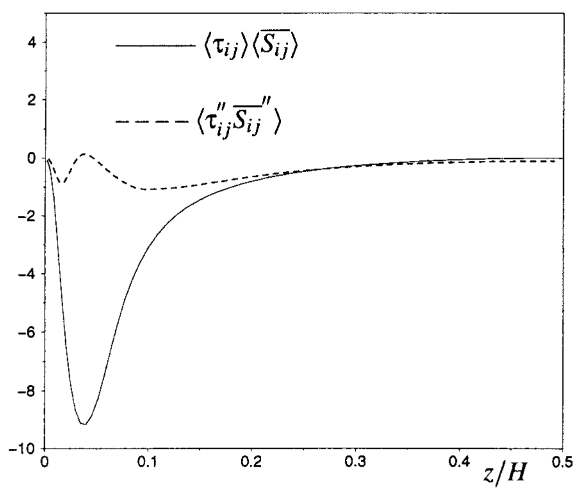

Figure 8: Dissipation of subgrid scale energy due to mean strain rates and fluctuating strain rates

While the first term of the right hand side of eq. (31) is negative for all z/H, as depicted in Fig. 8, the second term reveals a local maximum with positive dissipation rates due to fluctuating strain rates. This local maximum reflects the backscatter of energy. Therefore, it is concluded that in LES with the scale similarity model backscatter effects are predicted.

CONCLUSIONS

Two new subgrid scale turbulence models are proposed. The first one is represents a modification to the classical structure function model by Métais and Lesieur (Métais 1980). The advantage of this modified structure function is, that it can be used within the dynamic process by Germano (Germano 1992). The model was validated comparing results obtained in LES of turbulent channel flow at Reynolds number $Re_\tau = 360$ to results obtained in LES with the dynamic Smagorinsky model (Smagorinsky 1963), in underresolved DNS and with Kim et al.'s (Kim 1987) spectral DNS data. This comparison reveals, that the new structure function model and the Smagorinsky model lead to similar results. Both are too dissipative leading to mean velocity which are too high.

The second model is a scale similarity model. A new approach to compute the scaling constant in a given scale similarity model by Liu et al. (Liu 1994) is proposed. Again the LES results obtained with this model are compared to Kim et al. DNS data. Excellent agreement was obtained. It is further shown, that the model reliably predicts backscatter effects.

REFERENCES

Arnal, M. and Friedrich, R., 1992, "Large-eddy simulation of a turbulent flow with separation", *In: Turbulent Shear Flows VIII*, U. Schumann, F. Durst, B.B. Lander, F.W. Schmidt, J.H. Whitelaw (eds), Springer Verlag, Berlin, pp. 169.

Bardina, J., Ferziger, J.H. and Reynolds, W.C., 1980, "Improved subgrid scale models for large eddy simulation, *AIAA*, Pap. 80-1357.

Batchelor, G.K., 1953,"The theory of homogeneous turbulence", *Cambridge University Press*.

Boussinesq, J., 1925, "Essai sur la Theorie des Eaux Courantes", *Mém. Prés. par div. savant à làcad. sci. Paris*, Vol. 23, pp. 136-139.

Clark, R.A., Ferziger, J.H. and Reynolds, W.C., 1979, "Evaluation of subgrid scale turbulence models using an fully simulated turbulent flow, *Journal of Fluid Mech.*, Vol. 91, pp. 1-16.

Comte, P., 1994, "Structure-function based models for compressible transitional shear flows", *ERCOFTAC Bull.*, Vol. 22, pp. 9-14.

Germano, M., 1992, "Turbulence: the filtering approach", *Journal of Fluid Mech.*, Vol. 238, pp. 325-336.

Härtel, C., 1994, "Analyse und Modellierung der Feinstruktur im wandnahen Bereich turbulenter Scherströmungen", Dissertation, Department of Fluid Mechanics, Munich University of Technology.

Kim, J., Moin, P. and Moser, R., 1987, "Turbulence statistics in fully developed channel flow at low Reynolds number", *Journal of Fluid Mech*, Vol. 177, pp. 133-166.

Kraichnan, R.M., 1976, "Eddy viscosity in two and three dimensions", *Journal of Atmos. Sci.*, Vol. 33, pp. 1521-1536.

Leslie, D.C. and Quarini, G.L., 1979, "The application of turbulence theory to the formulation of subgrid modelling procedures", *Journal of Fluid Mech.*, Vol. 91, pp. 65-91.

Lilly, D.K., 1965, "On the computational stability of numerical solutions of time-dependent non-linear geophysical fluid dynamics problems", *Monthly Weather Rev.*, Vol. 93, no. 1, pp. 11-26.

Lilly, D.K., 1992, "A proposed modification of the Germano subgrid scale closure method", *Phys. of Fluids A*, Vol. 4, pp. 633-635.

Liu, S., Meneveau, D. and Katz, J., 1994,"On the properties of similarity subgrid-scale models as deduced from measurements in a turbulent jet", *Journal of Fluid Mech.*, Vol. 275, pp. 83-119.

Métais, O. and Lesieur, M., 1992, "Spectral large-eddy simulation of isotropic and stably-stratified turbulence", *Journal of Fluid Mech.*, Vol. 239, pp. 157-194.

Moin, P. and Kim, J., 1982, "Numerical investigation of turbulent channel flow", *Journal of Fluid Mech.*, Vol. 118, pp. 341-377.

Schumann, U., 1973, "Ein Verfahren zur direkten numerischen Simulation turbulenter Strömungen in Platten- und Ringspaltkanälen und über seine Anwendung zur Untersuchung von Turbulenzmodellen", Dissertation, University of Karlsruhe.

Smagorinsky, J., 1963, "General circulation experiments with the primitive equations, I, The basic experiment", *Mon. Weather Rev.*, pp. 99-165.