A DYNAMIC SUBGRID SCALE MODEL OF EDDY VISCOSITY TYPE DEDUCED FROM A LOCAL INTER-SCALE EQUILIBRIUM ASSUMPTION OF ENERGY TRANSFER

Tomoya Murota
Power & Industrial Systems R & D Laboratory, Hitachi, Ltd.
832-2 Horiguchi, Hitachinaka-shi, Ibaraki-ken 312-0034, Japan

ABSTRACT

A new dynamic subgrid scale (SGS) stress model of the eddy viscosity type is proposed. This dynamic model depends on the assumption of energy transfer, *i.e.* the local inter-scale equilibrium, instead of the Germano identity on which traditional dynamic models depend. The choice of the energy transfer theory as the resolution principle for the dynamic procedure is natural because the main purpose of eddy viscosity models is to simulate the energy transfer between the grid scale and subgrid scale accurately. One of the advantages of the new model is no necessity for an *ad hoc* averaging treatment over a statistically homogeneous region, which has limited the application of dynamic models only to simple geometries. The validity of the proposed model is shown in the analysis of the plane channel flow.

INTRODUCTION

In fluid machine design, engineers often need to estimate the unsteady turbulent flow field for which they have high expectations of computational fluid dynamics (CFD). Large eddy simulation (LES) will be a promising approach to meet these expectations, but not yet.

The Smagorinsky model proposed by Smagorinsky (1963), which is a classical SGS stress model of the eddy viscosity type, has uncertainty of the model parameter, *i.e.* the Smagorinsky constant C_S. Germano *et al.* (1991) proposed a dynamic Smagorinsky model (DSM) to overcome this defect. This model derives the optimum value of C_S from the Germano identity. The least square method proposed by Lilly (1992) is used as a dynamic procedure to calculate the value of C_S from the Germano identity.

In spite of advantages, the DSM has not been put to practical use in fluid machine design yet. This is because applicable geometries of the DSM are limited to simple ones. To avoid numerical instability, averaging treatments over statistically homogeneous regions are introduced into Lilly's dynamic procedure. This prevents the application of the DSM to complex geometries.

A dynamic SGS model of the eddy viscosity type without *ad hoc* averaging treatments over homogeneous regions is proposed in this paper. The main difference between the proposed model and the traditional DSM is that the proposed model calculates the optimum value of the model parameter based on a theory about energy transfer instead of the Germano identity.

TRADITIONAL SGS MODELS OF EDDY VISCOSITY TYPE

Basic Equations for Large Eddy Simulation

The basic equations for incompressible flows are the grid-filtered continuity and Navier-Stokes equations given as follows.

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \tag{2}$$

Here u_i is the velocity component of x_i direction, p is the pressure divided by the density, $\sigma_{ij} \equiv 2\nu S_{ij}$ is the viscous stress tensor, ν is the kinematic viscosity, S_{ij} is the strain rate tensor defined as follows,

$$S_{ij} \equiv \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

and $\tau_{ij} \equiv \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ is the SGS stress tensor. The overbar $\bar{\cdot}$ denotes the grid-filtering operation.

Energy Transfer Effect by SGS Stress

The grid scale kinematic energy corresponding to the grid scale (GS) velocity field is affected by the SGS stress. The effect of the SGS stress is understood through the transport equations for the kinematic energy of the grid scale and the subgrid scale.

The transport equation of the grid scale kinematic energy defined as $K_{gs} \equiv \bar{u}_k\bar{u}_k/2$ is expressed as follows.

$$\frac{\partial K_{gs}}{\partial t}+\frac{\partial \bar{u}_j K_{gs}}{\partial x_j}=-\frac{\partial \bar{u}_i\bar{p}}{\partial x_i}+\frac{\partial \bar{u}_i\bar{\sigma}_{ij}}{\partial x_j}-\frac{\partial \bar{u}_i\tau_{ij}}{\partial x_j} -\varepsilon_{gs}+\tau_{ij}\bar{S}_{ij} \tag{4}$$

$$\varepsilon_{gs}\equiv\bar{\sigma}_{ij}\bar{S}_{ij}=2\nu\bar{S}_{ij}\bar{S}_{ij} \tag{5}$$

Here ε_{gs} is the viscous dissipation rate from the grid scale. The transport equation of the subgrid scale kinematic energy defined as $K_{sgs}\equiv\left(\overline{u_k u_k}-\bar{u}_k\bar{u}_k\right)/2$ is expressed as follows.

$$\frac{\partial K_{sgs}}{\partial t}+\frac{\partial \bar{u}_j K_{sgs}}{\partial x_j}=-\frac{\partial}{\partial x_j}\left(\overline{u_j K}-\bar{u}_j\bar{K}\right) -\frac{\partial}{\partial x_i}\left(\overline{u_i p}-\bar{u}_i\bar{p}\right)+\frac{\partial}{\partial x_j}\left(\overline{u_i\sigma_{ij}}-\bar{u}_i\bar{\sigma}_{ij}\right) +\frac{\partial \bar{u}_i\tau_{ij}}{\partial x_j}-\varepsilon_{sgs}-\tau_{ij}\bar{S}_{ij} \tag{6}$$

$$\varepsilon_{sgs}\equiv\bar{\varepsilon}-\varepsilon_{gs} \tag{7}$$

Here $K\equiv u_k u_k/2$ is the kinematic energy corresponding to the non-filtered velocity field, ε_{sgs} is the viscous dissipation rate from the subgrid scale and $\varepsilon\equiv\sigma_{ij}S_{ij}$ is the total viscous dissipation rate.

The SGS stress appears in Eqs.(4) and (6) in the same form of $-\tau_{ij}\bar{S}_{ij}$ with opposite signs. Energy is transferred between the grid and subgrid scales in the wave number space by these terms. The term $-\tau_{ij}\bar{S}_{ij}$ can be positive (forward scatter) or negative (back scatter) instantaneously. In the case of forward scatter, part of the energy is pumped out of the grid scale by the SGS stress and an equal amount of energy is provided to the subgrid scale. On the average, energy flows from the grid scale into the subgrid scale.

This energy transfer effect is very important for SGS modeling from the viewpoint of numerical stability as well as computational accuracy because underestimation of the average energy dissipation from the grid scale often causes numerical instability. So SGS models should represent the energy transfer effect of the SGS stress.

Eddy Viscosity Models

Eddy viscosity models generally approximate the SGS stress tensor as follows.

$$\tau_{ij}^{*}\equiv\tau_{ij}-\frac{1}{3}\delta_{ij}\tau_{kk}=-2\nu_e\bar{S}_{ij} \tag{8}$$

Here δ_{ij} is the Kronecker delta and ν_e is the eddy viscosity. The asterisk "*" denotes the trace free operation for a tensor.

Eddy viscosity models generally assume that the trace free operated SGS stress tensor is proportional to the strain rate tensor. But the assumption is not always true. Consequently eddy viscosity models show low correlations with the SGS stress by nature.

Nevertheless eddy viscosity models are widely used in engineering because these models have abilities to represent the energy dissipation effects of the SGS stress. Or rather it would be better to say that eddy viscosity models are designed to represent the energy dissipation behavior of the SGS stress. So the main issue of eddy viscosity models becomes how to determine the eddy viscosity to simulate the energy dissipation accurately.

For example, the Smagorinsky model approximates the eddy viscosity as follows.

$$\nu_e=\left(C_S\Delta_g\right)^2|\bar{S}| \tag{9}$$

Here $|\bar{S}|\equiv\left(2\bar{S}_{ij}\bar{S}_{ij}\right)^{\frac{1}{2}}$, C_S is the Smagorinsky constant and Δ_g is the characteristic length of the grid-filter.

Although there is a slight difference from the optimum value utilized actually, the theoretical value of the Smagorinsky constant can be derived from some assumptions about energy transfer in the wave number space. The first assumption is the local energy equilibrium within the subgrid scale. The second is that the viscous dissipation from the subgrid scale is almost the same as the total viscous dissipation. The last is that the characteristic wave number of the grid-filter is within the inertial sub-range and the energy spectrum function obeys the Kolmogorov 5/3-law. From all these assumptions, the theoretical value of the Smagorinsky constant is given as follows.

$$C_S=\frac{1}{\pi}\left(\frac{2}{3C_K}\right)^{3/4} \tag{10}$$

Here C_K is the Kolmogorov constant. With $C_K=1.5$, Eq. (10) gives $C_S\approx 1.73$.

Dynamic Smagorinsky Model

The dynamic Smagorinsky model derives the value of the Smagorinsky constant using the Germano identity (Germano *et al.*, 1991), which is expressed as follows.

$$L_{ij}\equiv\widehat{\bar{u}_i\bar{u}_j}-\hat{\bar{u}}_i\hat{\bar{u}}_j=T_{ij}-\hat{\tau}_{ij} \tag{11}$$

Here L_{ij} is a resolved turbulent stress tensor and T_{ij} is a subtest scale (STS) stress tensor. The hat $\hat{}$ denotes the test-filtering operation.

Generally the modeled SGS and STS stress tensors, denoted as τ_{ij}^m and T_{ij}^m respectively, do not satisfy the Germano identity. Substitution of τ_{ij}^m and T_{ij}^m into Eq.

(11) yields the error tensor γ_{ij} as follows.

$$\gamma_{ij} \equiv L_{ij} - T_{ij}^{m} + \hat{\tau}_{ij}^{m} \tag{12}$$

The model parameters of SGS models are determined so as to minimize the inner product of the error tensor, $\gamma_{ij}\gamma_{ij}$. When a SGS model has n-parameters, each parameter C_k can be calculated from the following system of equations.

$$\frac{\partial \gamma_{ij}\gamma_{ij}}{\partial C_k} = 0, \quad (k = 1, \cdots, n) \tag{13}$$

The above procedure is the general extension of the least square method proposed by Lilly (1992). Application of this dynamic procedure to the Smagorinsky model gives the value of the Smagorinsky constant as follows.

$$(C_S \Delta_g)^2 = -\frac{1}{2}\frac{\langle L_{ij}^{*} M_{ij}\rangle}{\langle M_{ij} M_{ij}\rangle} \tag{14}$$

$$M_{ij} \equiv (\Delta_{gt}/\Delta_g)^2 |\hat{\bar{S}}|\hat{\bar{S}}_{ij} - \widehat{|\bar{S}|\bar{S}_{ij}} \tag{15}$$

Here Δ_{gt} is the characteristic length of the test-grid-filter $\hat{\bar{\cdot}}$, and $\langle\cdot\rangle$ denotes the averaging operation over a statistically homogeneous region. The averaging operation has the effect of preventing numerical instability, but limits the application of the DSM to simple geometries.

There is some doubt about the reasonableness of application of Lilly's dynamic procedure to eddy viscosity models. When it comes to eddy viscosity models, model parameters must be determined so that energy dissipation from the grid scale is simulated correctly. But the traditional dynamic procedure using the Germano identity does not guarantee accuracy for the energy dissipation because there is no explicit relation to energy dissipation.

PROPOSAL OF A NEW DYNAMIC PROCEDURE FOR SGS MODEL OF EDDY VISCOSITY TYPE

Local Inter-Scale Equilibrium Assumption

Instead of the Germano identity, an alternative resolution principle utilized in the dynamic procedure for eddy viscosity models has to be found. Such a principle is derived from the theory about energy transfer.

Flow fields filtered by the test-grid-filter are considered. A schematic of energy spectrum in the wave number space is shown in Fig.1. Here k_g and k_{tg} are the characteristic wave numbers of the grid-filter $\bar{\cdot}$ and the test-grid-filter $\hat{\bar{\cdot}}$ respectively. The region $k < k_{tg}$ is called the test scale, $k > k_{tg}$ is the subtest scale, $k_{tg} < k < k_g$ is the intermediate scale and so on.

The intermediate scale is a remarkable region because both the SGS and STS stresses have a role in the energy transfer of this region. And information for this region is available in calculation of LES because this region is within the grid-filter resolved scale.

When the kinematic energy corresponding to the test

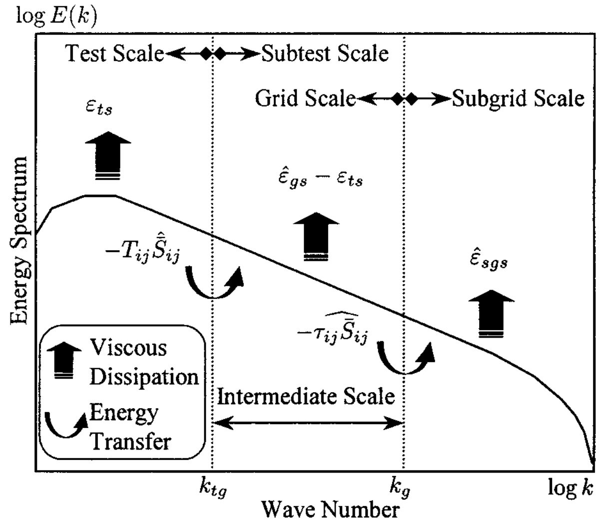


Fig.1 Energy transfer in the wave number space.

scale is defined as $K_{ts} \equiv \hat{\bar{u}}_k\hat{\bar{u}}_k/2$, the intermediate scale kinematic energy can be expressed as $K_{is} \equiv \hat{K}_{gs} - K_{ts}$. The transport equation of K_{is} can be derived as follows.

$$\begin{aligned}\frac{\partial K_{is}}{\partial t} + \frac{\partial \hat{\bar{u}}_j K_{is}}{\partial x_j} &= -\frac{\partial}{\partial x_j}\left(\widehat{\bar{u}_j K_{gs}} - \hat{\bar{u}}_j \hat{K}_{gs}\right) \\ &- \frac{\partial}{\partial x_i}\left(\widehat{\bar{u}_i \bar{p}} - \hat{\bar{u}}_i \hat{\bar{p}}\right) + \frac{\partial}{\partial x_j}\left(\widehat{\bar{u}_i \bar{\sigma}_{ij}} - \hat{\bar{u}}_i \hat{\bar{\sigma}}_{ij}\right) \\ &- \frac{\partial}{\partial x_j}\left(\widehat{\bar{u}_i \tau_{ij}} - \hat{\bar{u}}_i T_{ij}\right) \\ &- \left(\hat{\varepsilon}_{gs} - \varepsilon_{ts} - \widehat{\tau_{ij}\bar{S}_{ij}} + T_{ij}\hat{\bar{S}}_{ij}\right)\end{aligned} \tag{16}$$

$$\varepsilon_{ts} \equiv 2\nu \hat{\bar{S}}_{ij}\hat{\bar{S}}_{ij} \tag{17}$$

Here ε_{ts} is the viscous dissipation rate from the test scale.

By assuming energy balance within the intermediate scale, the following relationship is derived from Eq. (16).

$$\varepsilon_{ts} - T_{ij}\hat{\bar{S}}_{ij} = \hat{\varepsilon}_{gs} - \widehat{\tau_{ij}\bar{S}_{ij}} \tag{18}$$

Eq.(18) means that a portion of the energy transported from the test scale to the intermediate scale by the STS stress is dissipated by the viscous stress and the rest is transported to the subgrid scale by the SGS stress. This energy balance between the test-grid-filtered flow field and the grid-filtered one is called a "local inter-scale equilibrium".

Dynamic Procedure for Eddy Viscosity Model

The SGS and STS stress tensors are unknowns in Eq.(18). Here the SGS stress is approximated using Eq.(8) which is the generalized eddy viscosity model. The STS stress is approximated as T_{ij}^{A} using the Taylor expansion instead of the eddy viscosity model, because application of the eddy viscosity model both to the SGS and STS stresses only gives an indefinite solution. Substitution of the models for the SGS and STS stresses into Eq.(18) yields the following expression for the eddy viscosity.

$$\nu_e = -\frac{T_{ij}^{A*}\hat{\bar{S}}_{ij} - \left(\nu|\hat{\bar{S}}|^2 - \nu\widehat{|\bar{S}|^2}\right)}{\widehat{|\bar{S}|^2}} \tag{19}$$

This is the core equation of the dynamic procedure for the SGS stress model of the eddy viscosity type proposed in this paper. Other expressions for the individual eddy viscosity models such as the Smagorinsky model can be derived in the same way, but they are not adopted here because Eq.(19) gives the eddy viscosity which is the very goal of the eddy viscosity models.

No averaging operations over statistically homogeneous regions are introduced into the new dynamic procedure so as not to limit applicable geometries. But to avoid negative viscosity, which violates stable calculation by nature, the clipping technique is introduced as follows.

$$\nu_e = \max\left(0, -\frac{T_{ij}^{A*}\hat{\bar{S}}_{ij} - \left(\nu|\hat{\bar{S}}|^2 - \nu\widehat{|\bar{S}|^2}\right)}{\widehat{|\bar{S}|^2}}\right) \tag{20}$$

Approximation of Subtest Scale Stress

In the proposed dynamic procedure, the concrete expression of the approximated STS stress tenor T_{ij}^A must be defined. Additionally before derivation of T_{ij}^A, the expression of the grid-filter and the test-filter must be defined.

Here a usage of a finite volume method (FVM) for space discretization is assumed. The volume averaging operation corresponds to the grid-filtering operation in the FVM. A scalar f in three-dimensional space can be expressed as follows using the Maclaurin expansion.

$$f_{(x_1,x_2,x_3)} = \sum_{l=0}^{\infty}\left(x_k\frac{\partial}{\partial a_k}\Big|_{a_1=0,a_2=0,a_3=0}\right)^l f_{(a_1,a_2,a_3)} \tag{21}$$

Integration of Eq.(21) over a computational cell yields the expression of the grid-filtering operation as follows.

$$\bar{f} = f + \frac{h_k^2}{24}\frac{\partial^2 f}{\partial x_k^2} + (1-\delta_{mn})\frac{h_m^2 h_n^2}{1152}\frac{\partial^4 f}{\partial x_m^2 \partial x_n^2} + \frac{h_k^4}{1920}\frac{\partial^4 f}{\partial x_k^4} + O(h_k^6) \tag{22}$$

Here it is supposed that the cell geometry is a rectangular parallelepiped, each edge of which is parallel to one of the coordinate axes and the centroid of which is on the coordinate origin. Here h_i is the length of the edge parallel to the x_i coordinate axis.

A formulation of the test-filtering operation is not unique. The test-filtering operation is defined here as follows.

$$\hat{f}_{(i,j,k)} = f_{(i,j,k)} + \frac{\beta^2}{24}\Big(f_{(i+1,j,k)} + f_{(i-1,j,k)} + f_{(i,j+1,k)} + f_{(i,j-1,k)} + f_{(i,j,k+1)} + f_{(i,j,k-1)} - 6f_{(i,j,k)}\Big) \tag{23}$$

Here the subscripts (i,j,k) denote computational cell indexes and β is a parameter related to the characteristic length of the test-filter. The Taylor expansion gives the following equation.

$$f_{(i+1,j,k)} - 2f_{(i,j,k)} + f_{(i-1,j,k)} = h_1^2\frac{\partial^2 f}{\partial x_1^2} + \frac{h_1^4}{12}\frac{\partial^4 f}{\partial x_1^4} + O(h_1^6) \tag{24}$$

Substitution of Eq.(24) into Eq.(23) yields the test-filtering operator in differential form as follows.

$$\hat{f} = f + \beta^2\left(\frac{h_l^2}{24}\frac{\partial^2 f}{\partial x_l^2} + \frac{h_l^4}{288}\frac{\partial^4 f}{\partial x_l^4}\right) + O(h^6) \tag{25}$$

The application of Eqs.(22) and (25) to the STS stress tensor yields the following equation.

$$\begin{aligned} T_{ij} &= 2(1+\beta^2)\frac{h_l^2}{24}\frac{\partial u_i}{\partial x_l}\frac{\partial u_j}{\partial x_l} \\ &+ \left(\frac{h_l^2}{24}\right)^2\left[\left(\frac{4}{5} + 16\beta^2 - \beta^4\right)\frac{\partial^2 u_i}{\partial x_l^2}\frac{\partial^2 u_j}{\partial x_l^2} + \left(\frac{6}{5} + 12\beta^2\right)\left(\frac{\partial u_i}{\partial x_l}\frac{\partial^3 u_j}{\partial x_l^3} + \frac{\partial^3 u_i}{\partial x_l^3}\frac{\partial u_j}{\partial x_l}\right)\right] \\ &+ (1-\delta_{mn})\frac{h_m^2}{24}\frac{h_n^2}{24}\left[(2+4\beta^2)\left(\frac{\partial^2 u_i}{\partial x_m \partial x_n}\frac{\partial^2 u_j}{\partial x_m \partial x_n} + \frac{\partial u_i}{\partial x_m}\frac{\partial^3 u_j}{\partial x_m \partial x_n^2} + \frac{\partial^3 u_i}{\partial x_m \partial x_n^2}\frac{\partial u_j}{\partial x_m}\right) - \beta^4\frac{\partial^2 u_i}{\partial x_m^2}\frac{\partial^2 u_j}{\partial x_n^2}\right] + O(h^6) \end{aligned} \tag{26}$$

It is not easy to handle Eq.(26) directly. For the sake of programming convenience, Eq.(26) is approximated using L_{ij}, $\hat{L}_{ij}$ and $(\hat{\bar{u}}_i - \bar{u}_i)$ as follows.

$$T_{ij}^A = \frac{1+\beta^2}{\beta^2}L_{ij} + \frac{5\beta^2 - 22}{5\beta^4}\left(\widehat{L_{ij}} - L_{ij}\right) + \frac{5\beta^2+32}{5\beta^4}(\hat{\bar{u}}_i - \bar{u}_i)(\hat{\bar{u}}_j - \bar{u}_j) \tag{27}$$

This approximate equation covers all terms of $O(h^2)$ and a part of the terms of $O(h^4)$ on the right hand side of Eq.(26), and its error is estimated as follows.

$$\begin{aligned} T_{ij} - T_{ij}^A = (1-\delta_{mn})\frac{h_m^2}{24}\frac{h_n^2}{24}\Bigg[&\frac{98}{5}\frac{\partial^2 u_i}{\partial x_m \partial x_n}\frac{\partial^2 u_j}{\partial x_m \partial x_n} + \frac{44}{5}\left(\frac{\partial u_i}{\partial x_m}\frac{\partial^3 u_j}{\partial x_m \partial x_n^2} + \frac{\partial^3 u_i}{\partial x_m \partial x_n^2}\frac{\partial u_j}{\partial x_m}\right) \\ &- \frac{32}{5}\frac{\partial^2 u_i}{\partial x_m^2}\frac{\partial^2 u_j}{\partial x_n^2}\Bigg] + O(h^6) \end{aligned} \tag{28}$$

A lower order approximation, which covers only all terms of $O(h^2)$ on the right hand side of Eq.(26), also can be given as follows.

$$T_{ij}^A = \frac{1+\beta^2}{\beta^2}L_{ij} = T_{ij} - O(h^4) \tag{29}$$

VERIFICATION OF PROPOSED MODEL

Numerical Method

The proposed model is built in the simulation code with the following specifications. The FVM is adopted for space discretization. The collocated grid system is used for the allocation of the velocity and the pressure. For the convection term, a central differencing scheme with fourth order accuracy in space and the 4-step Runge-Kutta scheme with fourth order accuracy in time are applied. For the diffusion term, the central differencing scheme with second order accuracy and the Crank-Nicolson scheme are applied. A MAC like scheme is applied for the divergence free correction of the velocity and the pressure.

Test Cases

Simulations of the fully developed plane channel flow with Re_τ=395 are performed to verify the proposed model. Here Re_τ is the Reynolds number based on the wall friction velocity u_τ and the channel half width δ. The computational domain is set up as $6.4\delta \times 2\delta \times 1.6\delta$ and periodic treatments are applied to streamwise (x) and spanwise (z) directions. Coarse grids (case 1) and fine grids (case 2) are tested. The resolutions of computational grids are summarized in Table 1.

Computational Results

Two variations of the proposed dynamic SGS model of the eddy viscosity type, *i.e.* the model with Eq.(29) that is a lower order approximation of the STS stress and the model with Eq.(27) that is a higher order approximation, are verified with β=1. Hereafter the first and the second models are named LISEA2 and LISEA4 respectively.

For comparison the Smagorinsky model with C_S=0 (NOSGS), C_S =0.1 (SM010) and C_S =0.15 (SM015) is verified. All these cases adopt the wall dumping function of the Van Driest (1956) type.

In spite of no averaging operation, all calculations of LISEA2 and LISEA4 are numerically stable. The results are shown below together with DNS results obtained by Moser *et al.* (1999).

Fig.2 shows the mean streamwise velocity profiles in the wall unit. Fig.2(a) shows the results of case 1. All models without NOSGS overestimate the velocity at the log-law region and it is because of the discretization error. The result of LISEA4 is close to the result of SM010, which is regarded as the best-tuned version of the Smagorinsky model for plane channel flow. LISEA2 gives a slightly lower velocity than the results of SM010 and LISEA4 at the log-law region.

Table 1 Resolutions of computational grids. N is a grid number and h^+ is a computational cell size in the wall unit.

case	Nx	Ny	Nz	h_x^+	h_y^+	h_z^+
1	32	64	32	79	1.1-45.8	19.8
2	64	64	64	39.5	1.1-45.8	9.9

Fig.3 shows the profiles of the velocity fluctuations. LISEA2 and LISEA4 give almost the same results. Clear differences between LISEA2,4 and SM010 are shown in Fig.3(a) and LISEA2,4 give closer results to the DNS data than SM010.

Fig.4 shows the mean energy transfer rate from the grid scale to the subgrid scale, $\langle \varepsilon^+_{\rm gs-sgs} \rangle_t$. For LISEA2 and LISEA4, $\varepsilon_{\rm gs-sgs}$ is given as follows.

$$\varepsilon_{\rm gs-sgs} = -T^A_{ij}\hat{\bar{S}}_{ij} + \nu|\hat{\bar{S}}|^2 - \nu\widehat{|\bar{S}|^2} \qquad (30)$$

For SM010 and SM015, $\varepsilon_{\rm gs-sgs}$ is give as follows.

$$\varepsilon_{\rm gs-sgs} = (C_S\Delta_g)^2|\bar{S}|^3 \qquad (31)$$

SM010 and SM015 dissipate much more energy from the grid scale than LISEA2 and LISEA4 in case 1. The difference between LISEA2 and LISEA4 is small. LISEA2 and LISEA4 give a negative value near the wall both in case 1 and case 2, and this suggests the occurrence of backscatter. But in this study the effects of backscatter are not reflected in the calculation because of the clipping treatment.

CONCLUSIONS

A new dynamic SGS stress model of the eddy viscosity type was proposed. This model applies the energy balance assumption named the "local inter-scale equilibrium" as the resolution principle of the dynamic procedure instead of the Germano identity. This model does not need an *ad hoc* averaging treatment over a homogeneous region to avoid numerical instability.

The analysis of the fully developed plane channel flow using the proposed model showed the numerical stability of the model. The present model with higher order approximation of the STS stress gave the mean streamwise velocity close to the results of the Smagorinsky model with the wall function and with C_S=0.1, which is the best-tuned version of the Smagorinsky model for plane channel flow.

REFERENCES

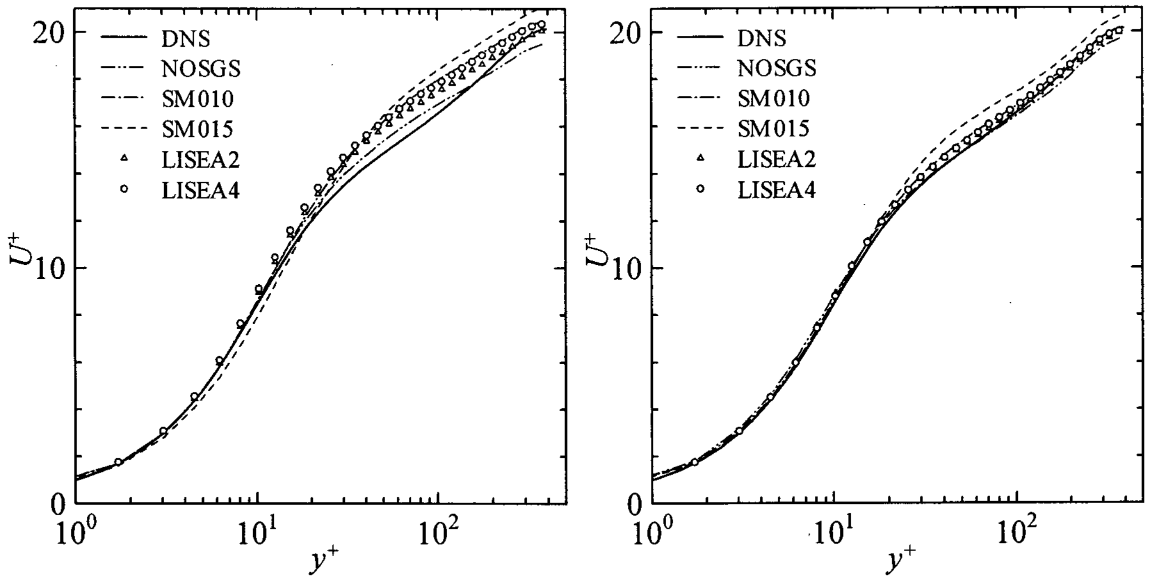
Germano, M., Piomelli, U., Moin, P., and Cabot, W. H., 1991, "A dynamic subgrid-scale eddy viscosity model", *Phys. Fluids* A 3, pp. 1760-1765.

Lilly, D. K., 1992, "A proposed modification of the Germano subgrid scale closure method", *Phys. Fluids* A 4, pp. 633-635.

Moser, R. D., Kim, J., and Mansour, N. N., 1999, "Direct numerical simulation of turbulent channel flow up to Reτ=590", *Phys. Fluids* 11, pp. 943-945.

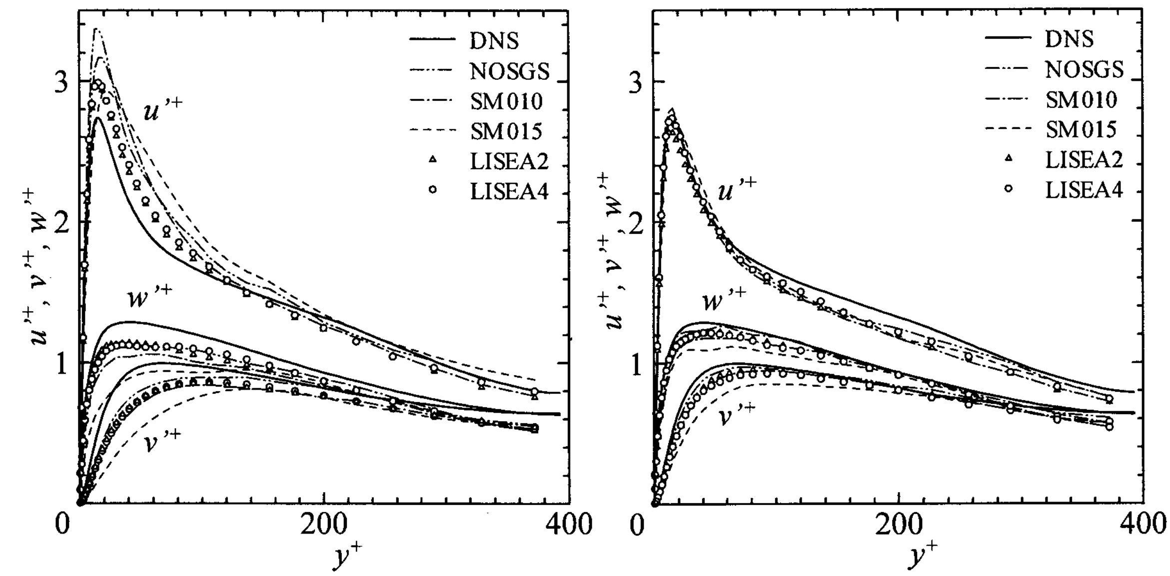
Smagorinsky, J., 1963, "General circulation experiments with the primitive equations. I. The basic experiment", *Mon. Weather Rev.* 91, pp. 99-164.

Van Driest, E. R., 1956 "On turbulent flow near a wall", *J. Aeronaut. Sci.* 23, p.1007



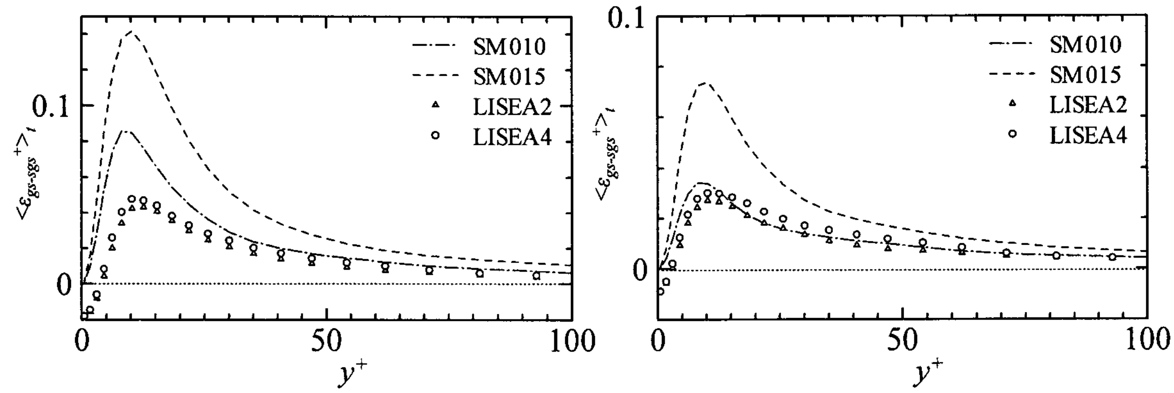
(a) Case1 (b) Case2

Fig.2 Profiles of the mean streamwise velocity



(a) Case1 (b) Case2

Fig.3 Profiles of the velocity fluctuations



(a) Case1 (b) Case2

Fig.4 Profiles of the mean energy transfer from grid scale to subgrid scale by SGS stress, $<\varepsilon_{gs\text{-}sgs}{}^+>_t$.