MODELING EQUATIONS FOR TURBULENT FLOWS WITH FREE-SURFACE FLUCTUATION

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ABSTRACT

Computation of turbulent flows with free-surface fluctuations is considered within the framework of the Reynolds-averaged equations of motion and the continuity equation. The mean position of the free surface may be either steady or chaging slowly but the instantaneous position is assumed fluctuating in space and time. The computation of the flow in changing flow domain in the framework of Eulerian Reynolds-averadged equations requires a special treatment as to the Eulerian averages as well as computation of the moving boundary. A method is proposed to incorporate the effects of randomly moving free surface in the widely used $k-\omega$ two equation turbulence modeling. Calculations are conducted and the results are compared with experimental results wherever possible. Lack of accurate data of the free-surface position and fluctuation amplitude prevents definite evaluation, but generally the employed two-equation scheme is satisfactory.

INTRODUCTION

Turbulent flows with moving boundaries such as the free surface of an open-channel flow are influenced not only by the existence of the free boundary but also by its motion both turbulent and non-turbulent. The position and the shape of the free surface are also, influenced by the turbulent motion within the flow, and they depend on each other. Turbulent flows with free surfaces must therefore be calculated along with the mean position and the fluctuation of the free surface when its movement is not small. The present work is concerned with a method of how the turbulent fluctuations of the free surface and the related statistical quantities may be modeled so that open-channel flows with significant surface movement can be calculated.

As to the models for the Reynolds stresses, the present authors (nakayama & Yokojima, 2001a) have tested recent two-equation turbulence models developed for shear flows without a free boundary to calculation of open-channel flows, both near equilibrium and widely deviated from equilibrium but the with small variations of the position and the shape of the free surface. One result of that study is that the $k-\omega$ model modified for free-surface effects was found to do well for various flows including cases with rough surfaces. We use the same model in computing open channel flows that involve large changes of the shape and the position of the free surface both in the time-averaged and instantaneous flows. Surface waves of large amplitudes induce strong local acceleration and deceleration and the streamlines may have large curvature, demanding flexible and generally applicable turbulence models. The details of these rapidly varying flows are not easily analyzed in practical engineering problems. We propose a method based on the Reynolds averaged equation with the free surface represented by a single-valued continuous function of the horizontal coordinates. There are more elaborate methods that allow discontinuous and even disperse interfaces that are being developed, but the performance of the overall numerical method including the turbulence models must first be examined with simpler but sufficiently accurate methods such as the one used in the present work. The test flows we consider are those over a sudden drop and those over trenches with a wide range of Froude numbers. These are the flows that have been experimentally investigated by the latest Particle Image Velocimetry (PIV) techniques and accurate data of mean and turbulence quantities are available even in recirculating regions.

EQUATIONS FOR FLUCTUATING FREE SURFACE

The definition and the representation of the moving interface may be done in a few different ways. Elaborate methods are needed for flows that involve a wildly moving interface with loquid drops dispersing or gas bubbles entrained. However, for open-channel flows where fluid is assumed only on one side of the interface, the orientation of the free surface is nearly horizontal, and it can be conveniently represented it by its vertical position, $x_2 = \tilde{h}(x_1, x_3, t)$, where x_1, x_3 are the horizontal coordinates, x_2 is the vertical coordinate and $\tilde{h}$ is the instantaneous height of the free surface. With this notation, $\tilde{h}$ satisfies the following equation:

$$\frac{\partial \tilde{h}}{\partial t} + \tilde{u}_\alpha\Big|_{\tilde{h}} \frac{\partial \tilde{h}}{\partial x_\alpha} = \tilde{u}_2\Big|_{\tilde{h}} \tag{1}$$

where $\tilde{u_i}$ is the instantaneous velocity component in the x_i direction, $\Big|_{\tilde{h}}$ means the value at $x_2 = \tilde{h}$ and the Greek indecies imply summation over the indecies for the horizontal directions x_1 and x_3 only. It is a kinematic equation requiring that the fluid particles on the free surface stay on it. In the method of calculation based on the Reynolds-averaged equations, we solve for the averaged quantities within the averaged flow domain. This means that the average position of the free surface $x_2 = H = \overline{\tilde{h}}$ must also be solved by taking the average of the equation for $\tilde{h}$. The average of the nonlinear term on the left-hand side of this equation is like a Lagrangian correlation since the position of the velocity component $\tilde{u}_\alpha$ moves with the free surface. In our computation with Eulerian variables we approximate as follows

$$\begin{aligned} \tilde{u}_\alpha\Big|_{\tilde{h}} &= \tilde{u}_\alpha\Big|_H + h\frac{\partial \tilde{u}_\alpha}{\partial x_2} \\ &= U_\alpha\Big|_H + u_\alpha\Big|_H \end{aligned} \tag{2}$$

where $h = \tilde{h} - H$, $U_\alpha = \overline{\tilde{u}}_\alpha$ and $u_\alpha = \tilde{u}_\alpha - U_\alpha$. The second term on the right hand side in the first line is dropped since it is zero or very close to zero on the shear-free surface. Using this, the Reynolds average of Eq.(1) becomes

$$\frac{\partial H}{\partial t} + U_\alpha\Big|_H \frac{\partial H}{\partial x_\alpha} = U_2\Big|_H - \frac{\partial}{\partial x_j}\overline{u_j h}\Big|_H \tag{3}$$

It is noted that in this equation that the term containing the correlation between the fluctuating velocity component and the fluctuating depth of the free surface appears. This term is a contribution of the surface fluctuation to the net mean mass flux. It has been found experimentally (Nezu & Nakayama, 1998) that the correlation between velocity components and the surface position are nonzero in fully developed open channel flow and in noniform flows its streamwise gradient can have nonzero values. Similar equation has been used by Hodges & Street(1999) in the LES calculation of flows with nonlinear waves.

Generally the above correlation terms need to be directly modeled or calculated by a modeled transport eqaution. One way is to first calculate the magnitude of the surface fluctuation $\overline{h^2}$. Its transport equation can be obtained in a similar way as that for turbulent kinetic energy and the results is

$$\frac{\partial \overline{h^2}}{\partial t} + U_\alpha\Big|_H \frac{\partial \overline{h^2}}{\partial x_\alpha} = -2\overline{u_\alpha h}\Big|_H \frac{\partial H}{\partial x_\alpha} - \frac{\overline{u_j h^2}}{\partial x_j}\Big|_H + 2\overline{u_2 h}\Big|_H \tag{4}$$

The equations for the velocity-elevation correlations may be used and can be derived but it is too much a complication as the present stage without a definite need for it.

TURBULENCE MODEL

The test calculations by the present authors using various two-equation models in calculation of various open-channel flows indicated that the low-Reynolds number $k - \omega$ model worked uniformly for a few test flows with small variations of the free surface. It is a two-equation model with the linear eddy viscosity whose defficiency is that the Reynolds normal stresses that become important in flows with complex mean-flow strain field are not well represented. Also the history and nonlocal effects that most turbulence has are not accounted for. The most important aspect is that the free surface acts as a turbulence damper. It is stronger for low Froude number but is weaker as the free surface looses regidity as the Froude number increases. In view of this damping effects of the free surface, we use the eddy viscosity ν_t as given by the original $k - \omega$ model but it is modified near the free surface to account for the damping effects as the amplitude of the free surface fluctuation as a parameter

$$\nu_t = \alpha^* f_s \frac{k}{\omega} \tag{5}$$

where α^* is the parameter used in the original $k - \omega$ model, and f_s is a damping factor effective only near the free surface.

$$f_s = 1 - exp\left(-C_h\frac{h'}{L_s}\right) exp\left(-C_h\frac{y'}{L_s}\right) \tag{6}$$

where h' is the RMS fluctuation of h, y' is the vertical distance from the free surface, L_S is the turbulent length scale at the mean free surface defined in the present work by

$$L_s = 0.75 k_S^{1/2}/\omega_S \tag{7}$$

where k_S and ω_S are the values of k and ω at $x_2 = H$. h' in a fully-developed flow is a function of the Froude number. It is based on the experimental observation and the examination of the DNS data that the eddy viscosity and turbulence production are reduced near the free surface over distances scaled by the turbulent scale, and that the degree of damping is more for small Froud numbers with smaller fluctuation h'.

It has been estimated by Borue et al.(1995) but it is much smaller than experiments of Nezu & Nakayama(2001) or Direct Nymerical Simulation (DNS) of Nakayama & Yokojima(2001b). We use a relation based on these measurements and DNS data

$$\frac{h'}{H} = Fr^2 \frac{U_S\sqrt{k_S}}{U_m^2} \tag{8}$$

where the subscript S is used for the values on the free surface, and Fr is defined by the mean velocity U_m and the mean depth H.

In the general case of spacially varying flows, h' must be computed via Eq.(4). Here we show a method of modeling the terms in this equation. The correlations between veloity components and the surface fluctuation appearing may be done by a gradient-diffusion type model, as has been done by Shen & Yue (2001) in a large-eddy simulation. Based on this we have

$$-\overline{u_\alpha h}\Big|_H = \frac{\nu_t\big|_H}{\sigma_{h1}} \frac{\partial H}{\partial x_\alpha} \tag{9}$$

and

$$-\overline{u_\alpha h^2}\Big|_H = \frac{\nu_t\big|_H}{\sigma_{h2}} \frac{\partial \overline{h^2}}{\partial x_\alpha} \tag{10}$$

where σ_{h1} and σ_{h2} are constant, presently set equal to 1. The last two terms involving the gradients in the vertical direction may be modeled by the concept of relaxation from the fully-developed equilibrium flow so that we have

$$\begin{aligned} \frac{\partial \overline{h^2}}{\partial t} + U_\alpha\Big|_H \frac{\partial \overline{h^2}}{\partial x_\alpha} &= -2\frac{\nu_t\big|_H}{\sigma_{h1}}\left(\frac{\partial H}{\partial x_\alpha}\right)^2 \\ &- \frac{\partial}{\partial x_\alpha}\left(\frac{\nu_t\big|_H}{\sigma_{h2}} \frac{\overline{u_j h^2}}{\partial x_j}\right) \end{aligned}$$

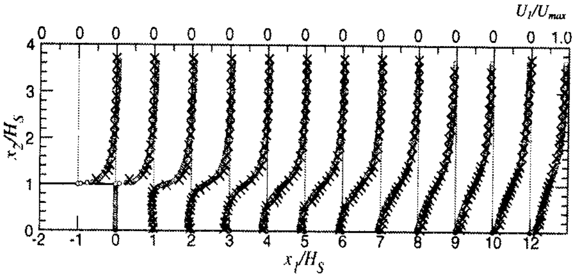

Figure 1: Mean velocity profiles in subcritical flow over backward-facing step compared with experiment. o; calculation; ×; experiment(Nakagawa & Nezu (1982)

$$- \frac{C_e}{\tau_S}\left(\overline{h^2} - Fr^4 \frac{U_S^2 k_S}{U_m^4} H^2\right) \quad (11)$$

where C_e is additional model constant, also presently set equal to 1 and τ_S is the time constant at the free surface and is $1/C_\mu \omega_S$.

NUMERICAL METHOD

All transport equations are solved in time-developing form by starting computation from an assumed initial state. This allows calculation of possible unsteadiness of the real flow that is of larger time scale than that of turbulence. This is important in the present calculation of high-speed flows in which generally large surface waves are generated if motion is started from a horizontal free surface which needs to be calculated in time-advancing manner.

The variables are descretized using the staggered grid arranged on the rectangular coordinates. The free surface position H is defined at the center of the vertical grid-lines. Time advancement for all velocity components, the free surface position H, k and ω are done by the second-order Adams-Bashforth method, in which the viscous and turbulence terms are evaluated by the second-order central difference formula and the convective terms are calculated using the third-order upwind differencing scheme. H is calculated assuming that the mean velocities are known at the new time step. Calculation of $\overline{h^2}$ is done with velocities and H all known. The pressure is calculated by the HSMAC procedure of simultaneously correcting the velocity components and the pressure. The boundary condition for pressure is not needed in the HSMAC procedure, and the Dirichlet condition for pressure at the free surface is enforced by setting the pressure at the points closest to the free surface to be equal to the hydrostatic pressure there. The method is similar to the one used by Nakayama & Yokojima(2003).

CALCULATION RESULTS

Subcritical flow over backward-facing step

The calculation method is first applied to a low Froude-number subcritical flow over a backward- facing step. The step height H_S is about 1/4 of the flow depth at the step. This flow is very close to internal flow past a step in a channel with small change in the surface elevation making it very close to horizontal. For this flow, Nakagawa & Nezu(1982) made an accurate measurement of the free surface and the calculation results for it can be verified. Fig.1 shows the calculation results of the mean velocity distribution compared with the measurement. The results are normalized by the

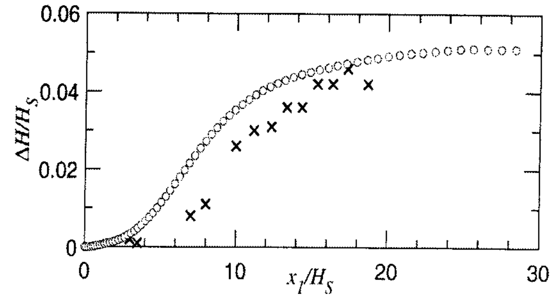

Figure 2: Free surface elevation increment, ΔH downstream of backward-facing step. o; calculation; ×; experiment (Nakagawa & Nezu(1982)

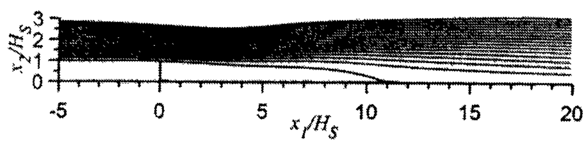

Figure 3: Computed mean streamlins in high-Froude-number flow over a backward-facing step.

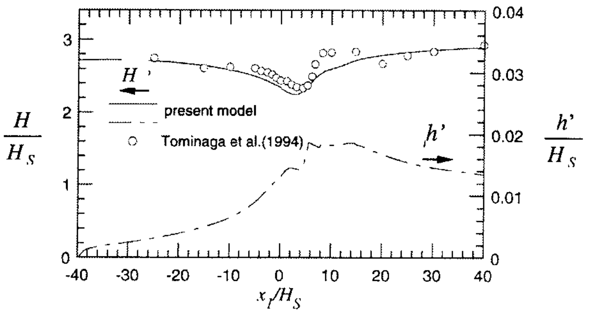

Figure 4: Free surface elevation and surface fluctuation in High-Froude number flow over a backward-facing step.

maximum velocity U_{max} at the step. It is seen that the calculation is in good agreement with the measurement. Fig.2 is a comparison of the free surface elevation represented as the difference ΔH from the elevation at the position of the step. The calculation tends to be slightly larger than the measurements, but the magnitudes are too small to make a definite statement.

A larger Froude-number flow in which the free surface varies by larger magnitudes has been measured by Tominaga et al.(1994). The step height is more than 1/2 of the flow depth at the step and in this case the flow accelerates just downstream of the step and the free surface dipps a little then increases like a weak hydraulic jump. There is a unsteady free-surface fluctuation in this region. The present method has also been applied to this flow. Fig.3 shows the mean flow calculation results in terms of the streamlines. The reattachment point reported by Tominaga et al.(1994) is at about $x_1/H_S = 10$ and the calculation result is very close to it. The detailes of the free surface elevation and the intensity of the fluctuation is shown in Fig.4. H agrees very well except the sharp free surface rise near the reattachment point is calculated slightly milder. There is no quantitative data of the free-surface fluctuation, but Tom-

Table 1: Parameters for flow over stepped slope.

discharge Q (l/s)	13.0
depth midpoint between steps H_1(cm)	2.9
$Re = U_m H_1/\nu$	32500
$Fr_1 = U_m/\sqrt{gH_1}$	2.10
mean velocity U_m (cm/s)	11.2
maximum U_{max} (cm/s)	120.0
step height H_S (cm)	1.0

inaga et al.(1994) comments that the flucuation was large near the reattachment point. The present calculation shows this trend also.

Supercritical flow past stepped slope

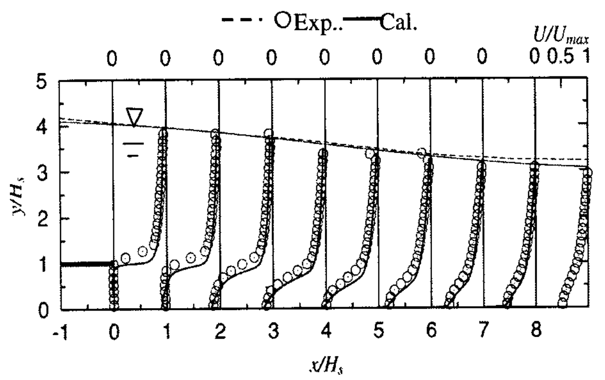

Figure 5: Mean velocity profiles and the free-surface elevation in stepped slope flow.

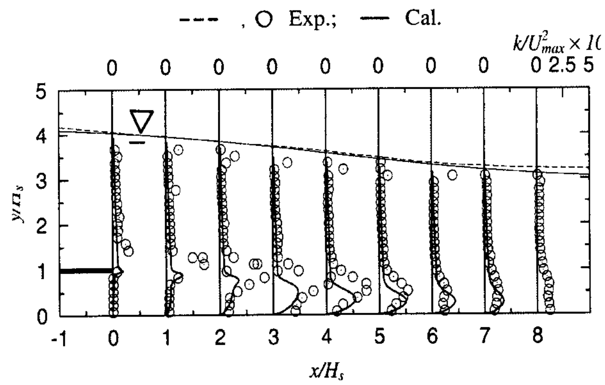

Figure 6: Turbulent kinetic energy distribution in stepped slope flow.

Next calculation case is a high-speed supercritical flow. Recently Ohmoto et al.(2001) made measurements using the PIV technique in a supercritical flow over a series of steps. The ratio of the step height H_S to the horizontal channel distance is 1/100 and there are ten steps in the experimental channel. The flow is steady and around the last few steps the flow could be considered fully developped and periodic with respect to the length between consecutive steps. The experimental conditions are shown in Table 1. The Reynolds number Re and the Froude number Fr_1 are defined by the cross sectional mean velocity U_m and the mean depth H_1 at a position between two steps. The measurements are made around the step that is located near the end of the channel and the flow over this step may be considered fully developed and is nearly periodic except for the head drop of 1cm over one step. Calculation is done from 50 step heights upstream and 50 step heights downstream of the step. A fully-developed uniform-flow profiles for the velocity and equilibrium profiles for k and ω at the inflow plane, and horizontal free surface were assumed initially. After the large waves generated by the initial stage of the flow development have translated past the downstream boundary, the profiles at the downstream end are equated to the inflow profiles assuming periodicity with the period equal to the distance between the steps.

Table 2: Parameters for flow over a drop with a trench.

discharge Q (l/s)	0.00227
$Re = U_m H_1/\nu$	7590
$Fr_1 = U_m/\sqrt{gH_1}$	0.889
unstream mean velocity U_m (cm/s)	38.9
step height H_S (cm)	2.0
depth of trench H_d (cm)	2.0
length of trench L (cm)	6.0

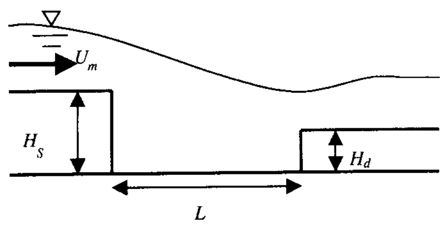

Figure 7: Sckematic of flow over a drop with a trench.

Fig.5 shows the profiles of the mean streamwise velocity component together with the free-surface elevation after the steady-state has been reached. First, the free-surface position and the shape are very well predicted by the present method. The velocity profiles appear a little off the experimental profiles but it is seen to be due to a slight shift at the step edge where the optical measurement may be influenced by the step itself. The reattachment point is predicted very well as well.

Fig.6 is a comparison of turbulent kinetic energy distribution. Here we see that the calculation underpredicts considerably downstream of the step. Particularly the shear layer coming off the step corner is seen to spread below the step while the measurement indicates larger spread above it. Some erratic-looking values near the surface is not also predicted.

Flow over a drop with trench

The next case is a flow over a drop with a trench at the bottom of the drop measured by Fujita & Maruyama (2001). Skematic of this flow is shown in Fig.7. This is a new type of falling work that is proposed to implement in steep sloped channels running in urban areas located in a hilly region. It will work as an energy dissipater and at the same time will provide a better aquatic space. It has been found experimentally that depending on the ratio of the length to depth of the trench, the flow can be very different and can exhibit distinctively periodic nature. In the computation we take the case in which the free surface deforms significantly but the unsteadiness does not set in. The experimental condi-

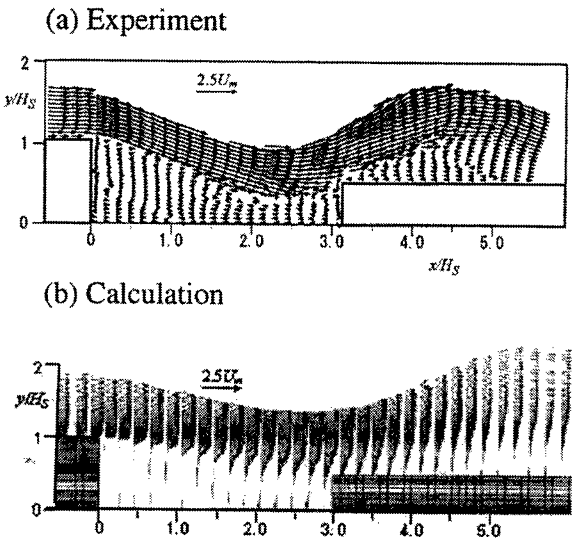

Figure 8: Sckematic of flow over a drop with a trench.

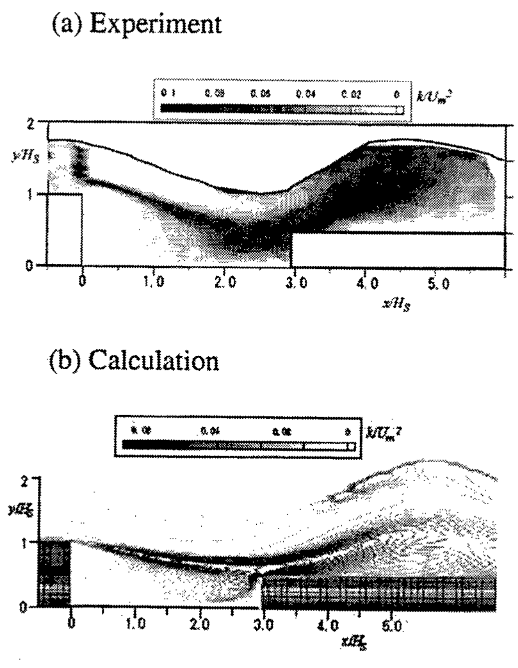

Figure 9: Sckematic of flow over a drop with a trench.

tion is shown in Table 2. Since the test flume has sufficient length before the drop where measurements are taken, the flow approaching the drop is assumed to be fully developed in the calculation. The calculation is conducted for the region from $90H_S$ upstream to $105H_S$ downstream from the drop.

In this case, the flow developes to almost the uniform flow at the position of the drop. Therefore, in this case also the inflow is self generated. The calculated mean velocity distribution is compared with the experiments in Fig.8. It is seen that the overall pattern of the flow is computed very well. The main difference is in the flow over the downstream step. The calculated flow here shows small back flow somewhat downstream of the step, while the experiment shows a strong back flow right at the corner. One reason for this is the difference in the flow depth at the top of the upstream step. In the calculation the depth is larger and hence the separated flow does not drop as much and the bulk of the flow reattaches on the downstream channel just past the trench. The main flow running off the step in the measurements, however, impinges on the upper corner of the forward step that cause a second separation.

The turbulent kinetic energy distribution is compared in Fig.9. Here only the distribution is shown by the darkness of the shades. Again the overall pattern is seen to be well predicted, but the magnitudes and detailed distribution do not agree well with the measurement. It has been stressed that the free surface has damping effects that cannot be neglected. Here we have a free surface that rises and falls sharply causing strong acceleration and deceleration along with large streamline curvature. These distortions are largely due to an irrotational wave motion. The eddy-viscosity models are known to predict incorrect turbulence production in an irrotational strain field. This calculation case demonstrates importance of modeling of this aspect. For further improvements, the effects of the fluctuation of the free surface should be treated in more direct way. The transport equation for the second-order correlation terms should then be modeled. The surface fluctuation has not been computed for this case, in which the changes are too large to make adequate calculation with the present equation for h^2.

CONCLUSIONS

Computation of turbulent flows with free-surface fluctuations has been done within the framework of the Reynolds-averaged equations of motion and the continuity equation. The mean position of the free surface may be either steady or chaging slowly but the instantaneous position is assumed fluctuating in space and time. The computation of the flow in changing flow domain in the framework of Eulerian Reynolds-averadged equations requires a special treatment as to the Eulerian averages as well as computation of the moving boundary. A method of computing the time average and the amplitude of fluctuation of the free-surface position had been presented and used to compute various turbulent open channel flows including rapidly varying flows. The basic turbulence model used is built around the widely used $k-\omega$ two equation turbulence modeling. Calculations results indicate generally good, particularly the high-speed flows that have not hitherto been computed well with differential methods are well predicted.

While a lack of accurate data of the free-surface position and fluctuation amplitude prevented detailed evaluation of the calculation of the free-surface fluctuations, but they can be made as experimental data and DNS data become available.

REFERENCES

Borue, V. , Orszag, S. A. and Staroselsky, I., 1995 "Interaction of surface waves with turbulence: Direct numerical simulation of turbulent open-channel flow," *J. Fluid Mech.*, Vol.286, pp.1-23.

Fujita, I. and Maruyama, T., 2001 "Hydraulic characteristics of open-channel flow downstream of a falling works with a trench," *Annual J. Hydraulic Eng.*, JSCE, Vol.45, pp.403-408.

Hodges, B. R., and street, R. L., 1999, "On simulation of turbulent nonlinear free-surface flows,", *Comp. Physics*, Vol.151, pp. 425-457.

Nakagawa, H. and Nezu, I., 1982 " Experimental investigation on turbulent sdtructure of backward-facing step flow in an open ahcnnel," *J. Hydraulic Reserch*, Vol.25, pp.67-88.

Nakayama, A. and Yokojima, S., 2001(a) "General applicability of low-Reynolds-number two-equation turbulence models as applied to the prediction of open channel flows," *J. Hydraulic, Coastal and Environmental Eng.*, JSCE, No.628/II-56, pp.141-153.(in Japanese)

Nakayama, A. and Yokojima, S., 2001(b) "Direct numerical simulation of the fully developed open-channel flow at subcritical Froude numbers," DNS/LES Progress and Challenges, Third AFOSR International Conference, Arlington, Texas, pp.569-576.

Nakayama, A. and Yokojima, S., 2003 "Modeling Free-Surface Effects for Calculation of Turbulent Open-Channel Flows," *Env. Fluid Mech.*, Vol.3, No.1, pp.1-21.

Nezu, I. and Nakayama, T., 1998 "Relationship between turbulent structures near the free-surface and surface-wave-fluctuations," *J. Hydraulic, Coastal and Environmental Eng.*, JSCE, No.593/II-43, pp.69-78.

Ohmoto, T., Nariai, N. and Yakita, K., 2001 "Turbulent flow structure of separated shear layer in stepped open channel," *J. Applied Mech.* JSCE, Vol.4, pp.541-548.

Shen, L. and Yue, D. K. P., 2001 "large-eddy simulation of free-surface turbulence," *J. Fluid Mech.*, Vol.440, pp.75-116.

Tominaga, A., Nezu, I., Liu, K. and Kamiya, M., 1994 "Numerical calculation of open-channel flow over abrupt change in the bed shape taking into account the free-surface variation," *Proc. 8th Symp. on Computational Fluid Dynamics*, pp.337-340. (in Japanese)

Wilcox, D. C. 1998, Turbulence modeling for CFD(2nd ed.), DCW Industries, La Canada, California.