# LES ANARYSIS ON DISPERSION IN SURFACE LAYER OVER RANDOMLY ARRANGED ROUGHNESS BLOCKS

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## ABSTRACT

We perform Large Eddy Simulation (LES) for analysis of the dispersion in the spatially developing turbulent boundary layer over uniformly and randomly arranged roughness blocks. The objective of this paper is to pursue the detailed characteristics of turbulence structures and dispersion process in the surface layer over the roughness blocks with various types of arrangement on the ground. We often encounter the complex arrangement of roughness such as buildings in urban area, but almost studies have dealt with uniform arrangement. Especially it can be considered that the dispersion behavior is much influenced by the pattern of arrangement. Two LES cases are carried out. One is an uniform (staggered) array for roughness blocks. The other is a random array for them in a horizontal plane without changing averaged height. But each roughness block has a different height. We obtain the profile of turbulent statistics in a statistical manner in order to avoid local effects near the roughness blocks. LES of concentration fluctuations are carried out over smooth wall at first to validate the numerical model by comparing with Fackrell's experimental results and Sykes's LES analysis data, then are done over rough wall. Consequently, near the roughness height, touchdown in the random array is earlier than that in the uniform (staggered) array because of the strong local effect in the random case. On the other hand, in the upper part, although there is no clear difference between velocity fields of uniform and random arrays, a great difference of mean as well as fluctuating concentration between both cases arises.

## INTRODUCTION

For the estimation of urban air quality, it is important to predict turbulent flows over buildings and structures, which are densely arranged in the urban area, and to clarify the dispersion process in the surface layer over very roughened surface. Especially we have to bring into focus unsteady phenomena of air pollution close to the ground surface, because the air pollutants are mainly emitted from urban canopies formed at the center of city where there exist many types of buildings or transportation systems. For detailed investigation for fluctuating characteristics of diffusion from an environmental point of view, LES technique is very expecting as a numerical approach. Also, for LES of surface layer, it is required to install the configuration of roughened surface in actual city to numerical model. While, recent study makes it possible to numerically simulate the spatially developing turbulent boundary layer over a rough surface by using the pseudo-periodic boundary conditions based on the re-scaling technique for inflow and outflow of the computational domain (Nozawa and Tamura, 2001). This technique realizes much reduction of computational power and memories by the small computational region. This paper applies the LES technique of rough-wall turbulent boundary layer to the spatially developing turbulent boundary layer over randomly arranged roughness blocks. In the turbulent flows formed at near-ground region of such an urban-like area, dispersion characteristics of plumes emitted from a point source is discussed, including concentration fluctuations. Especially the effects of arrangement pattern for roughness blocks on the dispersion process as well as the turbulence characteristics are examined by using the numerical model for rough surface which is formulated by actually placing rectangular blocks on the flat plate. Useful information might be provided for predicting the dispersion behavior in actual urban area with various shapes of roughness blocks (buildings, artificial structures or trees).

## PROBREM FORMULATION

### Governing equations for Large Eddy Simulation

The turbulent flow over roughness blocks is simulated with large eddy simulation (LES). The governing equations are given by the continuity, the incompressible Navier-Stokes equations and the scalar (concentration) evolution equation as follows.

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial \bar{u}_i}{\partial t}+\frac{\partial \overline{u_i u_j}}{\partial x_j}=-\frac{\partial \bar{p}}{\partial x_j}+\frac{\partial}{\partial x_j}\left[\frac{1}{\mathrm{Re}}\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)-\tau_{ij}\right] \quad (2)$$

$$\frac{\partial \bar{c}}{\partial t}+\frac{\partial \overline{c u_i}}{\partial x_i}=\frac{\partial}{\partial x_i}\left[\frac{1}{\mathrm{Re}}\frac{1}{Sc}\left(\frac{\partial \bar{c}}{\partial x_i}\right)-\xi_i\right] \quad (3)$$

where $t,u,p,c$, $Re$ and $Sc$ denote time, velocity, pressure, concentration, the Reynolds number and the Schmitt number respectively. $\bar{u}_i=(\bar{u},\bar{v},\bar{w})$ are the filtered components of the velocity vector. The quantities $\tau_{ij}$ and $\xi_i$ are the subgrid stress and the subgrid concentration flux, respectively as follows.

$$\tau_{ij}=\overline{u_i u_j}-\bar{u}_i\bar{u}_j \quad (4)$$

$$\xi_i=\overline{c u_i}-\bar{c}\bar{u}_i \quad (5)$$

The Smagorinsky model with Van-Driest type is employed for the subgrid closure model as follows.

$$\tau_{ij}-\frac{1}{3}\delta_{ij}\tau_{kk}=-2\nu_e\bar{S}_{ij} \quad (6)$$

$$\nu_e=(C_s\Delta)^2(2\bar{S}_{ij}\bar{S}_{ij})^{1/2} \quad (7)$$

$$\xi_i=-\frac{\nu_e}{Sc_t}\frac{\partial \bar{c}}{\partial x_i} \quad (8)$$

The model coefficient $C_s$ and the turbulent Schmitt number $Sc_t$ set to be 0.1 and 0.5, respectively. At the current stage, we use a basic subgrid closure model, because this model brings about reasonable results for boundary-layer type of flows according to the previous paper (Nozawa and Tamura, 2001). However, focusing on local turbulence structures in the near wake, there remains a discussion about the necessity of the more sophisticated model such as the dynamic model so on.

## Numerical model for spatially developing rough-wall turbulent boundary layer

Spalart's method (Spalart and Leonard, 1985) or its simplified version - Lund's method (Lund et al, 1998) is effective to simulate the turbulent boundary layer over a smooth wall. In Lund's method, the velocity field at a downstream station is rescaled, and then reintroduced as a boundary condition at the inlet, to allow for the calculation of spatially developing boundary layer in conjunction with pseudo-periodic boundary conditions applied in the streamwise direction. The parameters in the rescaling procedure, $\gamma(u_\tau(inlt)/u_\tau(recy))$ and $\beta$ ($\theta(inlt)/\theta(recy)$), are not independent of each other, and Lund gave the relation between them from Blasius's equation which is valid for smooth wall. When the rough-wall boundary layer is simulated, the relation between $\gamma$ and $\beta$, should be re-deduced (Nozawa and Tamura, 2001). Here, we employ the relation between $\gamma$ and $\beta$ from Eq. (9). To solve Eq. (9), the empirical relation obtained experimentally by Prandtl is used. Figure 1 shows the schematic for simulating rough-wall turbulent boundary layer.

$$\frac{\theta_{recy}}{\theta_{inlt}}-1=\frac{c_f'(x)}{2\theta_{inlet}}\frac{x}{1-r}\left\{\left(\frac{u_{\tau inlet}}{u_{\tau recy}}\right)^{\frac{2r-2}{r}}-1\right\} \quad (9)$$

$$r=\frac{3.95}{2.87+1.58\log(x/k_s)} \quad (10)$$

where, $c_f'$ is local skin friction coefficient, $k_s$ is the equivalent sand roughness and $x$ is the distance from the leading edge of the surface.

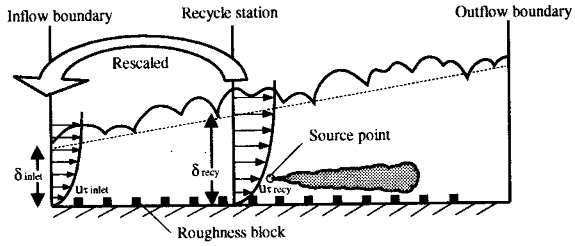


Fig.1 Numerical model for rough-wall turbulent boundary layer

## Numerical discretization and boundary conditions

For the numerical procedure a fractional-step method is used. Concerning time advancing of the momentum equation (2), the Adams-Bashforth method and the Crank-Nicolson method are employed for the convection terms and diffusion terms. Basically the spatial derivatives are approximated by the second-order central difference. On the other hand, the diffusion equation (3) is advanced in time by the Adams-Bashforth method. In order to avoid the numerical instability near the source point of diffusion, a third-order upwind scheme is used for the convection term. The computational domain is set to be $5.4\delta_0$ *$2.5\delta_0$ *$2.1\delta_0$, with corresponding grid numbers of 150, 90 and 120 in the streamwise, wall-nomal and spanwise directions(see Fig.2).

The boundary conditions are assumed to be as follows:

Bottom surface and block surfaces:
No slip condition for velocity and zero-gradient condition for pressure and concentration;
Top surface:
Free slip condition for stremwise and spanwise component of velocity, $v=u_\infty d\delta^*/dx$ for wall-nomal component of velocity and zero-gradient condition for pressure and concentration;
Spanwise:
Periodic condition for velocity and pressure and zero-gradient condition for the concentration;
Outflow boundary:
Convective boundary condition of form $\partial/\partial t+c\partial/\partial x=0$ is applied for velocity and zero-gradient condition for pressure and concentration;
Inflow condition:
The inflow condition is presented in a previous section.

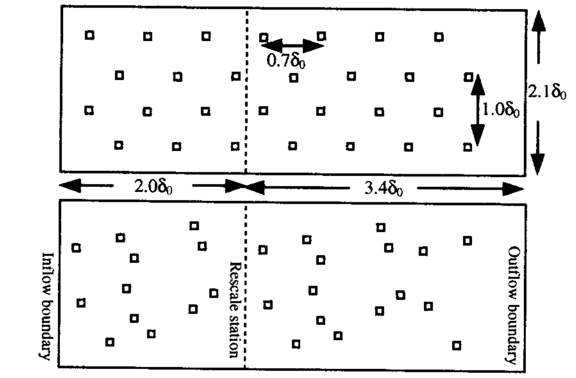


Fig.2 Computational domain and distribution of roughness blocks, upper: uniformly arranged roughness blocks; lower: random arranged roughness blocks

## Modeling for boundary surface with roughness blocks

A method proposed by Goldstein (1993) is used to impose the no-slip boundary condition at surfaces of a roughness element. Goldstein provided Eq. (6) for the stable condition of $\Delta t$ and the parameters of oscillation equation, $\alpha$ and $\beta$, when the time marching of the forcing term is done with a second order accurate Adams-Bashforth scheme.

$$\Delta t < \frac{-\beta - \sqrt{(\beta^2 - 2\alpha k)}}{\alpha} \qquad (6)$$

where, $k$ is a problem dependent constant of order one. In our case, it is appropriate to set $k$ equal to 1. $\alpha$ and $\beta$ are selected so that damping ratio derived from $\alpha$ and $\beta$ equals 1, that is to say, to reach the critical damping condition. In this way, the right hand side of the equation (6) can be prescribed as one parameter. $\Delta t$ can be decided from numerical stability of flow computation. We have to give one parameter to satisfy Eq. 6. For faster convergence of an oscillation system, we should decide one parameter to minimize the right hand side of the equation (6) as small as possible.

## TURBULENCE STRUCTURES IN SURFACE LAYER OVER ROUGHNESS BLOCKS

In this paper, two cases of LES are carried out to investigate turbulence structures which are affected by the arrangement patterns of roughness blocks, including a densely-scattering pattern like urban area. The first case is an uniform (staggered) array for roughness blocks, which is called UF case hereafter. The second case is a random array for them in a horizontal plane without changing average height, which is called RD case hereafter. But each roughness block has a different height. (see Fig.2 and Fig.3) In two cases, roughness density is fixed, $1.3*10^{-3}$. The Reynolds number based on friction velocity and boundary layer height is 2200. Ratio of roughness-block height to boundary layer thickness is about 0.05. Figure 4 shows an instantaneous streamwise velocity in RD case. Bulge-like flow structures can be recognized at upper region of turbulent boundary layer, so the present numerical model can produce reasonable results for spatially developing type of boundary layer.

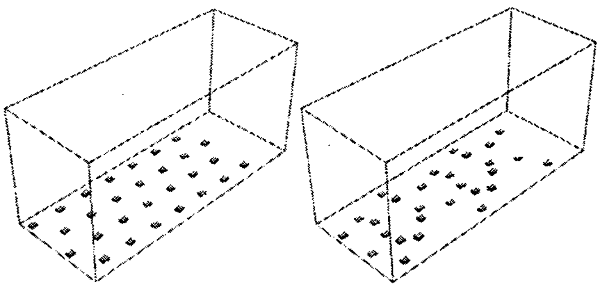

Fig.3 A bird's-eye view of the roughness blocks, left: UF case; right: RD case

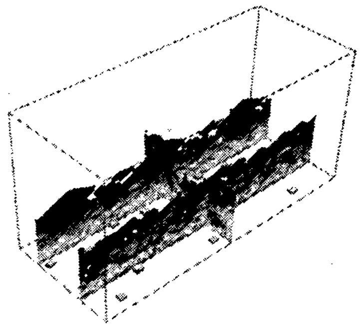

Fig.4 Instantaneous streamwise velocity in RD case

## Mean profiles and $z_0$ estimation

Figure 5 shows mean velocity profiles normalized by friction velocities $u_\tau$. The horizontal axis is the height $y$ shifted with zero displacement height $d$. Both values of $u_\tau$ and $d$ are deduced from the Reynolds stress profiles (Raupach, 1981). Since the present LES can compute the wake flow of roughness blocks without any modeling, the profiles of turbulent flow statistics are much influenced locally by the roughness blocks. Now we propose to obtain the profile of turbulent statistics in a statistical manner. First, a moving average in the streamwise direction is performed. Secondly, all cases at each representative point are plotted together on a graph. According to Fig.5 and Fig.7, it is possible to estimate a deviation of turbulent flow statistics.

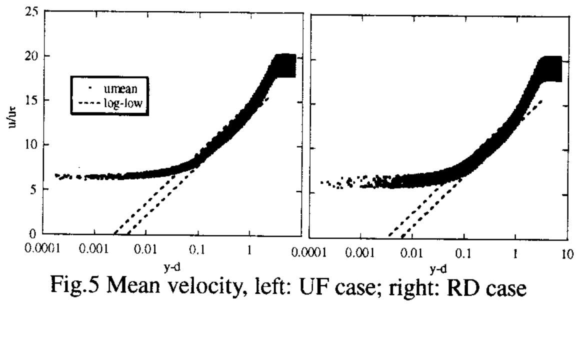


Fig.5 Mean velocity, left: UF case; right: RD case

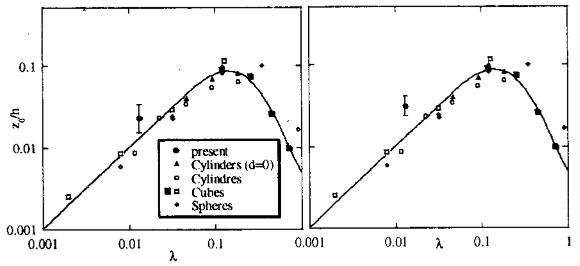

Fig.6 Comparison of normalized roughness length as a function of roughness density with Raupach's data, left: UF case; right: RD case

Roughness length $z_0$ can be estimated by extrapolation of mean profiles. Figure 6 show normalized roughness length as a function of roughness density with experimental data collected by Raupach (1991). The present results for uniformly-arranged roughness blocks agree with the experimental data better than those for the randomly -arranged roughness blocks. By now, it is unknown whether Raupach's result, that the roughness length depends only on roughness density and roughness height, can be extended to the random roughness case to some extent or not. A future examination will be required.

## Turbulence statistics

Figure 7 shows fluctuating velocities and Reynolds stress profiles normalized by $u_\tau$. Raupach's data are also displayed for comparison (Raupach, 1991). Although different symbols were used for different roughness density in Raupach's paper, only a symbol of round mark is used to grasp a tendency and a scatter of a present data qualitatively. The computed fluctuating velocities are overestimated in comparison with the experimental data beneath $(y\text{-}d)/\delta = 0.1$. In this region, it can be thought the roughness height is an important factor to determine turbulence characteristics. Our roughness heights are $(y\text{-}d)/\delta = 0.035$ for UF case, $(y\text{-}d)/\delta = 0.033$ for RD case, while are $0.008 < (y\text{-}d)/\delta < 0.052$ for five roughness types of Raupach's data. Raupach's data show fluctuating velocities sift slightly to high values as the roughness height increases. Nevertheless, the computed fluctuating velocity would be large. This is due to discrepancy of a flow near the roughness blocks at different *Re* number or deformation of turbulence structures by using pseudo -periodic boundary conditions.

Next let us enumerate discrepancies between UF case and RD case. First of all, we bring into focus the region beneath $(y\text{-}d)/\delta = 0.1$. In comparison with UF case, the streamwise fluctuating velocities in RD case are much scattered up to a higher point. It means that the roughness layer has reached to the upper part in RD case. The thickness of roughness layers in UF case and RD case are roughly estimated, $1.7h$ and $3h$, respectively, where $h$ is the averaged roughness height. We can recognize the same results about scattered phenomena from vortical structures. Figure 8 shows mean vorticity contours. As opposed to vortex formation being regular in UF case, vortex formation itself is irregular or random in RD case.

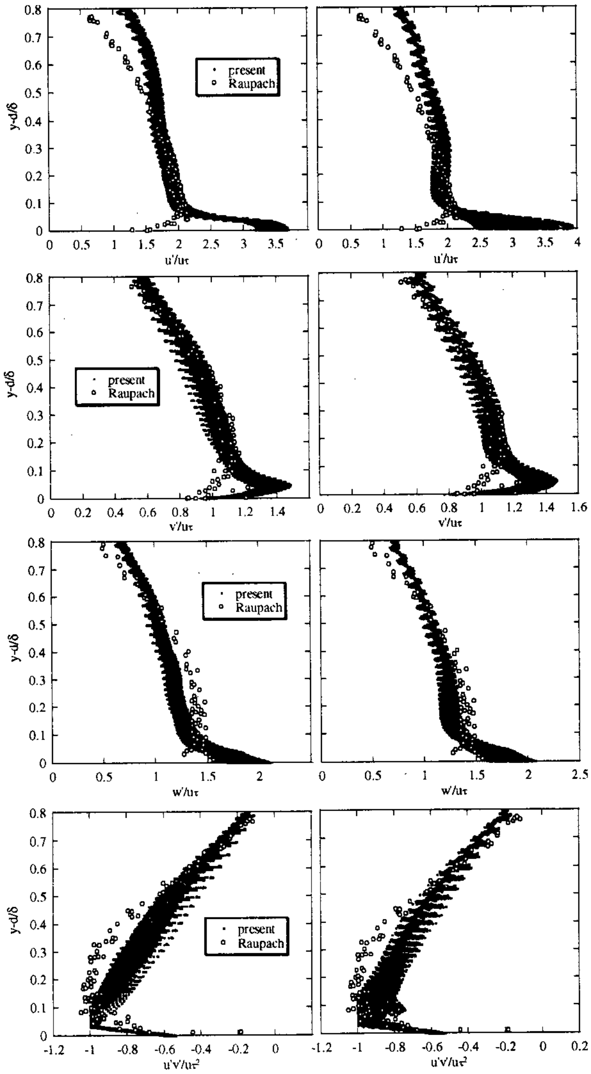

Fig.7 Fluctuating velocities and Reynolds shear stress, left: UF case; right: RD case

Secondly, in the region of $0.1 < (y\text{-}d)/\delta < 0.4$, the vertical profiles of streamwise fluctuating velocities in RD case is almost constant at $u'/u_\tau = 2.0$. Other components have a same tendency. It means that fluctuating velocities in RD case are slightly lager than those in UF case in the region of $0.3 < (y\text{-}d)/\delta < 0.4$. Thirdly, maximum value in the deviation of fluctuating velocities in RD case can be larger than that in UF case. Fourthly, the constant flux layer in RD case can be estimated to become thicker compared with UF.

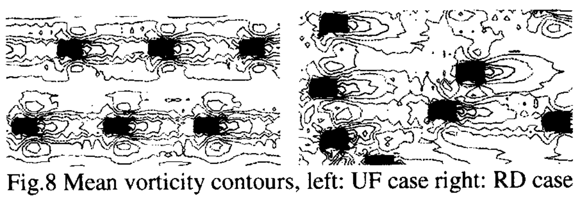

Fig.8 Mean vorticity contours, left: UF case right: RD case

## DISPERSION PLUMES FROM A POINT SOURCE IN URBAN-LIKE AREA

### Validation of numerical model

First LES of concentration fluctuations over smooth wall are carried out to validate the numerical model, by comparing with Fackrell's the experimental results (Fackrell and Robins, 1982) and Sykes's LES data (Sykes and Henn, 1992). The Reynolds number based on friction velocity and boundary layer height is 13000. A height of a point source is $y/\delta$ = 0.2. Lund's method is applied to simulate the spatially turbulent boundary layer flow, while the boundary condition on bottom is modeled with wall function for the high Reynolds number. Figure 9 compares the computed results for the vertical profiles of mean velocity, fluctuating velocities and Reynolds stress with Fackrell's and Sykes's data. In regard to profiles of mean velocity, good agreements are achieved. But fluctuating velocities are underestimated compared with the experimental data especially in the region of 0-0.4 for $y/\delta$. In this computation, we do not use any special technique to develop fluctuating velocities and to thicken a boundary layer. Only natural flow is merely considered. Therefore, it has been underestimated. But concerning the wall-nomal and spanwise fluctuating velocities, as opposed to the Sykes's LES results showing a sudden decrease beneath $y/\delta$ = 0.2, our computed results reproduce the Fackwell's experimental data qualitatively.

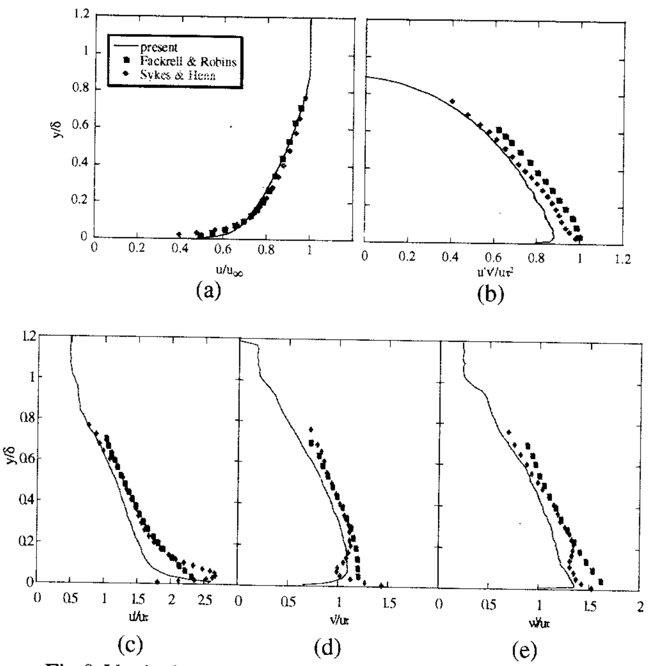


Fig.9 Vertical profiles of turbulence statistics over smooth wall (a) Mean velocity; (b)Reynolds stress; (c),(d),(e) fluctuating velocities

Figure 10 compares the computed results for the vertical profiles of mean and fluctuating concentrations with Fackrell's and Sykes's data. In our results, the diffusion width is underestimated compared with the experiment data. This is a matter of course, considering the result that fluctuating velocities is underestimated.

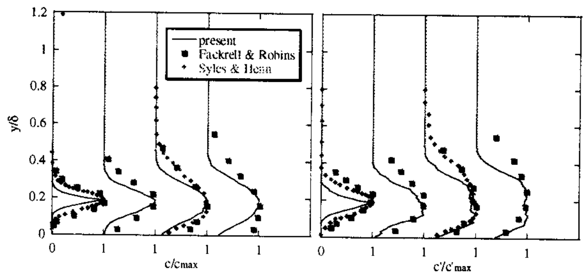


Fig.10 Comparison of vertical profiles of mean and fluctuating concentration with experimental data over smooth wall, From left: $x/\delta$= 0.96, 1.92, 2.88, 3,83

### Dispersion plumes over uniformly and randomly arranged roughness blocks

In previous section we examined the characteristics of turbulent structure in two cases, UF case and RD case. This section finally discusses the discrepancy in dispersion plumes between UF and RD cases. The position of source point is set to be at $2.3\delta_0$ downstream from an inlet and at a height of $0.13\delta_0(2.5h)$.

Figure 11 shows the instantaneous concentration field in vertical plane at center ($z$ =$1.1\delta_0$). Concentration has spread to the upper part immediately after the emission in RD case. Also earlier touchdown can be seen in RD case compared with UF case. We can recognize it more clearly in Fig. 12. Figure 12 shows the instantaneous concentration field in horizontal plane ($y$ =$0.021\delta_0$, $0.4h$). In RD case, it can be thought that touchdown became earlier due to down wash, which occurs in the wake of a roughness block immediately behind a source point.

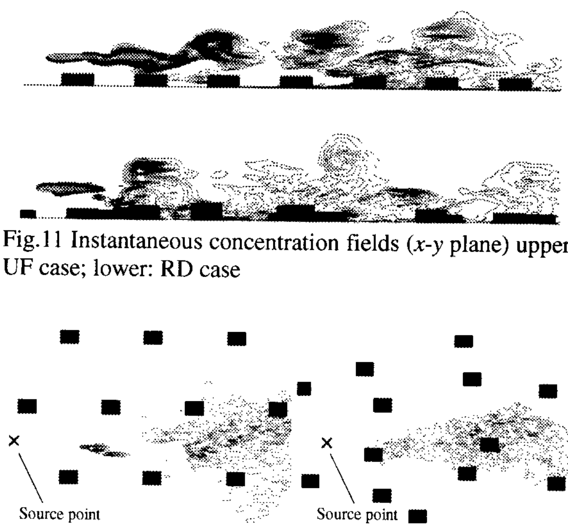

Fig.11 Instantaneous concentration fields ($x$-$y$ plane) upper: UF case; lower: RD case

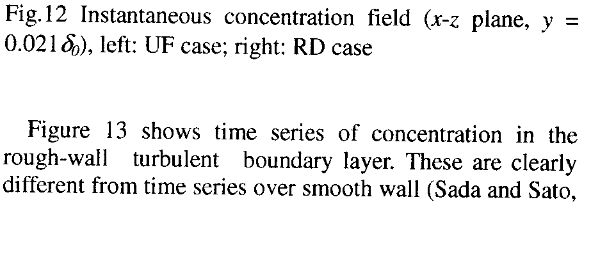


Fig.12 Instantaneous concentration field ($x$-$z$ plane, $y$ = $0.021\delta_0$), left: UF case; right: RD case

Figure 13 shows time series of concentration in the rough-wall turbulent boundary layer. These are clearly different from time series over smooth wall (Sada and Sato, 2000). In the case of smooth wall, spike-like peaks frequently appear in the almost zero concentration at the edge of the plume and the number of occurrence for these peaks increases close to the central axis of the plume. On the other hand, the computed concentrations in the rough-wall case become lower at $y/\delta_0 = 0.018$ than those at plume center, but are not intermittent. The number of peaks with high concentration is almost same at the plume center in both UF and RD cases. We can find an obvious discrepancy between UF case and RD case. Thick region with high concentration can be seen notably in RD case. The existence of downwash effect can be also recognized from the time series.

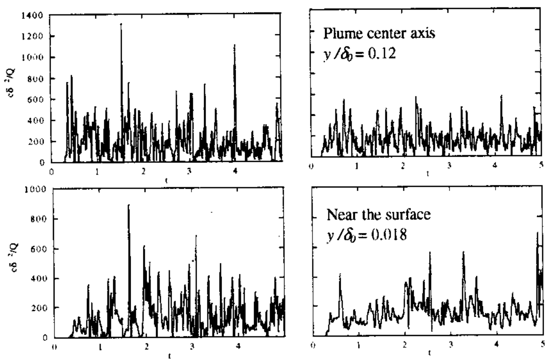


Fig.13 Time series of concentration left: UF case; right: RD case ($x = 1.25\delta_0$ from source point)

Figure 14 shows the vertical profiles of mean and fluctuating concentration. Compared with the result over smooth wall in Fig.10, mixture of concentration is rapidly performed in a short fetch under the influence of active velocity turbulence generated from near the rough surface. In comparison between UF and RD cases, although there is a small gap, they are almost the same at $x/\delta = 0.64$. At $x/\delta = 1.28$, 1.92 and 2.56, both mean and fluctuating concentrations in RD case are larger than those in UF case in the upper part. One of the reasons is obviously the influence of the fluctuating velocities. But a gap in the upper part is too large, taking into consideration the influence of the fluctuating velocities. Actually, there are the maximum gaps of about 20 % for mean concentration and of about 60 % for fluctuating concentration between UF case and RD case in spite of about 10% for fluctuating velocities.

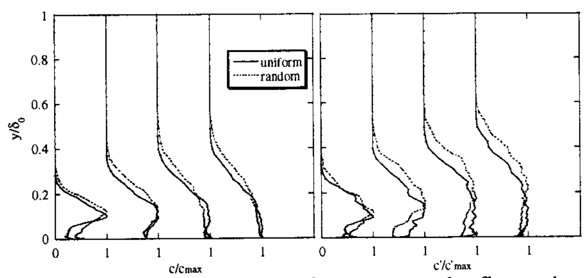


Fig.14 Vertical profiles of mean and fluctuating concentration in UF case and RD case, From left: $x/\delta =$ 0.64, 1.28, 1.92, 2.56

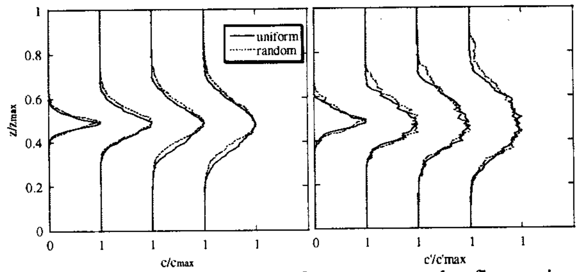


Fig.15 Lateral profiles of mean and fluctuating concentration in UF case and RD case, From left: $x/\delta =$ 0.64, 1.28, 1.92, 2.56

## CONCLUSIONS

This paper carried out LES of spatially developing turbulent boundary layer over roughened surface where various sizes of roughness blocks are directly arrayed at random locations on the flat plate. Also, LES analysis on dispersion in the surface layer is performed and its characteristics are discussed in comparison with the case of uniformly arranged roughness blocks.

As a result, near the roughness height, touchdown in a random array is earlier than that in an uniform (staggered) array because of the local turbulence characteristics over roughness blocks. On the other hand, in the upper part, although there is no large discrepancy of streamwise fluctuating velocity between the cases of uniform and random arrays, a great difference of mean and fluctuating concentrations between both cases arises.

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