INTERACTIONS BETWEEN PARTICLE CLUSTERS AND VORTICAL STRUCTURES IN HOMOGENEOUS TURBULENCE WITH AND WITHOUT UNIFORM SHEAR

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ABSTRACT

Numerical simulations have been conducted for decaying homogeneous isotropic turbulence laden with small heavy particles in order to investigate the interaction between vortical structures and particle clusters settling under the effect of gravity. Numerical results show that descending counter-rotating vortex pairs play an essential role in the interaction, where the settling velocity of the particles is remarkably increased. The increase in the settling velocity is caused by the combined effect of three mechanisms; the accumulation of particles in downward fluid flows induced by the vortex pair, the vortex motion triggered by its self-induction, and the two-way coupling effect of particles. A hemispherical or cone-like cluster of high particle concentration is generated through the interaction.

INTRODUCTION

Turbulent gas flows laden with small heavy particles are encountered in a variety of natural and engineering applications. Examples include combustion of sprays and ash from volcanic eruptions settling in the atmospheric turbulent boundary layer. Many studies have been carried out to understand the flow characteristics, and the interaction between particles and gas flow turbulence.

One of the key elements of the interaction is preferential concentration (or clustering) of particles. Squires and Eaton (1991) showed in their numerical simulation of homogeneous isotropic turbulence that particles tend to accumulate in low-vorticity or high-strain regions due to the particle inertia. Wang and Maxey (1993) demonstrated that the particles in gravity tend to accumulate in downward fluid flows, which leads to the significant increase in the mean settling velocity of particles.

Two-way coupling between carrier fluid and particles is another key factor. The effects of particles on the carrier fluid flow become important when the mean particle concentration exceeds $O(10^{-6})$. The two-way coupling effects modify not only turbulent kinetic energy of carrier fluid (Squires and Eaton, 1990, Elghobashi and Truesdell, 1993) but also the settling velocity of particles. Recent experimental (Aliseda et al., 2002) and numerical (Tanaka et al., 2000) studies indicate that particle clustering makes the two-way coupling more effective to increase the settling velocity of particles. However, the dynamic process in which the settling velocity increases has not yet been identified based on observations.

It has been demonstrated by numerical simulations of isotropic turbulence (Ferrante and Elghobashi, 2003) and homogeneous turbulent shear flows (Ahmed and Elghobashi, 2000, Tanaka et al., 2002) that vortical structures are also modified by the particle clusters. Tanaka et al. (2002) found that particle clusters tilt vortex layers with horizontal vorticity to generate the vertical vorticity in sheared turbulence. Ferrante and Elghobashi (2003) showed that they increase the magnitude of the horizontal vorticity in isotropic turbulence. However, it has not been clarified how the particle clusters are modified as a result of the interaction with vortical structures under the effect of two-way coupling.

In the present study, we investigate the interactions between particle clusters and vortical structures in decaying homogeneous isotropic turbulence by the use of a numerical simulation. We focus on the role which the interaction plays in the increase in the settling velocity. We also discuss how the mean shear affects the settling velocity of particles.

FORMULATION

Fluid and Particle Motions

We consider the motions of small heavy spherical particles under the gravitational force in the negative x_2 direction. The particle diameter, d_p, was assumed to be small compared with the Kolmogorov length-scale, η, of turbulence. The particulate phase is assumed to be dilute enough that the effects of particle-particle interactions are neglected though the two-way coupling between two phases is considered. Taking account of the fact that the particle (solid) density, ρ_p, is much higher than the fluid (air) density, ρ_f, only the Stokes drag and the gravitational forces were assumed to exert on the particles. Under the assumption, the particle motions are governed by the following equations (Maxey and Riley, 1983),

$$\frac{dv_i}{dt} = \frac{1}{\tau_p}\left\{u_i(\boldsymbol{x}_p) - v_i - V_S\delta_{i2}\right\},\ \frac{d\boldsymbol{x}_p}{dt} = \boldsymbol{v},\ (i = 1, 2, 3) \tag{1}$$

where $\boldsymbol{v}$ and $\boldsymbol{x}_p$ denote the velocity and position of the particle, and $\boldsymbol{u}$ and ν represent the velocity and kinematic viscosity of the fluid, respectively. Two parameters, $\tau_p = \rho_p d_p^2/18\rho_f\nu$ and $V_S = \tau_p g$, denote the particle inertia (or response time) and the still-fluid terminal velocity, respectively. g denotes the gravitational acceleration.

The motions of the carrier fluid are described by

$$\frac{\partial u_i}{\partial t} + u_k\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x_i} - g\delta_{i2} + \nu\nabla^2 u_i + \frac{1}{\rho_f}f_i \tag{2}$$

with the solenoidal condition $\partial u_j/\partial x_j = 0$, where p is the pressure. f_i represents a body force which is the sum of the reaction forces exerted by the particles on the fluid. Here,

Table 1: Parameters for particles. Here, N_p denotes the number of particles. ϕ_v and ϕ_m represent the volume fraction and mass loading of particles, respectively.

N_p	ρ_p/ρ_f	τ_p/τ_K	V_S/v_K
2^{19}	1000	$2.18 \rightarrow 0.32$	$0.89 \rightarrow 2.3$
ϕ_v	ϕ_m	α	d_p
0.8×10^{-4}	0.8×10^{-1}	3.7	7.11×10^{-4}

we assume that the mean body force $\langle \boldsymbol{f} \rangle$ is balanced by the mean pressure gradient, where $\langle\ \rangle$ denotes the spatial average. For later convenience, we introduce

$$\Delta V \equiv \langle -v_2 \rangle - V_S, \qquad (3)$$

which represents the increase in the mean settling velocity of particles from their still-fluid terminal velocity.

Numerical Method

The motions of the carrier fluid were solved on 64^3 grid points in a cubic box of sides of 2π by using the Fourier spectral/Runge-Kutta-Gill scheme. The initial velocity field was given by the Fourier coefficients with specified energy spectrum,

$$E(k) = c\, k \exp(-k/k_0), \qquad (4)$$

and with random phase. Here, $k_0 (= 4.2)$ is a wavenumber at which the energy spectrum takes the maximum and c is a normalization constant. We set it as $c = k_0^{-4}/6$ so that $\omega'(0) = 1$, where ω' is the vorticity magnitude. Here, a dimensionless time is introduced as $t^* = \omega'(0)t = t$. $\eta k_{\max}$ varied from 0.98 at $t^* = 30$ to 2.66 at the end of the simulation ($t^* = 200$). The Taylor microscale Reynolds number was $20 \sim 24$ during the simulation except the initial transient period.

We introduced particles randomly throughout the computational domain at $t^* = 30$ after the flow attained a fully turbulent state. Many simulations were conducted with different values of the particle response time τ_p and the still-fluid settling velocity V_S. Expecting a significant increase in the settling velocity (Wang and Maxey, 1993, Aliseda et al., 2002), we here focus on the case of $\tau_p \approx \tau_K$ and $V_S \approx v_K$, where τ_K and v_K are the Kolmogorov time and velocity scales of turbulence, respectively. The number of particles per grid cell should be high to conduct the simulations with high accuracy (Boivin et al., 1998). We increased the number α times by using the virtual particles which represent $1/\alpha$ real ones. The particle volume fraction was as low as 8.0×10^{-5}. The parameters employed in this simulation are summarized in Table 1.

The particles were tracked in the Lagrangian frame. The initial particle velocity was set to be the same as the sum of the surrounding fluid velocity and the still-fluid terminal velocity. Cubic spline interpolation was used for the evaluation of fluid velocity at the particle position from its neighboring grid points, whereas Taylor series 13 points method (Yeung and Pope, 1988) was used for the distribution of the reaction force to the grid points.

For comparison a one-way coupling simulation was conducted with the same initial condition. Another one-way coupling simulation was performed starting from the data at $t^* = 100$ of the two-way coupling simulation in order to clarify the two-way coupling effects on the interaction between vortex structures and a particle cluster.

RESULTS

In this section, we examine the interaction between the vortical structures and the particle clusters, focusing on the case which makes the largest contribution to the increase in the settling velocity of particles in this simulation.

Turbulence kinetic energy and enstrophy

The energy and the enstrophy evolves as t^{-1} and t^{-2}, respectively, in later times in the single-phase flow, as is expected from the form of the initial energy spectrum (Eq.(4)). Figure 1 shows the time evolution of turbulence kinetic energy and enstrophy of carrier fluid for the two-way coupling case. They are normalized by the counterparts in the single-phase flow to emphasize the two-way coupling effects. The relative magnitude of the energy slightly increases in later times in this simulation. It is seen that the enstrophy is more effectively increased by the two-way coupling. The horizontal components of vorticity are found to become greater than the vertical one though all components increase with time relative to their counterparts in the single-phase flow. Examination of each term in the vorticity equations has revealed that the horizontal vorticity is directly generated by the particles, while the vertical vorticity is intensified by the stretching-and-tilting terms.

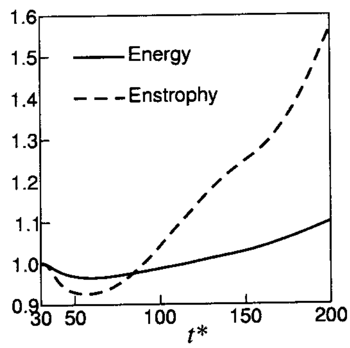

Figure 1: The effect of particles on the time evolution of turbulence kinetic energy and enstrophy of carrier fluid. The values were normalized by those in the single-phase flow.

Increase in settling velocity of particles

Figure 2 shows the time evolution of the mean settling velocity of particles for the one-way and two-way coupling simulations. In the figure, the increase in the settling velocity, ΔV (Eq.(3)), is normalized by the magnitude of the vertical component of velocity, u'_2. The increase in the mean settling velocity is about 10 percent of u'_2 in the one-way coupling simulation, which is consistent with the result obtained by Wang and Maxey (1993). In the two-way coupling simulation, the increase amounts to 35 percent of u'_2 at the end of the simulation. It is somewhat smaller than a 50% increase found in the experiment conducted by Aliseda et al. (2002). This is probablly due to the lower Reynolds number in our simulation.

Counter-rotating vortex pair

Now, we show a typical example of the interaction between particle clusters and vortical structures to understand the mechanism leading to the increase in the settling velocity. As was shown in Wang and Maxey (1993), particles tend to accumulate in the downward fluid flows. Here, we focus on a counter-rotating pair vortices which induce a downward fluid flow between them (Figs. 3). In Figs. 3, the vortex

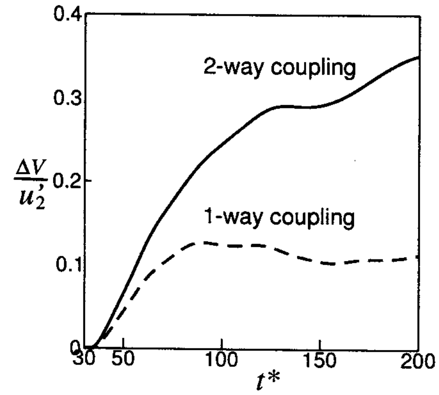

Figure 2: Time evolution of the increase in the settling velocity of particles.

tubes are represented by their central axes, which were extracted by tracing the loci of sectional local minimum of the pressure (Miura and Kida, 1998). The light axis is inclined at about $-30°$ to the $+x_3$ direction, while the dark one at about $30°$ to the $-x_3$ direction. As will be shown later, this vortex pair brings about a significant increase in the settling velocity of particles.

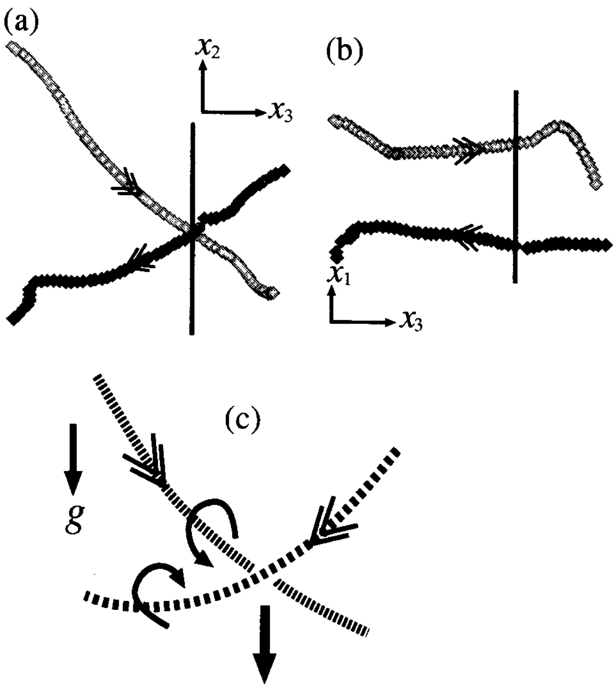

Figure 3: Vortex pair which is focused on ($t^* = 140$). (a) side view, (b) top view, (c) schematic of the vortex pair.

Figure 4 shows the time evolution of the vortex pair. Thick and thin lines denote the vortex pairs in the two-way and one-way coupling simulations, respectively (The latter started from the data of the former at $t^* = 100$). It is seen that the central part of the vortex pair moves in the gravitational direction as time elapses. This is caused by the self-induction of the vortex pair. It is interesting to note that the downward motion of the vortex pair is enhanced by about 30% in the two-way coupling case, indicating that the vortices are activated by the particle cluster. It is also interesting that the vortex tubes are finally stretched in the gravitational direction as a result of the two-way coupling effect.

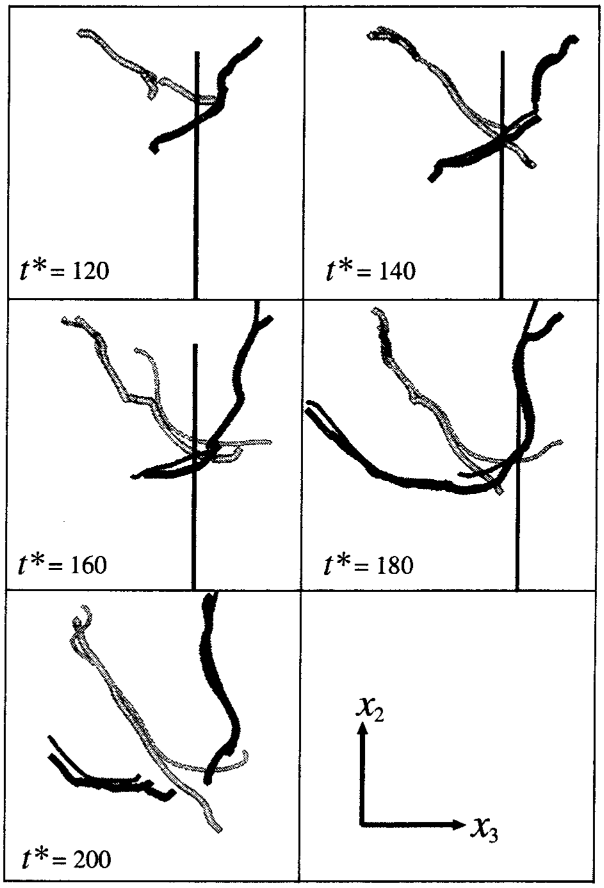

Figure 4: Time evolution of the vortex pair. Thick and thin lines for the two-way and one-way coupling simulations, respectively.

Interaction between the vortex pair and a particle cluster

Figure 5 shows how the vortex pair interacts with the particles in the two-way coupling case. It is found that a hemispherical or cone-like cluster of high paritcle concentration is created at the end of the interaction (see the bottom panels in Fig. 5 and its cross section shown in Fig. 9 below). The particles composing the cluster were traced back to the beginning of the interaction to examine how the particle cluster was generated. The counterpart in the one-way coupling case (at $t^* = 180$) is shown in Fig. 6 for comparison.

At $t^* = 120$, the parciles are distributed in a large area above the vortex pair. As time goes on, they are pulled down into the region between the pair vortices. It is seen that the width of the cluster in the x_1 direction rapidly decreases with time (see right pannels in Fig. 5 and also Fig. 7 below). This is due to their inertia. They accumulate in a narrow region of high downward fluid velocity between the vortices and are accelerated in the gravitational direction. It should be noted that the downward motion of the vortex pair due to the self-induction also contributes to the increase in the settling velocity. The settling velocity of the cluster is surprisingly high (about four times higher than u_2') during the interaction.

It is interesting that a vertical streak of high particle concentration comes out at $t^* = 140$ in the second left pannel of Fig. 5. This indicates that the particle cluster is contracted not only in the x_1 direction (perpendicular to the vortices) but also in the x_3 direction (nearly parallel to them). This particle concentration in the x_3 direction is triggered by the straining flow induced by the vortices (indicated by round

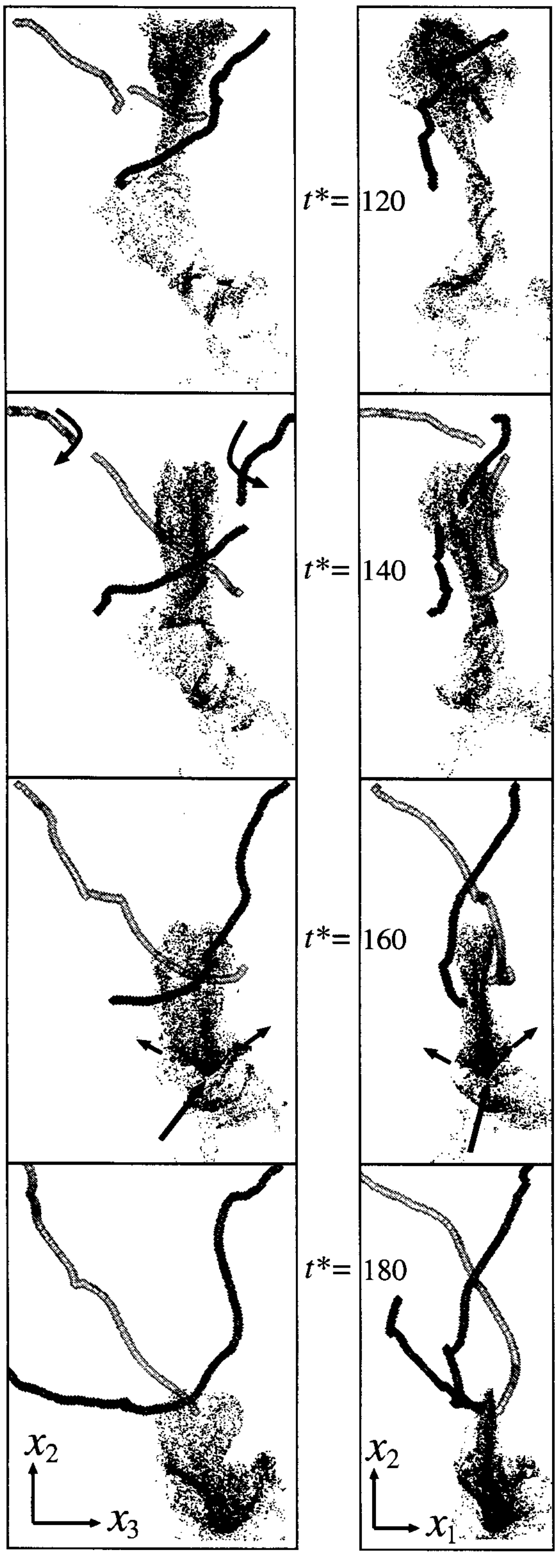

Figure 5: Interaction between the vortex pair and a particle cluster in the two-way coupling simulation.

arrows in Fig. 5). It is also found that the concentration is remarkably enhanced by the two-way coupling effect. As a result, the particles are concentrated on a small region at the bottom of the cluster as indicated by the long straight arrows in Fig. 5 (see also Fig. 7 below). Finally, they are turned to the horizontal directions to form the cone-like cluster. The cluster generated in the one-way coupling case is

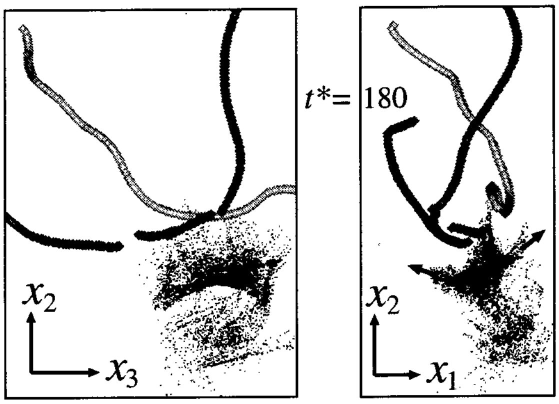

Figure 6: The same as the bottom panels in Fig. 5 but for the one-way coupling simulation.

rather horizontal and elongated in the x_3 direction (Fig. 6).

Modification of vortical structures

Figure 7 shows a time series of the vortical strucure and the local concentration of particles on the $x_1 - x_2$ planes indicated by the vertical lines in Fig. 4. The distribution of the x_3 component of vorticity is represented with solid lines for $\omega_3 > 0$ (counter-clockwise rotation) and broken lines for $\omega_3 < 0$ (clockwise rotation). Shades denote the regions of high particle concentration with darker shades representing higher concentration. The crossing point of the vortex axis is represented by ×. For comparison, the vorticity and particle concentration fields are shown for the one-way coupling case in Fig. 8.

As was shown before, the particles accumulate in the region between the pair vortices. Once the particle concentration becomes sufficiently high, the particle cluster begins to accelerate the downward flow locally between the vortices. Then, the pressure is lowered there, and the regions of positive and negative vorticities approach particularly on the lower side of the vortex pair. As a result, the downward flow converges into a narrow region, generating elongated high-vorticity regions on both sides of the flow (Ferrante and Elghobashi, 2003).

The particle concentration becomes very high at the bottom of the vortical structure in the above-mentioned process (see also Fig. 5). Finally, the particles are turned to the horizontal directions to form a hemispherical or cone-like cluster. At the same time, a region of high horizontal vorticity is generated by the cluster. This high-vorticity region is also cone-like as is indicated by Fig. 9.

Interaction in turbulent shear flows

Tanaka et al. (2002) examined the motions of small heavy particles in homogeneous turbulence subjected to the mean flow in the x_1 direction that was uniformly sheared in the x_2 direction by the use of numerical simulations. Their results show that particle clusters are generated along the shear layers, which are nearly perpendicular to the x_2 axis. The cross section of the particle cluster is similar to that observed at $t^* = 180$ in the one-way coupling simulation (see Fig. 8). However, they are more elongated in the spanwise (x_3) direction due to the motions of quasi-streamwise vortices. Since the particle clusters are rather horizontal, the two-way coupling is not effective to accelerate the fluid flow, and therefore the increase in the settling velocity of particles is not so significant as in isotropic turbulenece.

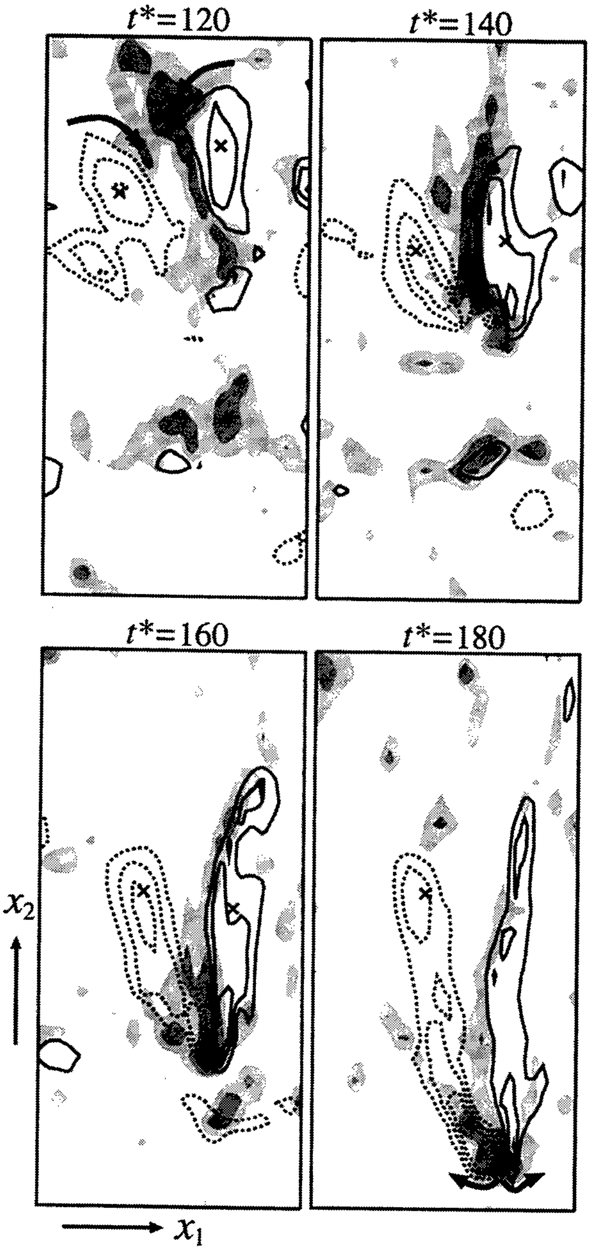

Figure 7: Spatial distributions of particle concentration and x_3 component of vorticity on the $x_1 - x_2$ plane. The domain of sides of $26\Delta x \times 53\Delta x$ is shown. $\times$ denotes the position of the vortex axis. Solid (broken) lines denote the contour lines of $\omega_3 = 0.1, 0.15, 0.2$ $(-0.1, -0.15, -0.2)$. The local particle concentration changes as $C/\langle C\rangle = 2, 4, 8, 16$ from the lightest to the barkest shades. Here, C is obtained by counting the number of the particles in each grid cell.

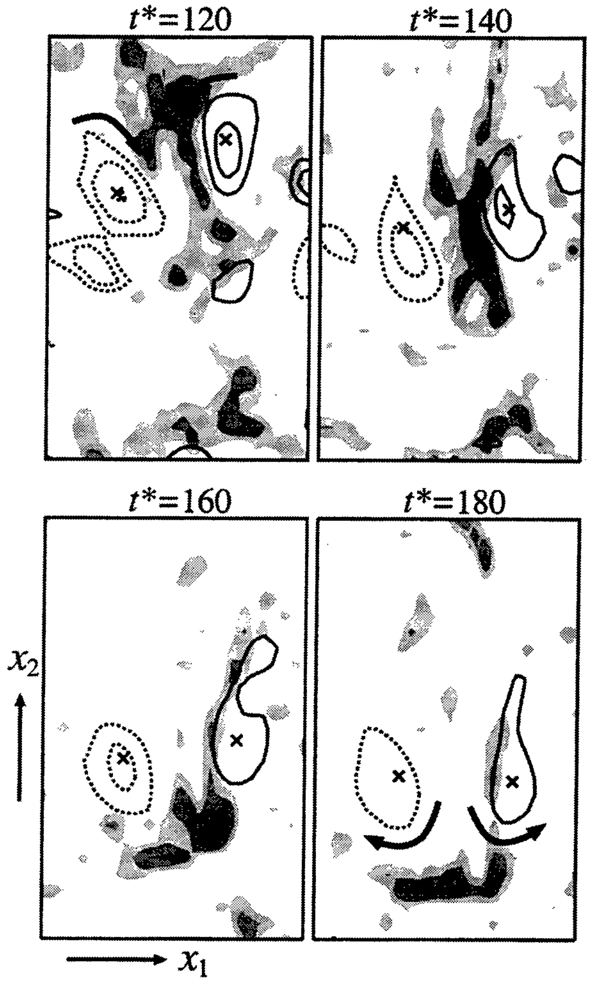

Figure 8: The same sa Fig. 7 but for the one-way coupling simulation. The upper part of the region $(26\Delta x \times 40\Delta x)$ is plotted.

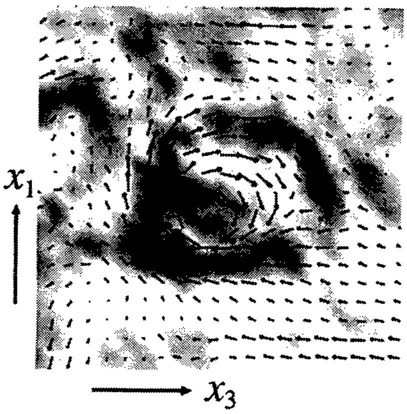

Figure 9: Concentrated region of particles and vorticity vectors on an $x_3 - x_1$ plane at $t^* = 180$ with black for $C/\langle C\rangle \geq 10$ and white for $C = 0$.

CONCLUSIONS

Numerical simulations have been conducted for decaying homogeneous isotropic turbulence laden with small heavy particles settling under the effect of gravity. We have focused on the interaction between vortical structures and particle clusters, and its role in the dramatic increase in the settling velocity of particles. It is found that the counter-rotating vortex pairs descending due to their self-induction play an essential role in the interaction. We have observed the following scenario of the interaction.

The particles located above the vortex pair are attracted to the region between the pair vortices. Because of their inertia, they tend to accumulate in a narrow region between the vortices to form a vertical sheet of high particle concentration. The particles are accelerated in the gravitational direction in this process. The downward motion of the vortex pair also paly an important role in the increase of the settling velocity.

As the particle concentration becomes high, the two-way copling effect begins to work. The downward fluid flow is locally accelerated by the cluster, which further increases the settling velocity of particles. The two-way coupling effect also enhances the concentration of the particles. As a result, a cluster of high particle concentration is generated in a small region below the vortex pair. Finally, the cluster is deformed into a hemispherical or cone-like shape due to the horizontal fluid flow at the bottom of the vortical structure.

It should be noted that the two-way coupling effect enhances the downward movement of the vortex pair due to the self-induction by activating the vortices. The vortex tubes are finally stretched out in the gravitational direction through the interaction. The cone-like structure of high horizontal vorticity is created by the cluster below the pair vortices.

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