TIME-DEPENDENT ISOTROPIC TURBULENCE

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ABSTRACT

Time dependent isotropic turbulence belongs to a class of problems for which modeling assumptions based on Kolmogorov's theory may not be appropriate: the Kolmogorov theory defines a steady state, or perhaps more generally a self-similar state, but during transient evolution, even the smallest scales of motion can well be very far from steady or self-similar. But spectral closures, which make no special assumptions about either small or large scales, are appropriate methods for such problems.

The transient evolution of turbulence driven by statistically steady forcing, the transient evolution of turbulence driven by linearly unstable large-scale modes, and the transition from one steady state to another in turbulence driven by a statistically unsteady force are analyzed using a recently developed spectral closure. Although DNS has an important role in validating spectral closures, because these analyses require considering a large number of test cases, often run to very large numbers of time steps, direct simulation alone is impractical.

INTRODUCTION

Recent demands on CFD have called attention to some limitations of current RANS and LES models in 'non-equilibrium' turbulent flows in which the assumption of a local Kolmogorov steady state is inappropriate. Examples include strongly statistically time-dependent turbulent flows and transitional flows (Rubinstein et al, 2001).

Simplified spectral closures are one way to address these problems: these models make no assumptions about Kolmogorov scaling, spectral self-similarity, or their equivalent, assumptions which are crucial to formulating simpler models, but which inevitably limit their applicability to complex flows. Simplified spectral closures describe interscale energy transfer more accurately than single-point models but are computationally more tractable than general analytical turbulence closures like the Direct Interaction Approximation (DIA; Kraichnan, 1959) and the Lagrangian Renormalized Approximation (LRA; Kaneda, 1981).

Since spectral closures are projection independent, they can be also used to formulate LES subgrid models, RANS transport models, and RANS-LES hybrids with arbitrary resolution within a unified analytical framework. Computationally successful examples of this approach include the LWN (local wavenumber) spectral closure model (Clark and Zemach, 1995) and the model SCIT (Bertoglio et al, 1994).

This paper describes one such closure, the CMSB model (Cartoon Model of Spectral Behavior; Rubinstein and Clark, 2003). This model is a generalized Heisenberg closure which includes the effects of local interactions and allows evolution of the turbulent time-scale. In the first respect, it resembles the model of Canuto and Dubovikov (1996). In comparison to EDQNM (Orszag, 1973), it trades a much coarser description of energy transfer for time-scale evolution. In this respect, it is a simplification of Kraichnan's (1971) Test Field Model (TFM).

The model will be applied to three problems of time-dependent isotropic turbulence: the transient evolution of turbulence under steady forcing, transient evolution due to large-scale linear instability, and transient evolution from one steady state to another. These problems all exhibit dynamically significant departures from Kolmogorov self-similarity during important phases of evolution: in the first two problems, a Kolmogorov steady state does not even exist initially, and the analysis is concerned only with how or whether a Kolmogorov steady state emerges through nonlinear interactions; in the third, a transient phase of nonlinear reorganization which may depart from the Kolmogorov picture connects two different steady or self-similar states.

The first two problems are intended as a means to understand spectral broadening in transitional flows, in particular, this intention motivates the study of turbulence development due to large-scale linear instability. The evolution due to steady forcing has a threshhold character: only after energy builds up sufficiently in the forced scales does nonlinearity become important. This transient evolution is strongly dependent on initial conditions.

Preliminary comparisons will be given with the transient evolution of forced DNS, although the radically different forcing mechanisms preclude using these comparisons for model validation, at least for now.

The third problem focuses on the nonlinear reorganization of turbulence between different steady or self-similar states. One example is isotropic turbulence with time-dependent forcing: the turbulence is initially in a forced steady state; subsequently, the force amplitude is increased linearly until a state of self-similar growth is reached. The rate of increase of the forcing is an important parameter in view of the intuitive expectation that slow increase allows a quasi-static evolution, whereas sufficiently rapid increase will induce transient dynamic effects.

We conclude that simplified spectral closures are a natural and computationally feasible means to study these problems. The calculations can be run to exremely long times and large numbers of alternate scenarios can be compared with negligible computational resources. Although validation by DNS studies of some cases is of course indispensible, comparably comprehensive studies of these problems by DNS alone would be infeasible.

THE CMSB MODEL

The model equations used are given here with a brief outline of the derivation; details are given in (Rubinstein and Clark, 2003). The CMSB model begins with a Markovianized form of the DIA, but with a single Lagrangian time scale instead of the two time-scales used in the TFM.

Next, nonlinear interactions are restricted to certain degenerate types of local and distant interactions. This step greatly simplifies the energy transfer model, which is analytically no more complex than the classical Heisenberg model (Monin and Yaglom, 1975). Finally, the triad relaxation time of the DIA is replaced by a single-mode relaxation time.

The model equations are (i) the spectral evolution equation

$$\dot{E}(k,t) = P(k,t) - S(k,t) - 2\nu k^2 E(k,t) \qquad (1)$$

where $E(k)$ is the energy spectrum, $P(k)$ is the production spectrum, and $S(k)$ is the nonlinear transfer, (ii) the closure equation for $S(k)$

$$S(k) = c\left\{ k^4 \int_k^\infty dp\ \theta(p)\frac{E(p)^2}{p^2} - k^2 E(k)\times \int_k^\infty dp\ \theta(p)E(p) - \theta(k)\frac{E(k)^2}{k^2}\int_0^k p^4 dp + \theta(k)E(k)\int_0^k dp\ p^2 E(p) + \frac{1}{5}\left[k^4 \times \int_k^\infty dp\ \theta(p)p\frac{dE}{dp}\frac{E(p)}{p^2} - k^2 E(k)\theta(k)\int_k^\infty dp\ p\frac{dE}{dp} - k\frac{dE}{dk}\frac{E(k)}{k^2}\theta(k)\int_0^k p^4\ dp + k\frac{dE}{dk}\times \int_0^k p^2 E(p)\theta(p)dp\right]\right\} \qquad (2)$$

and (iii) the relaxation time evolution equation

$$\dot{\theta}(k) = 1 - \eta(k)\theta(k) - \nu k^2\theta \qquad (3)$$

where the turbulence frequency η is defined by

$$\eta(k) = c'\theta(k)\int_0^k dp\ p^2 E(p) \qquad (4)$$

The model constants c and c' are determined by comparison with the theory of isotropic turbulence (Rubinstein and Clark, 2003).

Important features of this model include treatment of nonlocal interactions, the possibility of energy transfer from small to large scales ('backscatter'), and the existence of both equipartition ensembles and Kolmogorov steady states. The classical Heisenberg model is recovered by retaining only the second and fourth terms in the energy transfer model Eq. (2) and making the simple algebraic model

$$\theta(k,t) = [k^3 E(k,t)]^{-1/2} \qquad (5)$$

The Heisenberg model includes nonlocal interaction effects and admits Kolmogorov steady states, but does not allow backscatter or equipartition ensembles (Rubinstein and Clark, 2003). Unlike the CMSB model, its dissipation range predictions are unphysical (Monin and Yaglom, 1975).

TURBULENCE GENERATED BY STEADY FORCING: TRANSIENT EVOLUTION

This section considers the evolution of a Kolmogorov steady state under steady forcing at large scales. The goal is to understand the dynamics through which this steady state is established.

Suppose then that the energy production term in Eq. (1) has the form

$$P(k) = ak^2\exp[-b(k-k_0)^2] \qquad (6)$$

and that the initial spectrum $E(k,0) = 0$. The results of integrating the CMSB model equations are shown in Figures 1 and 2: the evolution of energy, production, and dissipation are shown in Figure 1, and the corresponding spectral evolution is shown in Figure 2.

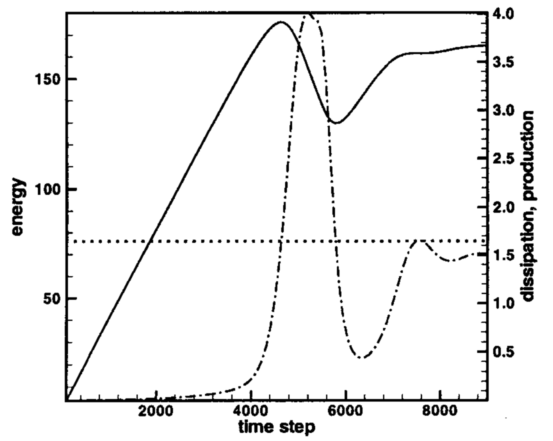

Figure 1: energy (solid), dissipation (dot-dash), production (symbols) in forced steady state closure simulation

The single-point statistics suggest that there are three distinct phases of evolution: (i) initial dynamically linear evolution dominated by energy pumping by the forcing. Nonlinearity is negligible and the energy grows linearly in time. This phase persists until about half of the total simulation time. It is followed by (ii) a 'transitional' phase beginning at about 4000 time steps marked by extremely rapid growth of dissipation. The dissipation overshoots the production, peaks and then relaxes; following relaxation, there follows (iii) a steady state characterized by nearly constant energy and balance between production and dissipation. Approximatly twice as many time steps as shown in Figure 1 are needed to achieve a robust steady state. The highly 'elastic' behavior exhibited in stage (ii) may be real, but is likely to be exaggerated by this model.

Figure 2 shows the corresponding spectral evolution at six different times, two each from each of the three phases just noted:

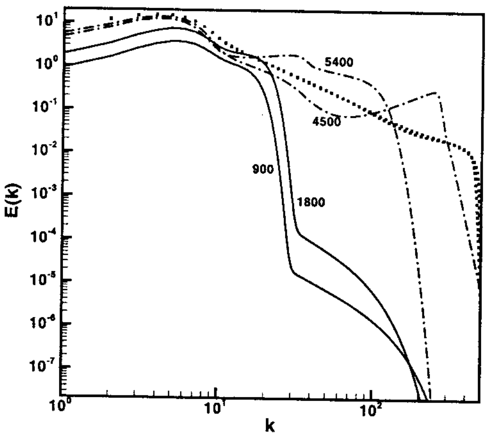

Figure 2: energy spectrum at short (solid), intermediate (dot-dash), and long (symbols) times in forced steady state closure simulation: numbers indicate the time steps corresponding to each curve

Corresponding to above are the spectral evolution phases: (i) energy growth limited almost entirely to the forced scales. Note that the forcing in this study peaked at $k = 5$ in the units of Figure 2. (ii) nonlinear reorganization with propagation of a wave-like spectral feature to small scales, leading finally to (iii) a steady Kolmogorov spectrum.

These three phases are quite similar to the evolution of decaying turbulence beginning from a Gaussian initial state (Clark and Zemach, 1995). The end of phase (ii) seems dynamically similar to the 'critical time' observed in decaying turbulence (Lesieur, 1990).

In practice, it would not be easy to implement statistically steady forcing of this type in DNS. At best, steady direct forcing f_i of the velocity would lead to the time-dependent production spectrum

$$P(k,t) = \langle u_i(\mathbf{k},t) f_i(-\mathbf{k}) \rangle \tag{7}$$

In fact, forcing in DNS is itself a significant problem (Chen et al, 1993). The goal of forcing schemes is to reach a steady state as quickly as possible, hence the transient evolution is generally not considered important.

Nevertheless, as a preliminary comparison, we show results comparable to Figures 1 and 2 in a 128^3 simulation using a spectral code developed by L.-P. Wang (Luo et al, 2002). The forcing was modified so that only the energy in the largest wavenumber shell was constant.

Figure 3 shows that the dissipation very closely follows the energy evolution with only a very small time lag, in

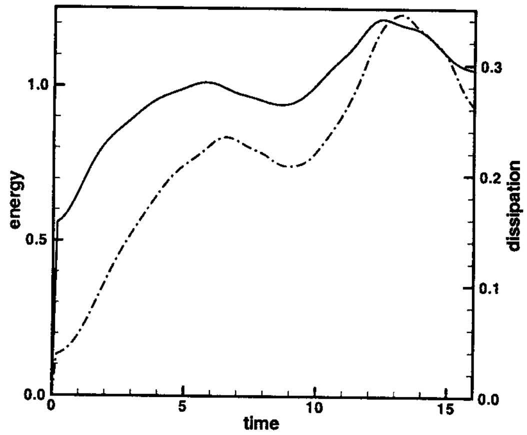

Figure 3: energy (solid), dissipation (dot-dash), in forced DNS

striking contrast to the result in Figure 1. This fact reflects the implicit coupling of all scales in this forcing scheme in which energy removed by any nonlinear interaction from the forced scales is immediately replaced at the next time-step. The corresponding spectral evolution in Figure 4 also contrasts with Figure 2: the energy spectrum fills up by simple growth at each scale.

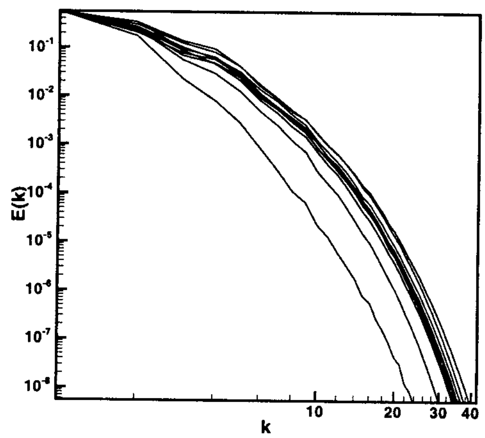

Figure 4: energy spectra at equal time intervals in forced DNS

Again, this forcing scheme is not intended as a realistic model of any dynamic process. In studies in progress, we are using direct random forcing of the largest scales to validate our closure models; in DNS practice, this scheme is impractical because of the large number of time-steps required to reach a steady state.

TURBULENCE GENERATED BY LARGE-SCALE LINEAR INSTABILITY: TRANSIENT EVOLUTION

Perhaps a more realistic model of spectral broadening in transitional flows is turbulence driven by linearly unstable large-scale modes. To study this problem, set the production term in Eq. (1) to

$$P(k) = \alpha(k) E(k) \tag{8}$$

where in this study,

$$\alpha(k) = a'k^2 \exp[-b'(k-k_0)^2] \quad (9)$$

Figure 5 shows the evolution of production, dissipation, and kinetic energy.

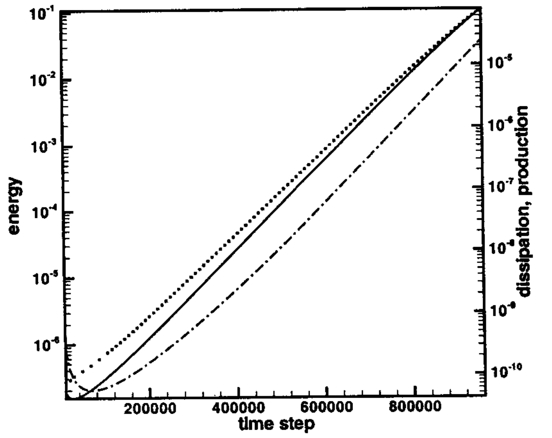

Figure 5: energy (solid), dissipation (dot-dash), production (symbols) in simulation of turbulence driven by large-scale linear instability

Evidently, following an initial transient, this system exhibits simple exponential growth. The spectral evolution in Figure 6 shows that the spectrum continues to fill smaller scales, indicating that nonlinearity is certainly active, but there is no tendency to establish Kolmogorov scaling.

This negative conclusion nevertheless reveals that boundary layer transition, for example, does not occur through the simple buildup of small scales by nonlinearity. Energy must be extracted from the mean flow by entirely different mechanisms.

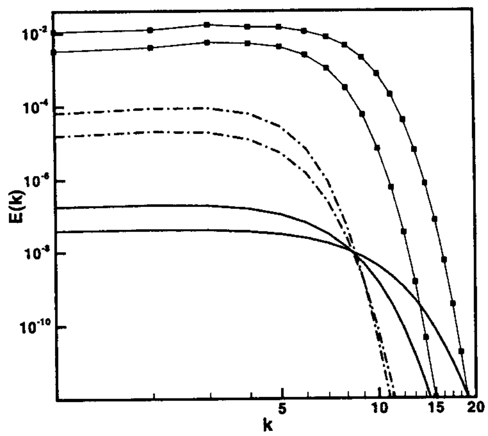

Figure 6: energy (solid), dissipation (dot-dash), production (symbols) in simulation of turbulence driven by large-scale linear instability

TURBULENCE DRIVEN BY TIME-DEPENDENT FORCING

In this study, the production spectrum is time dependent: we consider a ramp increase in forcing of the form

$$P(k,t) = P(k) \begin{cases} 1 & t \leq t_0 \\ 1 + r(t-t_0) & t \geq t_0 \end{cases} \quad (10)$$

where the steady part $P(k)$ is given by Eq. (6).

Simulations were begun from rest ($E(k,0) = 0$) as before. Figure 7 shows evolution of the energy, production, dissipation, and the ratio P/ϵ for a low ramp rate $r = 0.5$ in Eq. (10). The ramp begins at time step 16000, at which an approximately steady state exists. Production and dissipation appear to evolve together, and the energy spectra in Figure 8 appear essentially self-similar.

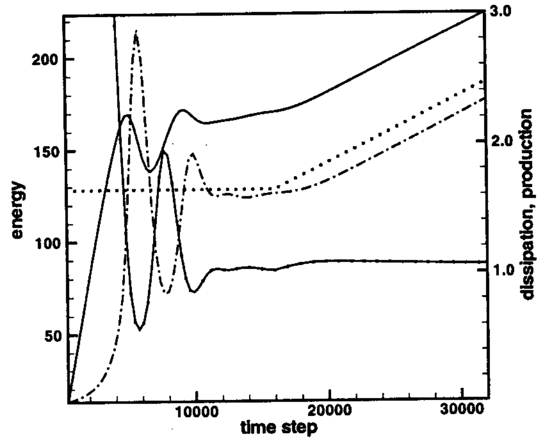

Figure 7: energy (solid), dissipation (dot-dash), production (symbols), and ratio P/ϵ (line plus symbols) in unsteady forcing with low ramp rate

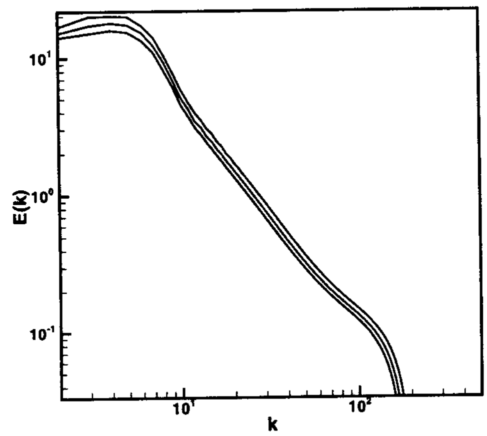

Figure 8: energy spectrum at three times following ramp in forcing, low ramp rate

For a higher ramp rate, $r = 8.0$ in Eq. (10), Figure 9 shows a more noticeable lag between the increase in production and the increase in dissipation. Nevertheless, a state of self-similar evolution appears to be established by 20000 time steps. In both the low and high ramp rate cases, the ratio P/ϵ relaxes to a value of about 1.17, however in more careful computations done for much longer times, we find that production and dissipation eventually equilibrate.

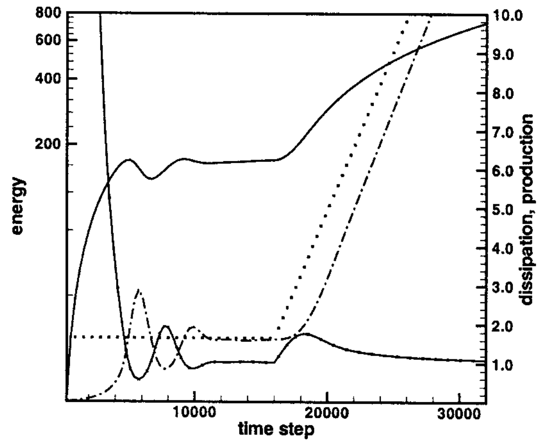

Figure 9: energy (solid), dissipation (dot-dash), production (symbols), and ratio P/ϵ (line plus symbols) in unsteady forcing with high ramp rate

The spectral evolution in Figure 10 shows, perhaps surprisingly, that the inertial range adjusts to the change in forcing more or less instantaneously, and that departures from self-similarity are pronounced only at large scales. This possibility reflects the role of distant interactions, which can cause rapid response at small scales to changes in much larger scales. Distant interactions are certainly required to bring about a balance of production and dissipation, even when the production is increasing. It will be important to confirm these results with DNS, and to compare with the results predicted by closures like those of Kovásznay and Leith (Monin and Yaglom, 1975), which only model local interactions.

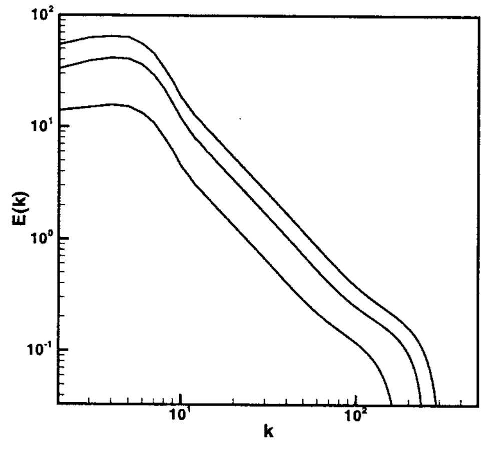

Figure 10: energy spectrum at three times following ramp in forcing, high ramp rate

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