A Singular Value Analysis of Boundary Layer Control

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There have been increased activities in developing efficient and robust controllers for viscous drag reduction in turbulent boundary layers. Many of these new approaches are quite different from the existing ones in that either they are directly utilizing modern control theory or they are derived from purely mathematical properties of the equations that govern the flow under investigation. Although turbulent flows are in general governed by nonlinear dynamics, some of these new approaches specifically target a linear mechanism that has been identified to be responsible for high-skin friction in turbulent boundary layers. The fact that a linear mechanism plays an important role in this nonlinear turbulent flow allows us to investigate the flow from a linear system theory point of view. Recently several investigators have reported successful applications of linear controllers derived from linear control theory, but many fundamental and challenging questions have been raised in the course of applying linear optimal control to turbulent boundary layers. In this study we have performed a singular value analysis of some existing controllers in order to gain new insights into the mechanism by which these controllers were able to accomplish viscous drag reduction in turbulent boundary layers.

The traditional eigenvalue analysis, which predicts whether a linear system is stable or unstable based on the eigenvalues of the system, is inadequate in explaining transient—nonetheless quite substantial—growth of the kinetic energy of certain disturbances in otherwise a stable system. This transient growth is due to non-normality of the linearized Navier-Stokes system. Eigenfunctions of a non-normal system are not orthogonal to each other, and as such the kinetic energy of certain disturbances can grow before its ultimate decay even in a linear system with no unstable eigenfunctions. Since the transient growth is due to a linear mechanism, its behavior can be analyzed through a singular value decomposition (SVD) of the system operator [1], with which the amplification factor of the so-called optimal disturbance could be determined. We hypothesize that the SVD analysis is also appropriate for examining performance of controllers for turbulent boundary layers. Effective controllers must reduce the non-normality of the flow system, since it is also believed to be responsible for sustaining near-wall turbulence structures, which in turn are responsible for high skin-friction drag in turbulent boundary layers.

We have performed the SVD analysis of various linear-quadratic-regulator (LQR) controllers we have developed as well as the so-called opposition control used by Choi *et al* [2], in the hope that it could shed new light into these control methods. Note that these methods have been designed to achieve a similar goal, *i.e.*, interfering the interactions between the near-wall streamwise vortices and the wall in order to reduce skin-friction drag in turbulent boundary layers. Despite the limitation that it cannot be easily implemented in practice, opposition control has been used as a reference case to which many other controllers have been compared. Although there have been some explanations now this simple control works, it is not completely understood, including what determines the optimal detection-plane location (currently observed to be around $y^+ = 15$ from the wall). With the SVD analysis, we have also examined whether it could provide a guideline of choosing optimal parameters in designing these controllers.

The first step to apply the SVD analysis is to formulate the governing equations in terms of the state-space representation as is done for other linear optimal controller design. The linearized Navier-Stokes equations with control input can be written in the following state-space representation:

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + B\mathbf{u} \tag{1}$$

$$\mathbf{u} = -K\mathbf{x} , \qquad (2)$$

where $\mathbf{x}$ and $\mathbf{u}$ represent the state-space and control input vectors, respectively, and A, B and K represent the system, actuation and control-gain matrices, respectively. The control-gain matrix K for an LQR controller is determined by solving an algebraic Riccati equation, while that for the opposition control is easy to construct (especially when $\mathbf{x}$ represents a collocation vector) once the detection-plane location is known.

Figure 1 shows distributions of singular values for the disturbance energy growth with an appropriate time scale. The flow system chosen here is a turbulent channel flow at Re_τ=100, where Re_τ is the Reynolds number based on the wall-shear velocity and the channel half-width, with various controllers. These singular values represent the amplification factor for each singular vector with which any disturbances can be expressed. As such, our concern here is whether there exist singular values much larger than one. It can be seen that the singular values corresponding to the opposition control (with the detection-plane location at $y^+ = 15$) and an LQR-controller (minimizing the disturbance energy growth) are much smaller than those corresponding to the uncontrolled system. The singular values corresponding to the opposition-controlled system with different detection-plane locations were higher than those shown here, corroborating with the numerical observation that $y^+ = 15$ is the optimal location. Also, shown are the singular values for a virtual flow [3], in which the linear coupling term between the Orr-Sommerfeld and the Squire equations was removed. Note that all singular values for this case are less than one, indicating that no transient growth in the virtual flow. From these distributions of singular values, one would expect that the virtual flow would be most effective in reducing the skin-friction drag in turbulent boundary layers.

In order to examine the applicability of the above SVD analysis, which is based on the linearized Navier-Stokes system, to fully nonlinear turbulent flows, we applied these controllers to direct numerical simulations of a turbulent channel flow at the same Reynolds number. Figure 2 shows the time evolution of mean skin-friction drag in the channel with various controllers. Note that the case without the linear coupling term (virtual flow)results in complete laminarization, consistent with the SVD analysis. Other results are also consistent with the SVD analysis, demonstrating that the SVD analysis is a viable tool in predicting the performance of a controller in the nonlinear turbulent channel flow.

We have shown that the SVD analysis could be used to gain useful information on the performance of certain controllers. It could be used in optimizing control parameters without actually performing expensive nonlinear computations. Other issues, such as the effect of using the evolving mean flow as control applied to a nonlinear flow system (also known as gain scheduling) and high Reynolds number limitation, will be investigated through the SVD analysis.

References

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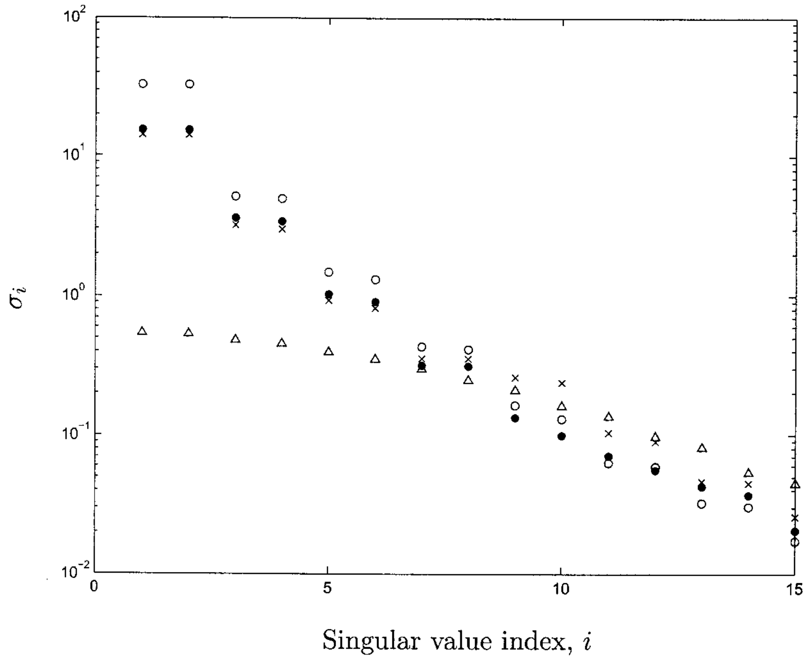

Figure 1: Singular values in a turbulent channel with different controllers: ○ , no control; •, opposition control; ×, LQR control; △, virtual flow. This is for the case of $k_x = 0$, $k_z = 6.0$ (corresponding to $\lambda_z^+ \approx 100$), and Re_τ=100.

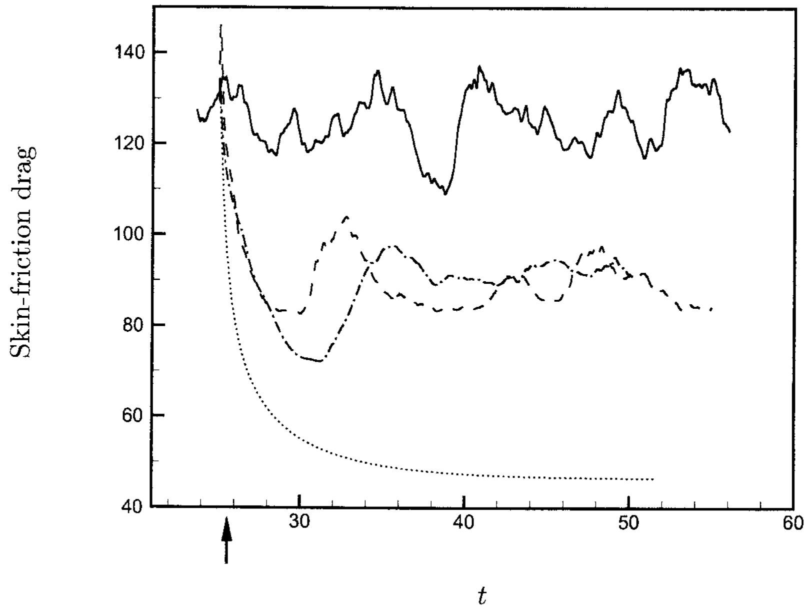

Figure 2: Mean skin-friction drag history with various control methods: ——— , no control; ---- , opposition control, -·-·- , LQR control; ········ , virtual flow.