TURBULENCE IN A STRATIFIED SHEAR FLOW WITH A VARIATION OF THE MEAN VELOCITY DIRECTION

Frank G. Jacobitz
Department of Mechanical Engineering
University of California
Riverside, California, USA
jacobitz@engr.ucr.edu

ABSTRACT

Direct numerical simulations are performed in order to investigate the turbulence evolution in stratified shear flow. Two different types of shear flows are compared. In the first flow, the magnitude of the mean velocity varies linearly in the vertical direction and the shear rate is constant. This flow has been studied in many previous investigations. In the second flow, the mean velocity describes a spiral. Here, the magnitude of the mean velocity is fixed but its direction varies in the vertical direction. The shear rate of this flow is also constant in the vertical direction. In both flows, a stable vertical density stratification is present. The Richardson number is varied from $Ri = 0$ to $Ri = 1$. The length of the vertical velocity spiral introduces a length scale that is not present in the linear velocity profile simulations. As long as the scale of the turbulent motion remains small compared to the length of the velocity spiral, the turbulent kinetic energy shows a similar evolution for both shear flows. Similarity is observed initially in all simulations, and throughout the simulations in the strongly stratified cases.

INTRODUCTION

Shear and stratification are ubiquitous features of turbulent flow in the geophysical environment (Caldwell 1987; Caldwell and Moum 1995). The prototypical example of stratified shear flow with uniform shear and uniform stratification has been studied extensively in the past. In this flow, the magnitude of the mean velocity varies in the vertical direction:

$$U = S_M z \qquad (1)$$

$$V = W = 0 \qquad (2)$$

$$\varrho = -S_\rho z \qquad (3)$$

Also, the mean density varies linearly in the vertical direction. The shear rate S_M and the stratification rate S_ρ are constant. A sketch of the mean velocity variation of this flow is given in figure 1. In the following, this type of shear flow with a variation of the mean velocity magnitude in the vertical direction will be referred to as the linear velocity profile case.

Using energy considerations, Richardson (1920) and Taylor (1931) established the Richardson number $Ri = N^2/S^2$ as the primary parameter to describe the stability of stratified shear flow. Here, $N = \sqrt{-gS_\rho/\rho_0}$ is the Brunt-Väisälä frequency and S is the shear rate. Miles (1961) and Howard (1961) showed that the flow is stable for $Ri > 1/4$ using linear inviscid stability analysis. Stratified shear flow has been investigated further, both experimentally (Komori, Ueda, Ogino & Mizushina 1983; Rohr, Itsweire, Helland & Van Atta 1988; Piccirillo & Van Atta 1997), as well as numerically (Gerz, Schumann & Elghobashi 1989; Holt, Koseff & Ferziger 1992; Itsweire, Koseff, Briggs and Ferziger 1993; Kaltenbach, Gerz and Schumann 1994; Jacobitz, Sarkar & Van Atta 1997; Jacobitz 2000).

Vertical shear present in geophysical flow can be due to a variation of the magnitude of the mean velocity, as well as a variation of the direction of the mean velocity. An important example for such a flow is the Ekman layer in the ocean (see for example Pedlosky 1986):

$$U = U_0(1 - \exp(-\frac{z}{\delta})\cos(\frac{z}{\delta})) \qquad (4)$$

$$V = U_0 \exp(-\frac{z}{\delta})\sin(\frac{z}{\delta}) \qquad (5)$$

Here, $\delta = \sqrt{(2\alpha/f)}$ is the Ekman layer thickness, α the vertical transport coefficient, and $f = 2\omega\sin(\theta)$ the local component of the planetary vorticity normal to the earth's surface (Coriolis parameter). In the Ekman layer, the magnitude as well as the direction of the mean velocity change in the vertical direction.

This contribution considers a different pro-

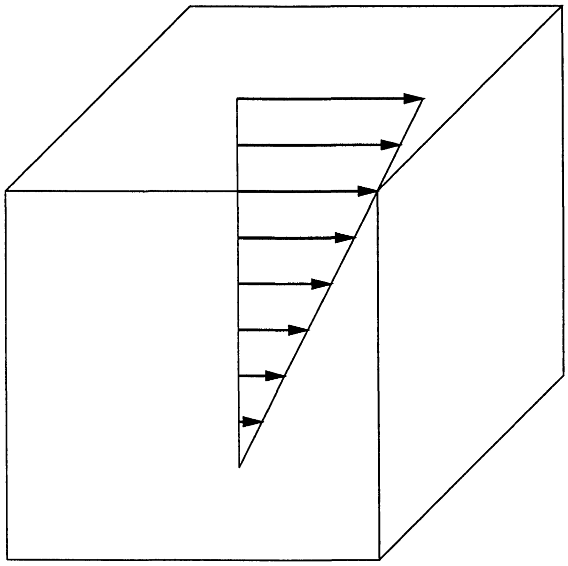

Figure 1: Sketch of the mean flow of the linear velocity profile case. The magnitude of the mean velocity varies in the vertical direction, but the direction of the mean velocity is fixed.

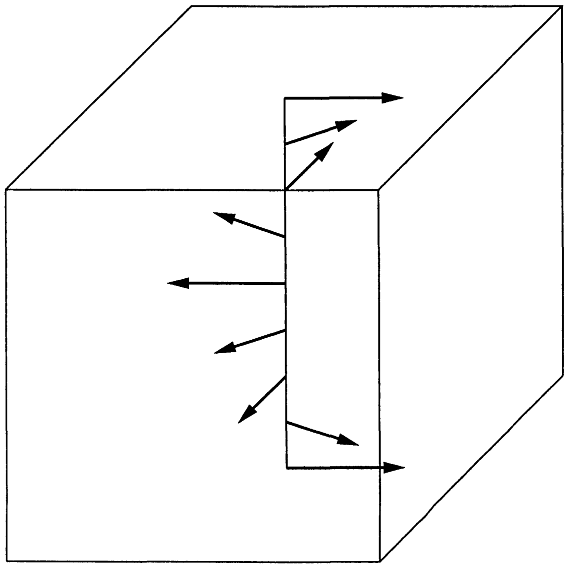

Figure 2: Sketch of the mean flow of the velocity spiral case. The direction of the mean velocity varies in the vertical direction, but the magnitude of the mean velocity is fixed.

totypical example of stratified shear flow, in which the magnitude of the mean velocity remains constant but its direction is varied:

$$U = U_0 \cos(k_z z) \qquad (6)$$

$$V = U_0 \sin(k_z z) \qquad (7)$$

$$W = 0 \qquad (8)$$

$$\varrho = -S_\rho z \qquad (9)$$

In this flow, the mean velocity describes a spiral of length $L_z = 2\pi/k_z$ in the vertical direction as sketched in figure 2. The total shear rate of this flow $S_D = kU_0$ is constant in the vertical direction. The mean density varies linearly in the vertical direction and the stratification rate S_ρ is constant. This type of shear flow with a variation of the mean velocity direction will be referred to as the velocity spiral case.

The two prototypes with a variation of the mean velocity magnitude and with a variation of the mean velocity direction are compared in this study to obtain a more complete understanding of turbulence in stratified shear flow.

NUMERICAL APPROACH

The direct numerical simulations presented in this study are based on the continuity equation for an incompressible fluid, the unsteady three-dimensional Navier-Stokes equation in the Boussinesq approximation, and an advection-diffusion equation for the density. In the direct numerical approach, all dynamically important scales of the velocity and density fields are resolved. A spectral collocation method is used for the spatial discretization and the solution is advanced in time with a fourth-order Runge-Kutta scheme.

The initial turbulence fields are taken from a simulation of isotropic turbulence without density fluctuations. The initial value of the Taylor-microscale Reynolds number $Re_\lambda = 35$ is fixed and values up to $Re_\lambda = 75$ are obtained in the simulations. The initial value of the shear number $SK/\epsilon = 2$ is also matched. The shear number assumes a value of about $SK/\epsilon = 6$ in all simulations. Here S refers to the shear rate S_M of the linear velocity profile case or S_D of the velocity spiral case. The simulations are performed on a parallel computer using a grid with up to $160 \times 160 \times 160$ points.

The shear rates S_M of the linear velocity profile simulations and $S_D = kU_0$ of the velocity spiral simulations are constant in the vertical direction. The mean velocity profile of the linear velocity profile simulations is fixed in time. Therefore, the shear rate S_M remains constant in these simulations. However, the turbulent fluctuations can interact with the mean velocity profile in the velocity spiral simulations. Therefore, the shear rate S_D will decay as these simulations advance in time.

RESULTS

In this section, the results of three series of simulations are presented. In the first two series, the spiral length L_z is varied for unstratified ($Ri = 0$) and stratified ($Ri = 0.1$) cases. In the last series, the spiral length is

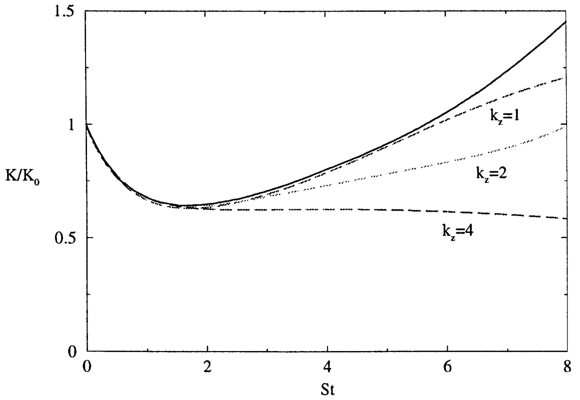

Figure 3: Evolution of the turbulent kinetic energy K for unstratified shear flow ($Ri = 0$) The solid line represents the linear velocity profile case and the dashed lines represent velocity spiral simulations.

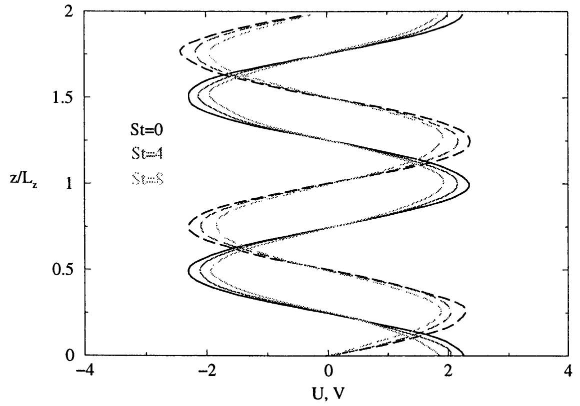

Figure 4: Decay of the mean velocity profile for an unstratified velocity spiral simulation with $Ri = 0$ and $k_z = 2$.

fixed ($L_z = 1$) and the Richardson number is varied from $Ri = 0$ to $Ri = 1$. The velocity spiral simulations are compared to simulations with a linear velocity profile.

Unstratified Simulations

In this section, the results from a series of unstratified simulations are presented. Figure 3 shows the evolution of the turbulent kinetic energy $K = \overline{u_i u_i}/2$ with non-dimensional time St. The solid line represents the evolution of K from a simulation with a linear velocity profile. Initially, the turbulent kinetic energy decreases due to the isotropic initial conditions until the shear production develops at about $St = 2$. Then, the turbulent kinetic energy grows exponentially. The exponential growth is possible in the simulation as there is no interaction between the turbulent fluctuations and the mean flow. A discussion of the energetics of this flow can be found in Jacobitz et al. (1997).

The dashed lines represent velocity spiral simulations with wavenumbers $k = 1$, $k = 2$, and $k = 4$. Initially, a remarkably similar evolution of the turbulent kinetic energy K compared to the simulation with a linear velocity profile (solid line) is obtained. For the simulation with $k = 1$ the similarity lasts to about $St = 6$. However, for the larger values $k = 2$ and $k = 4$, the similarity ends at about $St = 3$ and $St = 2$, respectively.

The spiral length $L_z = 2\pi/k_z$ introduces an additional length scale in the velocity spiral simulations that is not present in the simulations with a mean velocity magnitude variation. Similarity between the two cases is obtained as long as the integral scale of the motion is small compared to the spiral length L_z. Since the spiral length L_z is increased as the wavenumber k_z is decreased, the similarity between the two cases last longer for simulations with small values of the wavenumber k_z.

In the velocity spiral simulations, the turbulent fluctuations are allowed to interact with the mean velocity. The evolution of the mean velocity profiles with non-dimensional time St are shown in figure 4 for a simulation with $k_z = 2$. As time increases, the turbulent fluctuations extract energy from the mean flow. This extraction decreases the mean shear rate and the turbulence production. All simulations are started with the same fluctuation intensity $q = \sqrt{2K}$. Also, all simulations have the same initial shear rate $S_D = kU_0$. Therefore, the ratio q/U_0 increases as k_z is increased. This results in an earlier decrease of the mean shear and a slower turbulent kinetic energy growth with increasing k_z.

Stratified Simulations

In this section the results from a series of stratified simulations are presented. The Richardson number $Ri = 0.1$ is matched in all simulations. Figure 5 shows the evolution of the turbulent kinetic energy as a function of non-dimensional time St. The solid line corresponds to a simulations with a linear mean velocity profile. Compared to the unstratified case in figure 3, the growth of the turbulent kinetic energy is inhibited here due to the presence of stratification.

The dashed lines show the turbulent kinetic energy K of stratified velocity spiral simulations with $k_z = 1$, $k_z = 2$, and $k_z = 4$. The results of the stratified cases are qualitatively similar to the results of the unstratified cases shown in figure 3. Again, an initially close similarity between the two shear flows is obtained, in particular for the small wavenumber simulation with $k_z = 1$.

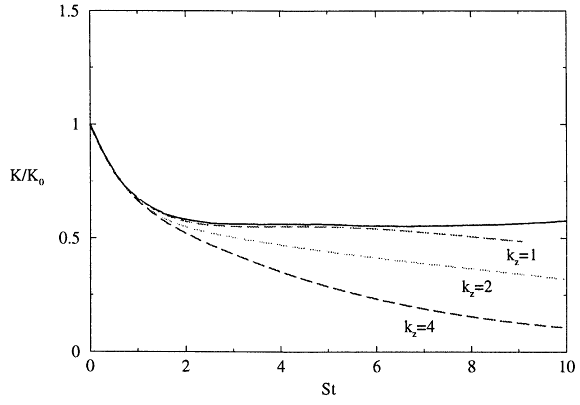

Figure 5: Evolution of the turbulent kinetic energy K for stratified shear flow ($Ri = 0.1$) The solid line represents the linear velocity profile case and the dashed lines represent velocity spiral simulations.

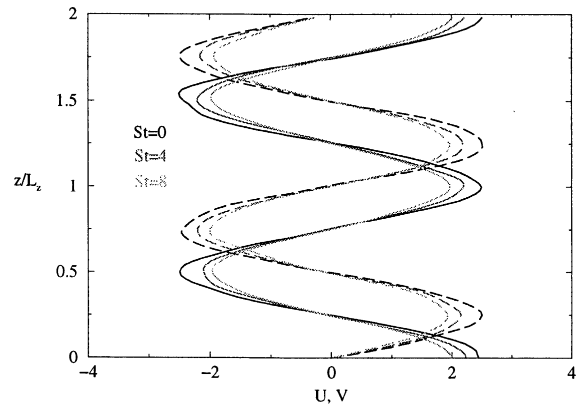

Figure 6: Decay of the mean velocity profile for a stratified velocity spiral simulation with $Ri = 0.1$ and $k_z = 2$.

The decay of the mean velocity for a stratified velocity spiral simulation with wavenumber $k_z = 2$ is shown in figure 6. Again a qualitatively similar decay is observed compared to the unstratified simulation shown in figure 4.

Richardson Number Variation

In this section, the results of a variation of the Richardson number Ri are presented. The value of the Richardson number is varied from $Ri = 0$ (corresponding to unstratified flow) to $Ri = 1$ (corresponding to strongly stratified flow) for simulations with a linear velocity profile and for velocity spiral simulations with a wavenumber $k_z = 1$.

Figure 7 shows the evolution of the turbulent kinetic energy K with non-dimensional time St. All simulations show an initial decay of K due to the isotropic initial conditions.

The solid lines represent the turbulent kinetic energy K of simulations with a vertical linear velocity profile. The weakly stratified cases with $Ri < 0.1$ show growth of K and the strongly stratified cases with $Ri > 0.1$ show decay of K. For the critical value of the Richardson number $Ri_{cr} = 0.1$, an approximately constant evolution of the turbulent kinetic energy is obtained.

The dashed lines represent the turbulent kinetic energy K of velocity spiral simulations. The wavenumber $k_z = 1$ is fixed in this series of simulations. For a given Richardson number Ri, the evolution of K is again remarkably similar for cases of both series of simulations. The similarity lasts up to a non-dimensional time of about $St = 6$ for the unstratified and weakly stratified cases. For the strongly stratified cases with $Ri = 0.5$ and $Ri = 1$ the similarity is qualitatively different. In the strongly stratified cases, the similarity is preserved throughout the simulations.

The decay of the mean velocity profiles of U and V of the velocity spiral simulations is shown in figure 8. The figure shows the mean velocity profile of the initial data at non-dimensional time $St = 0$ as well as the profiles at non-dimensional time $St = 8$ for three simulations with $Ri = 0$, $Ri = 0.1$, and $Ri = 0.5$. The strength of the decay decreases with increasing Richardson number Ri. The strongly stratified simulations therefore show the closest similarity with the velocity magnitude variation simulations that have fixed mean velocity profiles.

Another explanation for the close similarity of the large Richardson number cases can be given by a consideration of vertical length scales. Figure 9 shows the dependence of the Ellison scale $L_E = \rho/S_\rho$ on the Richardson number Ri at non-dimensional time $St = 8$. The Ellison scale is a measure for the size of vertical density overturns. It is compared to the Ozmidov scale $L_O = \sqrt{\epsilon/N^3}$ that gives an upper bound for the size of turbulent overturns in stratified flow. For Richardson numbers $Ri < 0.2$, the Ozmidov scale L_O is larger than the Ellison scale L_E and density overturns are not strongly affected by stratification. However, for $Ri > 0.2$, the Ozmidov scale and the Ellison scale are of comparable size. Here, the size of vertical density overturns is restricted by the Ozmidov scale.

In the velocity spiral simulations, the vertical scale of the flow is therefore restricted by stratification for large Richardson numbers. The vertical scale of the flow always remains small compared to the length of the velocity spiral. Therefore, the close similarity between the linear velocity profile simulations and the velocity spiral simulations observed in the large

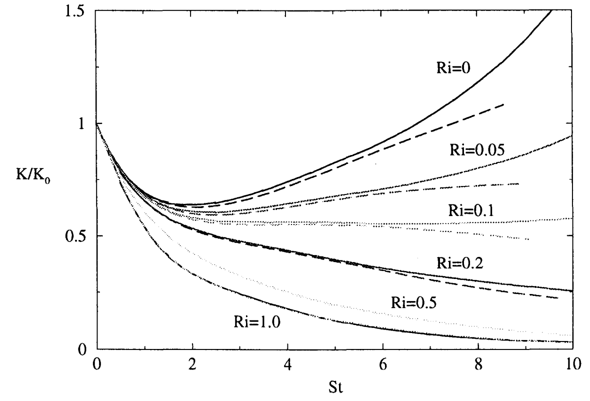

Figure 7: Evolution of the turbulent kinetic energy K for linear velocity profile simulations (solid lines) and velocity spiral simulations (dashed lines). The Richardson number is varied from $Ri = 0$ to $Ri = 1$. The wavenumber $k_z = 1$ is fixed in the velocity spiral simulations.

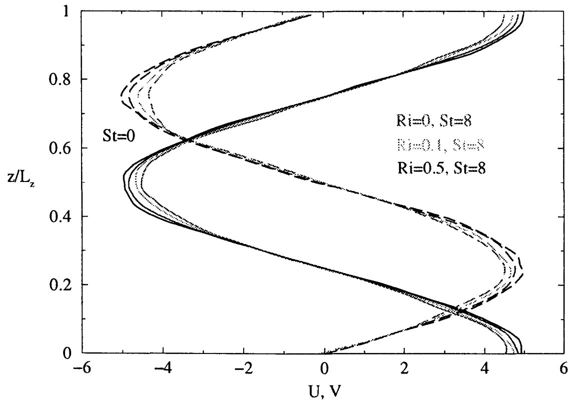

Figure 8: Decay of the mean velocity profiles of U and V. The figure shows the initial mean velocity profiles at $St = 0$ (largest amplitude) and the mean velocity profiles at $St = 8$ for simulations with $Ri = 0.5$, $Ri = 0.1$, and $Ri = 0$ (with decreasing amplitude).

Richardson number cases lasts throughout the simulations.

SUMMARY

Direct numerical simulations of turbulent stratified shear flow have been performed. Two different types of shear flows are compared. The first type has a linear velocity profile. In this flow, the magnitude of the mean velocity varies in the vertical direction, but the direction of the flow remains constant. This flow has been studied extensively in the past. The second type has a velocity spiral. In this flow, the magnitude of the mean velocity is constant, but the direction is varied in the vertical.

Similar results are obtained for both types of shear flows. The growth of the turbulent kinetic energy weakens as the Richardson number Ri is increased. The similarity of the evolution of K between the two cases was found to increase (1) for simulations with small

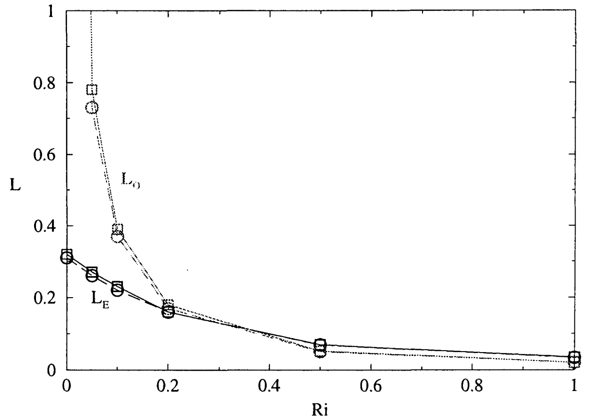

Figure 9: Dependence of the Ellison scale L_E and the Ozmidov scale L_O on the Richardson number Ri at non-dimensional time $St = 8$. The solid lines correspond to simulations with a linear mean velocity profile and the dashed lines correspond to velocity spiral simulations.

wavenumbers k_z and (2) for simulations with large Richardson numbers Ri.

For simulations with smaller wavenumbers k_z and therefore larger spiral lengths L_z, the vertical scale of the turbulent motion remains small compared to L_z for a longer time of the simulations. Therefore the similarity between the two cases is increased.

For simulations with larger Richardson numbers Ri and therefore stronger stratification, the vertical scale of turbulent density overturns is restricted by the Ozmidov scale. The vertical scale of the motion always remains small compared to the spiral length and strong similarity between the two cases is observed throughout the simulations.

References

Caldwell, D.R., 1987, "Small-scale physics of the ocean", *Rev. Geophys.*, Vol. 25, pp. 183–192.

Caldwell, D.R., and Moum, J.N., 1995, "Turbulence and mixing in the ocean", *Rev. Geophys.*, Vol. 33, pp. 1385–1394.

Gerz, T., Schumann, U., and Elghobashi, S.E., 1989, "Direct numerical simulation of stratified homogeneous turbulent shear flows", *J. Fluid Mech.*, Vol. 200, pp. 563–594.

Holt, S.E., Koseff, J.R., and Ferziger, J.H., 1992, "A numerical study of the evolution and structure of homogeneous stably stratified sheared turbulence", *J. Fluid Mech.*, Vol. 237, pp. 499–539.

Howard, L.N., 1961, "Note on a paper of John W. Miles", *J. Fluid Mech.*, Vol. 10, 509–512.

Itsweire, E.C., Koseff, J.R., Briggs, D.A., and Ferziger, J.H., 1993, "Turbulence in stratified shear flows: Implications for interpreting shear-induced mixing in the ocean", *J. Phys. Oceanogr.*, Vol. 23, pp. 1508–1522.

Jacobitz, F.G., Sarkar, S., and Van Atta, C.W., 1997, "Direct numerical simulations of the turbulence evolution in a uniformly sheared and stably stratified flow", *J. Fluid Mech.*, Vol. 342, pp. 231–261.

Jacobitz, F.G., and Sarkar, S., 1998, "The effect of nonvertical shear on turbulence in a stably stratified medium", *Phys. Fluids*, Vol. 10, pp. 1158–1168.

Jacobitz, F.G., 2000, "Scalar transport and mixing in turbulent stratified shear flow", *Int. J. Heat Fluid Flow*, Vol. 21, pp. 535-541.

Kaltenbach, H.-J., Gerz, T., and Schumann, U., 1994, "Large-eddy simulation of homogeneous turbulence and diffusion in stably stratified shear flow", *J. Fluid Mech.*, Vol. 280, pp. 1–40.

Komori, S., Ueda, H., Ogino, F., and Mizushina, T.,1983, "Turbulence structure in stably stratified open-channel flow", *J. Fluid Mech.*, Vol. 130, pp. 13–26.

Miles, J.W., 1961, "On the stability of heterogeneous shear flows", *J. Fluid Mech.*, Vol. 10, 496–508.

Pedlosky, J., 1986, *Geophysical fluid dynamics*, second edition, Springer-Verlag.

Piccirillo, P.S., and Van Atta, C.W., 1997, "The evolution of a uniformly sheared thermally stratified turbulent flow", *J. Fluid Mech.*, Vol. 334, pp. 61–86.

Richardson, L.F., 1920, "The supply of energy from and to atmospheric eddies", *Proc. Roy. Soc. A*, Vol. 97, 354–373.

Rohr, J.J., Itsweire, E.C., Helland, K.N., and Van Atta, C.W., 1988, "Growth and decay of turbulence in a stably stratified shear flow", *J. Fluid Mech.*, Vol. 195, pp. 77–111.

Taylor, G.I., 1931, "Effect of variation in density on the stability of superposed streams of fluid", *Proc. Roy. Soc. A*, Vol. 132, 499–523.