DETERMINING THE CASCADE OF KINETIC ENERGY AND SCALAR VARIANCE IN THE LOWER STRATOSPHERE

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ABSTRACT

This paper sum up some of the most importand results recently published in Cho & Lindborg (2001) and Lindborg & Cho (2000; 2001). We presents a successful observational determination of the cascade of kinetic energy and temperature and ozone variances in the mesoscale range (~ 1 to 500 km) of scales of motion in the lower stratosphere. The direction and strength of the cascade is determined by calculating third order structure functions using a huge data set (MOZAIC) of wind, temperature and ozone measurements taken from commercial aircraft. It is found that the cascade is downscale, *i.e.* energy and scalar variance are transferred from large to small scales.

INTRODUCTION

A central concept in the theory of nonlinear dynamics is the cascade, described in the famous poem by Richardsson (1922): "big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls, and so on to viscosity". The cascade concept was The paradigm of a cascade is three-dimensional high Reynolds number hydrodynamic turbulence, for which the turbulent kinetic energy spectrum in a broad range of wave numbers is determined by energy is transferred from large to small scales, also called the spectral energy flux. This flux must also be equal to the mean dissipation, ϵ, of kinetic energy into heat. From dimensional analysis we thus obtain

$$E(k) = C\epsilon^{2/3}k^{-5/3}\,, \qquad (1)$$

where C is a dimensionless constant.

Yaglom (1949), Obukhov (1949) and Corrsin (1951) developed the same type of the-

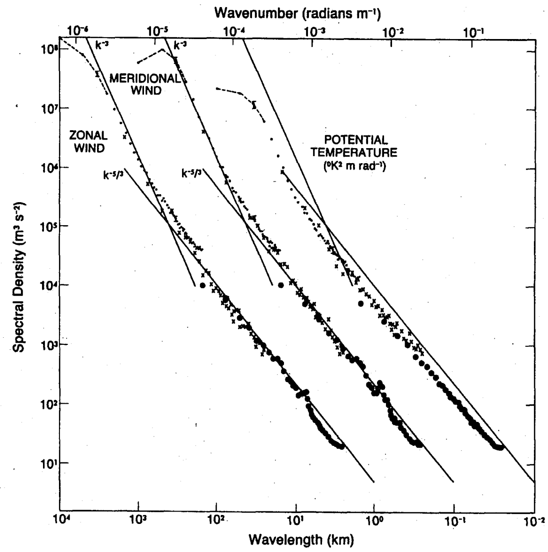

Figure 1: From the left to right: variance power spectra of zonal wind, meridional wind (m^3s^{-2}), and potential temperature (K^2m) near the tropopause from Globa Atmospheric Sampling Program aircraft data. The spectra for meridonal wind and temperature are shifted one and two decades to the right, respectively. Reproduced form Nastrom *et al.* (1996).

ory for a passive scalar, θ, in a turbulent hydrodynamic field. By similar arguments, the spectrum, F(k), of the passive scalar variance is obtained as

$$F(k) = C_\theta\epsilon_\theta\epsilon^{-1/3}k^{-5/3}\,, \qquad (2)$$

where C_θ is a dimensionless constant and ϵ_θ is the rate at which the passive scalar variance is smeared at the length scale of molecular diffusion.

Kinetic energy spectra, as well as spectra of temperature and ozone (Nastrom & Gage 1985, Nastrom *et al* 1986, Cho *et al.* 1999) from the upper troposphere and lower stratosphere, often exhibit a rather broad range with a $k^{-5/3}$-dependence, in the mesoscale region of length scales $\sim 10 - 500$ km. In fig. 1 the kinetic energy spectra and temperature spectra are reproduced from Nastrom & Gage (1985). Although these spectra cannot, in a simple way, be explained by classical three-dimensional turbulence, concepts from turbulence theory may be crucial to reach an understanding of the physics behind them, which is a problem that has been under scientific debate for over two decades. A key issue in this debate is the direction of the kinetic energy flux, Π_u, from small to large wave numbers, *i.e* the rate of kinetic energy which is transferred from large scale motions to small scale motions. Two opposite hypotheses have been put forward regarding the sign, or the direction, of the flux of kinetic energy.

First, there is the hypothesis (Bretherton 1969; Dewan 1979; 1997) that the kinetic energy flux is in the direction from small to large wave numbers. According to this hypothesis long gravity waves break down to shorter waves in a cascade process which is similar to three-dimensional Kolmogorov turbulence, resulting in a positive kinetic energy flux. The only parameter which can determine the spectrum is the energy flux and from dimensional considerations we obtain a $k^{-5/3}$-spectrum. At the shortest wavelengths, 100-1000 m say, a more violent instability occasionally sets in; the wave energy is broken down in intermittent spots of three-dimensional turbulence, and finally dissipated at scales of the order of 1 cm.

Secondly, there is the hypothesis (Gage 1979; Lilly 1983) that the $k^{-5/3}$-spectrum is the spectrum of two-dimensional turbulence with a negative energy flux, *i.e.* a flux from small to large scales, in accordance with Kraichnan's theory of two-dimensional turbulence (1967; 1970). Such a range naturally emerges in two-dimensional direct numerical simulations (DNS) with forcing at large wave numbers (Smith & Yakhot 1994; Maltrud & Vallis 1991).

Lindborg (1999) (hereafter L99) pointed out, that Kolmogorov's (1941) classical four-fifth law for the third order velocity structure function, may be the clue to an experimental determination of the flux of kinetic energy. This law is a consequence of a very special balance in the governing dynamic equations. The balance can be written (see Frisch 1995)

$$\nabla_r \cdot \langle \delta \mathbf{u} \delta \mathbf{u} \cdot \delta \mathbf{u} \rangle = -4\Pi \qquad (3)$$

where $\delta \mathbf{u} = \mathbf{u}' - \mathbf{u}$ is the difference between the velocities at two points separated by the vector $\mathbf{r}$ and $\langle\rangle$ is an ensemble average. In the three-dimensional case integration of (3) gives the four-fifth law for the third order longitudinal structure function. In the two-dimensional, or quasi-two-dimensional case, integration of (3) yields

$$\langle \delta u_L \delta u_L \delta u_L \rangle + \langle \delta u_L \delta u_T \delta u_T \rangle = -2\Pi r \qquad (4)$$

where L indicates the longitudinal direction, *i.e.* the direction of $\mathbf{r}$, and T a transverse direction, *i.e.* a direction perpendicular to $\mathbf{r}$. It can be argued (Lindborg 1999, Lindborg & Cho 2001) that (4) should hold both in the case when Π is positive (forward cascade) and in the case when Π is negative (inverse cascade).

For a scalar, θ, advected by a turbulent velocity field, Yaglom (1949) derived a relation similar to (3),

$$\nabla_r \cdot \langle \delta \mathbf{u} \delta\theta \delta\theta \rangle = -4\epsilon_\theta \, . \qquad (5)$$

In the three-dimensional case, integration of (5) yields the four-third law,

$$\langle \delta u_L \delta\theta \delta\theta \rangle = -4/3 \epsilon_\theta r \, , \qquad (6)$$

which has been experimentally confirmed in several studies, *e.g.* in an atmospheric boundary layer by Antonia *et al.* (1995) and in wind tunnel grid turbulence by Mydlarski & Warhaft (1998). In both these studies temperature served as the scalar. In the atmospheric boundary layer the typical length scale where (6) holds is $r \sim 1 - 10$ m, while in the wind tunnel, $r \sim 1 - 10$ cm. In the two-dimensional, or quasi two-dimensional case, integration of (5) yields

$$\langle \delta u_L \delta\theta \delta\theta \rangle = -2\epsilon_\theta r \, . \qquad (7)$$

In a highly stratified fluid, with a strong degree of system rotation, we find it reasonable to apply the two-dimensional, rather than the three-dimensional, form of the Yaglom relation.

METHODOLOGY

Using airplane data from the Measurements of Ozone and Water Vapor by Airbus In-Service Aircraft (MOZAIC) data set (see Marenco *et al.* 1996), we have calculated the the third order velocity structure function (4) and the velocity-scalar structure function (7) for temperature and ozone. To calculate the structure functions we closely followed the procedure outlined in L99. However, there were some differences such as the following. First, our data consisted of a somewhat expanded

MOZAIC set (August 1994 to December 1997, 7630 flights), whereas L99 used August 1994 to April 1997 with 5754 flights. Second, in order to avoid variability caused by sudden changes in altitude, we specified that the pairs of points used for the structure function computations lie within the same standard flight level. There were five flight levels from 9.4 to 11.8 km, each spaced 600 m apart. The adherence to the standard levels were quite strict. Third, we used the ozone measurement to differentiate between the troposphere and stratosphere. If the ozone levels associated with the pair of data points were both under 100 ppbv, then the result was binned as tropospheric. If the ozone concentrations for the data-point pair were both over 200 ppbv, then the result was classified as stratospheric. If one point was in the troposphere and the other point was in the stratosphere, the result was not included in either group. The two thresholds were chosen from the ozone concentration histograms for MOZAIC cruise levels (Thouret *et al.* 1996). There were few points in the stratosphere equatorward of 30°, since the tropical tropopause was usually above the flight altitude.

Note also that because the latitude-longitude coordinates for the Air France aircraft data were purposely degraded to about a 1-minute resolution (a result of pilot union concerns), we linearly interpolated the position to the original 4-s resolution. This was also done by L99 but was not stated in the paper. The data from the other airlines (Sabena, Lufthansa, and Austrian Airlines) were not altered.

Following L99 we used a set of 140 separation distances between 2 and 2510 km. For every data point the other end point was found by searching through the rest of the flight for the point coming closest to the specified r. If that distance was not within 1 km of the specified r, then the velocity difference was not computed. The separation distance r was calculated using the latitude-longitude coordinates, the altitudes, and an elliptical polar model of the Earth radius, $R = AB/(1-B\cos\phi)$, where $A = 1.8959460\times10^6$ km, $B = 3.3528129\times10^{-3}$, and ϕ is latitude. The separation distance was taken to be the arc length, not the straight-line distance.

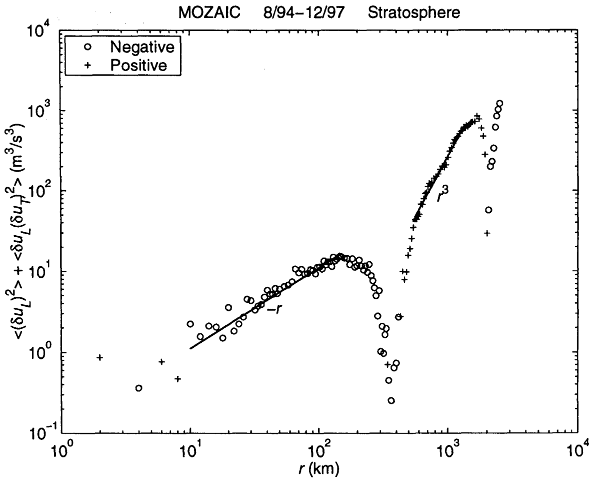

Figure 2: Third order velocity-temperature structure function versus separation distance

RESULTS

In fig 2. we have plotted the stratospheric third order structure function (4). The plot displays a negative linear range for 10km < r < 200km. By fitting the data to a straight line in this range, and using equation (4) we can estimate the flux Π of kinetic energy from large to small scales, alternatively the mean dissipation per unit mass to

$$\langle\epsilon\rangle = 6\cdot10^{-5}\,\mathrm{m^2/s^3}\,. \qquad (8)$$

This is a very reasonable value which falls somewhere in the middle of the wide range of previous estimates from various types of measurements. Reviews of previous measurements can be found in (Crane 1980, Hocking & Mu 1997, Vinnichenko 1969, Dewan 1997) .

Two different types of measurements of the mean dissipation in the stratosphere are reported in the literature. First, there is the method of radar observation (Crane 1980, Hocking & Mu 1997). The quantity directly measured by the radar is the scattering cross section per unit volume expressed as C_n^2. To estimate the dissipation from this quantity some rather elaborate model assumptions are required. Secondly there are airplane (Lilly *et al.* 1974, Vinnichenko 1969) or balloon (Cadet 1977) measurements of the three-dimensional turbulence inertial range energy spectra, alternatively second order structure functions, performed in more or less localized turbulence spots. If it is assumed that the Kolmogorov constant is known, then the mean dissipation can be estimated, either from the measured spectrum or from the structure function.

There are important principal differences between our method of estimating the mean dissipation and these two methods. To estimate an overall mean value of the dissipation either from radar measurements or from direct turbulence measurements, it is necessary to average over a wide range of subsets of data, for which the individual values of the dissipation can vary by over two orders of magnitude, due to the very large spatial and temporal variation of this quantity. It is not easy to estimate the relative weights of these subsets.

Our estimate is based on a time and space average of a quantity describing properties of the flow field at scales from 10 to 200 km. Whatever the nature is of the underlying dynamical process of our data, the process is definitely not as rarely occuring as three-dimensional turbulence. The quantity which we actually measure is the kinetic energy flux, at larger scales. We also use a huge data set, with samples from different locations. This should give us a more representative overall mean value.

As compared to radar measurements, our estimate rely on an analytical relation (4) with far more theoretical justification than the assumptions which are required to estimate the mean dissipation from radar data.

As compared to *in-situ* measurements from airplanes or balloons, there is another important difference. Our estimate relies on a relation (4) with a linear dependence of the dissipation. This means that we will measure a true mean value, even though there may be a very large spatial and temporal variation in the set of individual samples which the mean value is based on. The measured mean value over two or more subsets of data, for which the mean dissipation are very different, will actually be the true mean value. This is not true if we rely on non-linear relation such as (1) (see Landau & Lifshitz 1975, p. 140). A necessary condition to use such a relation for an estimate of the dissipation is that there is not to much variation of the turbulence intensity in each data set which is used.

In fig. 3 the third order temperature-velocity structure function, $\langle \delta u_L \delta T \delta T \rangle$, is plotted. There is a fairly clean negative linear range, $r \sim 20 - 400$km, which is the range corresponding to the observed $k^{-5/3}$-spectrum. Using the two-dimensional Yaglom relation (7) a linear fit in this range gives

$$\epsilon_T = 3.9 \cdot 10^{-6}\,\mathrm{K}^2/\mathrm{s}\,, \qquad (9)$$

for the flux, alternatively the dissipation, of temperature variance.

Temperature fluctations are associated with vertical displacements the flux of temperature

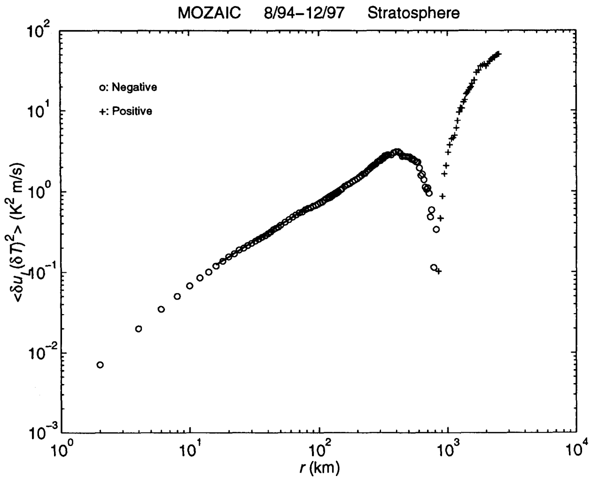

Figure 3: Third order velocity-temperature structure function versus separation distance

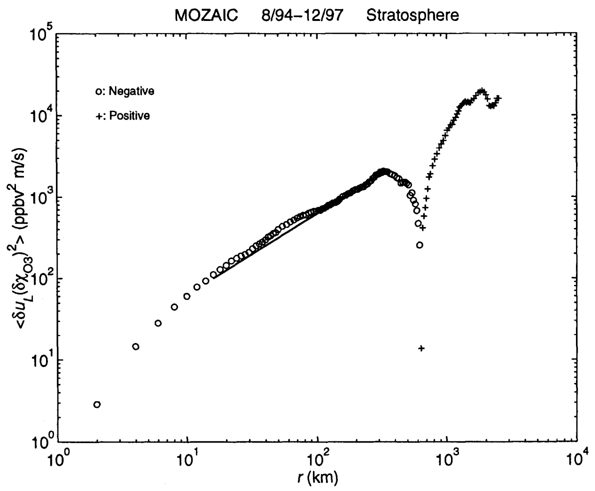

Figure 4: Third order velocity-ozone structure function versus separation distance

variance is most probably associated with a corresponding flux of potential energy, Π_{pot}, in the gravity field. This can be estimated to

$$\Pi_{pot} \sim \frac{g^2}{N^2 T_o^2}\epsilon_T \approx 2 \cdot 10^{-5}\mathrm{m}^2\mathrm{s}^3\,, \qquad (10)$$

where N is the Brunt-Visl frequency, g the gravity acceleration and T_o the mean temperature. The estimated value is comparable to the estimated flux of kinetic energy.

In fig. 4, we have plotted the third order ozone-velocity structure function, $\langle \delta u_L \delta O_3 \delta O_3 \rangle$. The shape of the function is very similar to the corresponding temperature curve, although the linear behaviour is not as clean. A linear fit gives

$$\epsilon_{O_3} = 3.1 \cdot 10^{-3}\,\mathrm{ppbv}^2/\mathrm{s}\,, \qquad (11)$$

for the flux, alternatively the dissipation, of ozone variance. All three curves show a rather abrupt change of sign at $r \sim 500 - 1000$ km, which is approximately the Rossby deformation radius. We shall not try to explain this, but only point out, first, that the Coriolis force probably is crucial for the understanding of this change of sign, and secondly, that the assumption of statistical homogeneity, which the Yaglom relation rests on, can be questioned at scales of the order of 1000 km.

CONCLUSIONS

By measuring third order structure functions of velocity, velocity-temperature and velocity ozone, from measurements reported in the MOZAIC data base, the direction and magnitude of the mesoscale cascade in the lower stratosphere, could be determined. The negative linear range in fig. 2, 3 and 4, show beyond all reasonable doubt that the previously reported $k^{-5/3}$ mesoscale horizontal spectra are associated with a downward energy flux, just as in three dimensional turbulence. However, the mechanism behind the cascade cannot be three-dimensional isotropic turbulence, since the horizontal scales are much larger than the relevant vertical scales. The gravity wave hypothesis seems to provide the most reasonable explanation of the cascade. In a future study this hypothesis will be further investigated by means of Direct Numerical Simulations of a stratified rotating fluid.

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