A ROLE OF ELASTIC ENERGY IN TURBULENT DRAG REDUCTION BY POLYMER ADDITIVES

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ABSTRACT

In the present study, the mechanism of turbulent drag reduction by polymer additives in a fully developed channel flow is investigated using direct numerical simulation. In order to see the elastic effect on turbulent drag reduction, the dilute polymer solution is expressed with an Oldroyd-B model which shows a linear elastic behavior. Simulations are carried out by changing the Weissenberg number at the Reynolds numbers of 3000 and 15000. The onset criterion for drag reduction predicted in the present study shows good agreement with previous theoretical and experimental studies. In addition, the turbulence statistics such as the mean streamwise velocity and rms velocity fluctuations are also in good agreements with previous experimental observations. The kinetic and elastic energy transport equations are derived to investigate the effect of elasticity on drag reduction. It is shown that the polymer stores the elastic energy from the flow in the sublayer and then releases again in the sublayer when the relaxation time is short (no drag reduction). However, when the relaxation time is long enough (drag reduction), the elastic energy is transported to and released in the buffer layer. Therefore, drag reduction occurs when the turbulent velocity scale is larger than the characteristic velocity scale of the polymer solution.

INTRODUCTION

Since Toms (1949) reported turbulent drag reduction by polymer additives, many studies on this phenomenon have been carried out. The first explanation about drag reduction is 'time criterion' (Lumley 1969) which says that drag reduction occurs when the relaxation time of the polymer solution is longer than the turbulent time scale of motion. In this scenario, drag reduction comes from the elongational viscosity which is increased greatly by 'coil-stretch' transition under the time criterion. However some studies (de Gennes 1990; Sreenivasan & White 2000) criticized the scenario using the elongational viscosity by arguing that the 'coil-stretch' does not occur in turbulent flow.

In addition, the same criterion as the time criterion is derived from the ingenious work of Goldshtik *et al.* (1980) where they applied a perturbation method to viscoelastic models (Maxwell and Oldroyd-B models) and showed that the time criterion might come from the elastic effect of dilute polymer solution.

In the mean time, Tabor & de Gennes (1986) thought that the elastic energy stored in polymer molecules causes drag reduction (elastic theory). That is the polymer molecules absorb the small-scale turbulence energy by prohibiting the turbulent cascade, which results in drag reduction. Joseph (1990) explained that the shear-wave speed from elasticity is associated with the cut-off criterion for turbulent cascade.

Recently, direct numerical simulations (DNS) have been performed to investigate the mechanism of drag reduction (Orlandi 1995; den Toonder *et al.* 1997; Sureshkumar *et al.* 1997; Dimitropoulos *et al.* 1998; De Angelis *et al.* 1999). Orlandi (1995) and den Toonder (1997) adopted the elongational viscosity models and obtained drag reduction. However, such models are based on inelastic constitutive equations so that they cannot predict the onset criterion for drag reduction. Sureshkumar *et al.* (1997), Dimitropoulos *et al.* (1998) and de Angelis *et al.* (1999) adopted viscoelastic models but used low-order spatial discretization schemes (artificial diffusion scheme) to avoid numerical instability. Due to the excessive numerical diffusion introduced by the artificial diffusion scheme, higher elasticity (or higher Weissenberg number) is required to produce drag reduction in these studies (see Min *et al.* 2000).

The objective of the present study is to propose the mechanism of drag reduction by conducting direct numerical simulation of turbulent channel flow with polymer additives. In order to investigate the effect of elasticity, an Oldroyd-B model (linear Hookean dumbbells) is used instead of the FENE-P model (nonlinear Hookean dumbbells). The elastic theory is combined with the kinetic theory (Bird *et al.* 1987), and then the kinetic and elastic energy transport equations are derived. The energy transfer between the flow and polymer is examined through these equations, from which the drag-reduction mechanism is elucidated.

GOVERNING EQUATIONS AND NUMERICAL METHOD

The non-dimensional governing equations of unsteady incompressible viscoelastic flow represented by an Oldroyd-B model are as follows:

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\beta}{Re}\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{1-\beta}{Re}\frac{\partial \tau_{ij}}{\partial x_j}, \quad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0, \quad (2)$$

$$\tau_{ij} + We\left(\frac{\partial \tau_{ij}}{\partial t} + u_m \frac{\partial \tau_{ij}}{\partial x_m} - \frac{\partial u_i}{\partial x_m}\tau_{mj} - \frac{\partial u_j}{\partial x_m}\tau_{mi}\right) = 0, \quad (3)$$

where u_i is the velocity, p is the pressure, τ_{ij} is the polymeric stress, $Re = U\delta/\nu$ is the Reynolds number, $We = \lambda U/\delta$ is the Weissenberg number, U is the centerline velocity of the fully developed laminar flow, δ is the channel half height, λ is the relaxation time and β is the ratio of solvent viscosity contribution to total viscosity of solution. In the present study, β is fixed at 0.9 for the case of viscoelastic fluid. For $Re = 3000$ ($Re_\tau = u_{\tau 0}\delta/\nu \simeq 135$), a calculation domain of $7\delta \times 2\delta \times 3.5\delta$ is chosen in the streamwise (x), wall-normal (y) and spanwise (z) directions, respectively, with $64 \times 97 \times 96$ grids. Here $u_{\tau 0}$ is the wall shear velocity for Newtonian fluid flow ($\beta = 1$). For $Re = 15000$ ($Re_\tau \simeq 530$), the minimal channel concept by Jiménez and Moin (1991) is adopted and a calculation domain of $2.4\delta \times 2\delta \times 0.9\delta$ is chosen with $128 \times 257 \times 96$ grids. We impose the periodic boundary condition in the streamwise and spanwise directions, and the no-slip boundary condition in the wall-normal direction. A fully developed turbulent flow field for a Newtonian fluid ($\beta = 1$) is used as an initial condition for the simulation of viscoelastic fluid flow.

The numerical algorithm is based on a semi-implicit, fractional step method. A fourth-order compact difference scheme (Lele 1992) is used for the polymer stress derivative $\partial\tau_{ij}/\partial x_j$ in (1), and a modified compact upwind difference scheme (MCUD3, Min *et al.* 2000) is used for the polymer stress convection term $u_m\partial\tau_{ij}/\partial x_m$ in (3). All other terms are discretized using the second-order central difference scheme.

There is no diffusion term in the constitutive equation (3) so that the amplification of small numerical disturbances breaks down numerical solutions even at modest Weissenberg numbers. In order to overcome the numerical breakdown, numerical methods involving low-order spatial discretization scheme (artificial diffusion method, AD) have been used in the previous direct numerical simulations (Sureshkumar *et al.* 1997; Dimitropoulos *et al.* 1998; de Angelis *et al.* 1999), even though they smear the steep gradients of the polymeric stresses. Figure 1 shows the time histories of the mean pressure gradient at $We = 2$ in the cases of using AD and MCUD3, respectively. It is clearly shown that, at $We = 2$, MCUD3 predicts drag reduction but AD does not due to excessive numerical diffusion.

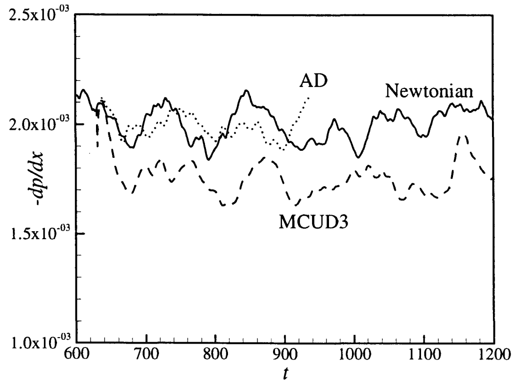


Figure 1: Time histories of the mean pressure gradient required to drive a fixed mass flow rate in a channel at $We = 2$ in the cases of AD and MCUD3.

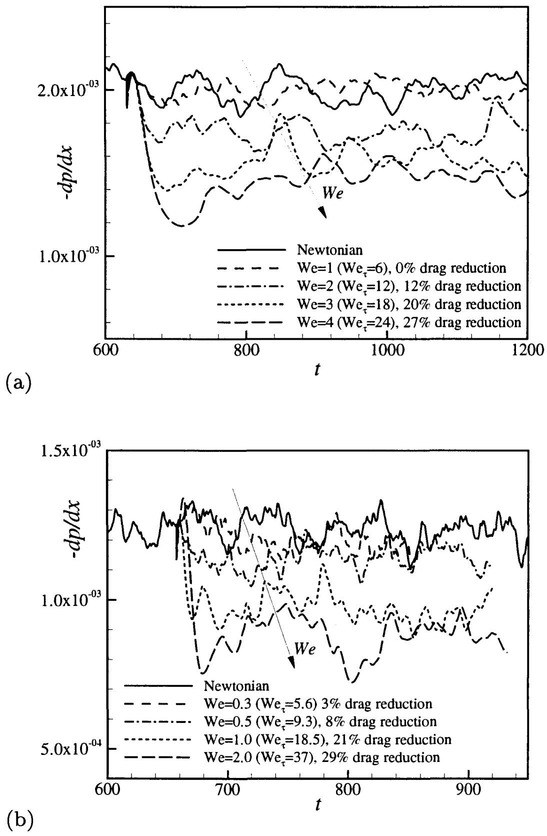


Figure 2: Time histories of the mean pressure gradient at different Weissenberg numbers: (a) Re=3000; (b) Re=15000.

RESULTS AND DISCUSSIONS

Changes in Drag and Velocity

Figure 2 shows the time histories of the mean pressure gradients at $Re = 3000$ and 15000. It is seen that drag decreases more with larger We and drag reduction occurs at $We_\tau = \lambda u_{\tau 0}^2/\nu > 5 \sim 6$.

Figure 3 shows the mean streamwise velocity and the rms streamwise and wall-normal velocity fluctuations in the cases of Newtonian ($\beta = 1$) and viscoelastic fluid flows ($We = 1$ and 2) at $Re = 15000$ ($Re_\tau \simeq 530$), together with the previous experimental results. The mean streamwise velocities show upward shifts at $We = 1$ and 2, as compared to the Newtonian case. The u_{rms} increases more with larger We but v_{rms} and w_{rms} (not shown here) decrease more with larger We. It is not easy to pick up the correct values of β and We from experimental results (see Joseph 1990). Since the experimental results of Luchik & Tiederman (1988, $Re_\tau \simeq 520$) and Wei & Willmarth (1992, $Re_\tau \simeq 570$) showed about 20% and 30% drag reduction, respectively, the present results at $We = 1$ and 2 (21% and 29% drag reduction, respectively) are compared with them. As shown in figure 3, present results are in good agreements with the experimental results of Luchik & Tiederman (1988) and Wei & Willmarth (1992).

Onset Criterion for Drag Reduction

The 'time criterion' suggested by Lumley (1969) indicated that the onset We_τ for drag reduction is 1. Berman (1977) showed that the onset We_τ ranges from 1 to 8 depending on the properties of polymers and solvents. Goldshtik *et al.* (1980) studied drag reduction theoretically and suggested that the onset We_τ is 1 for the Maxwell fluid and it is $5 \sim 6$ for the Oldroyd-B model with $\beta = 0.9$.

With DNS, Orlandi (1995) and den Toonder *et al.* (1997) could not suggest the onset We_τ because they used the inelastic constitutive models. Sureshkumar *et al.* (1997) and Dimitropoulos *et al.* (1998) showed that drag reduction occurred at $We_\tau = 25$ but not at $We_\tau = 12.5$. As shown in figure 1, when AD is used (as was done in Sureshkumar *et al.* 1997 and Dimitropoulos *et al.* 1998), drag reduction does not occur at $We_\tau = 12$ ($We = 2$ at

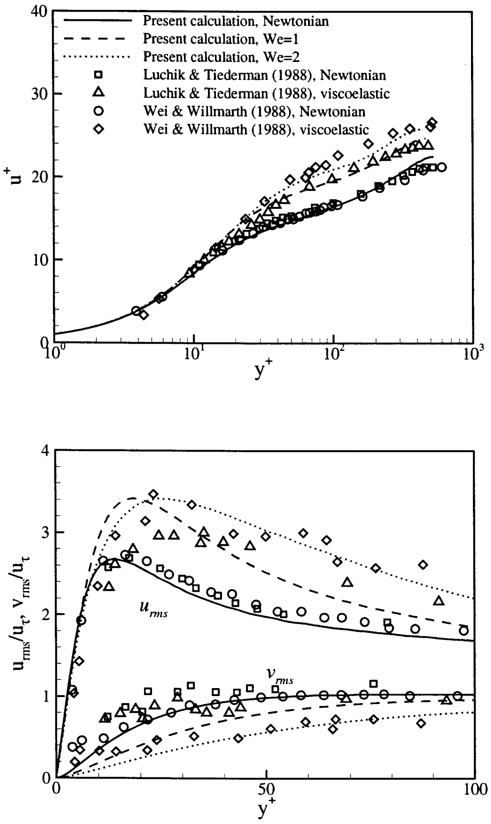


Figure 3: (a) Mean streamwise velocities; (b) rms velocity fluctuations. $Re = 15000$.

Re = 3000) while it occurs when MCUD3 is used. Therefore the onset criterion suggested by Sureshkumar *et al.* (1997) and Dimitropoulos *et al.* (1998) may have to be modified by considering the effect of the excessive numerical diffusion on drag.

Joseph (1990) suggested another onset criterion that drag reduction occurs when the velocity scale of turbulence u_τ exceeds the shear wave velocity $u_c = \sqrt{\nu/\lambda}$. Joseph's hypothesis is valid for Maxwell fluid and it can be expressed for the Oldroyd-B model (see Joseph 1990) as

$$u_\tau > u_c = \sqrt{\frac{\nu}{\lambda}} = \sqrt{\frac{1}{Re \cdot We \cdot (1-\beta)}}. \qquad (4)$$

At $Re = 3000$, $u_\tau/U = 0.04485$ and $u_c/U = 0.05774$ and 0.04082 for $We = 1$ and 2, respectively. At $Re = 15000$, $u_\tau/U = 0.03512$ and $u_c/U = 0.04714$ and 0.03641 for $We = 0.3$ and 0.5, respectively. Thus the present result at $Re = 3000$ shows in good agreement with the onset criterion suggested by Joseph (1990). However, the result at $Re = 15000$ does not satisfy the criterion (4). This may be due to the fact that the present simulation at $Re = 15000$ was conducted in a minimal channel. Thus, a further study is needed to clarify this issue at high Re.

Kinetic and Elastic Energy Transport Equations

The first idea of the elastic theory is found in Tabor & de Gennes (1986). They thought that polymers in solvents behave as an elastic spring. If the spring is a linear one, the elastic energy per unit volume stored by polymers can be expressed as

$$k_e = \frac{1}{2} nG \left(\left\langle Q^2 \right\rangle - \left\langle Q^2 \right\rangle_{eq} \right), \qquad (5)$$

where n is the number of polymer molecules per unit volume, G is the elastic modulus, $\langle Q^2 \rangle$ is the ensemble average of polymer length squared and the subscript eq denotes the equilibrium state. Applying the kinetic theory (Bird *et al.* 1987) to the elastic theory, one can obtain the kinetic and elastic energy transport equations as follows:

$$\left\langle \frac{Dk_m}{Dt} \right\rangle = -\langle P_k \rangle - \langle P_{e,m} \rangle + \frac{\beta}{Re} \left\langle \frac{d^2 k_m}{dy^2} \right\rangle, \qquad (6)$$

$$\left\langle \frac{Dk}{Dt} \right\rangle = \langle P_k \rangle - \langle \epsilon_k \rangle - \langle P_{e,t} \rangle + \frac{\beta}{Re} \left\langle \frac{d^2 k}{dy^2} \right\rangle, \qquad (7)$$

$$\left\langle \frac{Dk_e}{Dt} \right\rangle = \langle P_{e,m} \rangle + \langle P_{e,t} \rangle - \frac{1}{We} \langle k_e \rangle, \qquad (8)$$

where

$$P_k = -\overline{u'v'} \frac{d\overline{u}}{dy}, \; P_{e,m} = \frac{1-\beta}{Re} \overline{\tau}_{12} \frac{d\overline{u}}{dy},$$

$$\epsilon_k = \frac{\beta_2}{Re} \overline{\frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j}}, \; P_{e,t} = \frac{1-\beta}{Re} \overline{\frac{\partial u_i'}{\partial x_j} \tau_{ij}'}.$$

Here $k_m = \frac{1}{2}\overline{u}^2$, $k = \frac{1}{2}\overline{u_i' u_i'}$, $\langle \cdot \rangle = (1/V) \int \cdot dV$ and V is the total volume of the domain.

Equations (6)-(8) provide the information about the energy transfer between the polymer and the flow. The energy transfer between the mean kinetic energy k_m and the turbulent kinetic energy k is executed through P_k. The energy transfer between the mean kinetic energy k_m and the elastic energy k_e is done through $P_{e,m}$. The energy transfer between the turbulent kinetic energy k and the elastic energy k_e is done through $P_{e,t}$. The turbulent kinetic energy k is dissipated by ϵ_k and the elastic energy k_e is dissipated by itself.

Drag-Reduction Mechanism

Figure 4 shows the time histories of $\langle P_{e,m}\rangle$, $\langle P_{e,t}\rangle$ and $-\langle k_e\rangle/We$ at Re=3000. It is seen that $\langle P_{e,m}\rangle$, $\langle P_{e,t}\rangle$ and $-\langle k_e\rangle/We$ are nearly constant in time in the case of no drag reduction ($We = 1$). That is, the energy transfer between the flow and polymer is nearly steady. However, when the drag reduction occurs ($We = 4$), the energy transfer becomes quite unsteady.

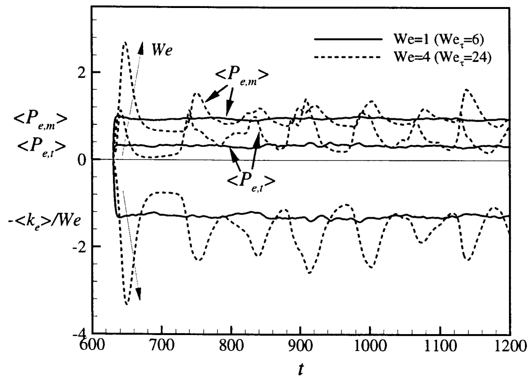


Figure 4: Time histories of $\langle P_{e,m}\rangle$, $\langle P_{e,t}\rangle$ and $-\langle k_e\rangle/We$ at Re=3000.

In order to find the reason that unsteady energy transfer occurs at $We = 4$, the spatial distribution of each term in the energy transport equations is shown in figure 5. It is seen that polymers store and release most of energy in the sublayer. As the We increases, $P_{e,m}$ decreases in the sublayer while it increases in the buffer layer. That is, the elastic energy stored in the sublayer is transported to and released in the buffer layer when drag reduction occurs.

In order to see that the near-wall elastic energy is transported to the buffer layer, an instantaneous field of velocity vectors and contours of polymer elastic energy at an $y-z$ plane is shown in figure 6. It is seen that the high elastic energy exists only near the wall when drag reduction does not occur ($We = 1$). However, when drag reduction occurs ($We = 4$), the high elastic energy is separated from the wall and transported to the buffer layer.

It is the relaxation time of the polymer solution that changes the behaviors of $P_{e,m}$ and $P_{e,t}$ at higher We. The fluid particle containing high elastic energy in the sublayer is lifted

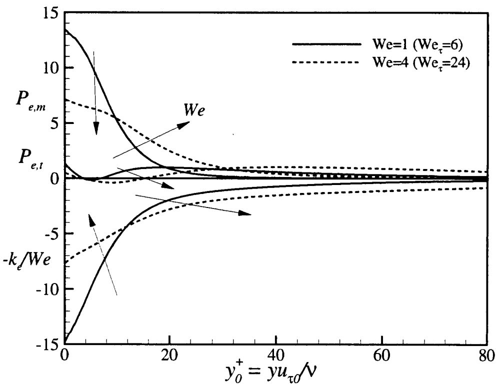
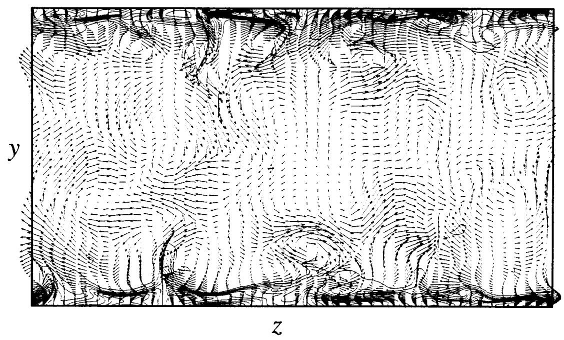
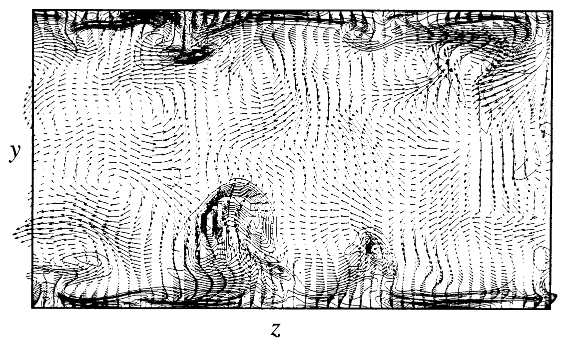


Figure 5: Spatial distributions of $P_{e,m}$, $P_{e,t}$ and $-k_e/We$ at Re=3000.



(a)



(b)

Figure 6: Instantaneous field represented by velocity vectors and contours of polymer elastic energy at an $y-z$ plane: (a) $We = 1$ (no drag reduction); (b) $We = 4$ (drag reduction).

away from the wall by near-wall turbulent motion. When the relaxation time is short, the particle releases the elastic energy in the sublayer before it reaches the buffer layer. When the relaxation time is long enough, however, the elastic energy is delivered to the buffer layer and released there. Therefore, it can be concluded that drag reduction occurs when the turbulent velocity scale is larger than the characteristic velocity of the polymer solution.

CONCLUSIONS

In the present study, direct numerical simulation of turbulent viscoelastic flow in a channel was conducted to investigate the drag-reduction mechanism by polymer additives. The viscoelastic fluid was expressed with the Oldroyd-B model. The onset criterion for drag reduction predicted in the present study was in good agreement with previous theoretical and experimental studies. In addition, the turbulence statistics such as the mean velocity and rms velocity fluctuations were also in good agreements with those in the previous studies.

The kinetic and elastic energy transport equations were derived to investigate the effect of elasticity on drag. The polymer stored the elastic energy from the flow in the sublayer and then released again in the sublayer when the relaxation time was short (no drag reduction). However, when the relaxation time was long enough (drag reduction), the elastic energy was transported to and released in the buffer layer. Therefore, drag reduction occurred when the turbulent velocity scale was larger than the characteristic velocity scale of the polymer solution.

Acknowledgment

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