A Diffusive Dispersion Model of Navier Stokes Chaos in Transition

By

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Abstract

The pressure and velocity field of the hyperbolic N.S. system in $(x.t)$ 4 space of a slightly compressible fluid is constructed asymptotically from a model derived from the N.S. [1,2]. The weak solution is for an arbitrary but given initial data with a barotropic relation $\rho(p)$. It includes the classical incompressible limit $\rho = 1$ without assuming "Divergence free vector velocity in the interior of a closed region" (D. Ruelle) [3]. Any residual divergence $\partial_x u$ is, however, equivalent to a Dirac Impulse of pressure (H. Lamb)[4]. It is suppressed here through the homogeneous pressure solution to secure "smooth" nonlinear wave limits observable along its real trajectory, while leaving integrally small dispersive transients as non-observable in its complex co-dimensional space. Some unstable ones may emerge remotely above observational norm bounds as nonlinear waves to interact with neighbours strongly, merging and bifurcating into singular envelopes to present spiral vortical strings and helical bands of KAM structures (Fig. 1) even in Hamiltonian or inviscid systems.

The model equation replacing the continuity relation is

$$\left[\partial_\tau + \left(u + \nabla^2 u\right)\partial_x - \varepsilon\partial^2_{xx} - \beta\partial^3_{xxx}\right]p(u) = 0 \qquad (1)$$

to be solved simultaneously with the Bernoulli's form of the momentum equations for $p(u)$ along the spiral real trajectory of $p(u)$ defined by $u + \nabla^2 u$. The space time similitude and a local similitude of diffusive dispersion render its asymptotic solutions manageable, via a polygonal approximation of the complex curvilinear $(x.t)$ space through "quasi-steady evolution of discrete nonlinear waves followed by their interaction to regenerate a new set of such waves at remote sites". In the limit of the smallest continuum scales, the physically reasonable "almost ergodic turbulent measures" are imposed, as suggested separately by Ruelle[3] and Smale[5]. The solutions display many features of globally coherent quasi-steady incompressible flows in

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physical space.Their images of evolution in solution phase space will bring out their ordered envelope as signatures of the transient "diffusive dispersion chaos." They are the "stationary" views of their respective local observers; but globally the laminar envelopes of the chaotic progressive transients in the neighboring traverse complex codimensions under the prevailing norm bound of the weak solution. They represent different embodiments of physical dynamic entities f(u), based on the fluid velocity relative to their zero reference state as observed by different observers in the curvilinear gravitational space. They include all the dynamic and thermodynamic properties, and their mixing characteristics of the "intermittent" chaotic fields.

To facilitate their analysis under arbitrary but given initial data and observational norm bound, a similarity parameter is introduced as $\lambda = \varepsilon/(M\beta)^{1/2}$, with the peak magnitude M of fluid velocity relative to a "zero reference state" where u = f(u) = 0 in the nearest far field of the local observer over the peak, travelling with the phase or group velocity c over the origin. The model equation (1) is reduced to a similar quasi-steady form for small λ and under space-time similitude $\xi = \lambda^{-1}(x - ct)\,\varepsilon/\beta$ as

$$[(u - c)\,\partial_{\xi} - \lambda\,\partial^2_{\xi\xi} - \partial^3_{\xi\xi\xi}]\,f(u) = 0 \qquad (2)$$

with $\lambda = \infty$ and o corresponding to classical diffusion and dispersion. The state function f(u) with finite λ are the transitional state of diffusive dispersion as defined quantum mechanically by the local mean disturbance field over the curvilinear complex space of an "anti-soliton". Its asymptotic limit with the same Reference State far up and downstream, is a symmetric soliton. Both types of stationary local views $f(\xi)$ are governed by equation (3) as the integral form of (2) from ξ to $\xi = \infty$ where u = f(u) = 0.

$$(\partial_{\xi} + \lambda\,\partial_{\xi})\,u = F(u) \qquad (3)$$

F(u) is nonlinear while (3) is quasilinear, amenable to asymptotic weak solutions for the evolution of u or f(u) under a specific norm bound separating the nonlinear from the linear stream. The convergence of the evolution solution of finite values of λ to the linear stream is complicated to be dealt with separately in abstract form. The convergence of $\lambda \to \infty$ to the diffusive shock wave was given by Hopf[6]. That of $\lambda \to 0$ with $\varepsilon \sim \beta \to 0$ as the train of solitons u~ sech $^2\,\xi$ of the KDV equation, approaches an open limit under the dubious proposition of "Benign pass-over" of solitons in strong interaction, however.

Novikov[8] studied the strong interactions of confluent logarithmic vortices and concluded their "collapse" into "screening similar structures". The present model elaborates how a nonlinear dispersive wave introduces a logarithmic singularity to excite chaos in the underlying linear chaotic field, possibly to emerge as new nonlinear waves remotely in the complex ($i\xi$, τ) space. The nonlinear and the linear streams in the weak evolution solution is coupled through their mutual interaction in the form of scalar velocity potential longitudinally and the vector potential transversally. Their crossing $\xi = 0\pm$ via complex space presents an image of "almost sphere". Its planar sections of Poincaré appear transversally as repeated or continual merging and splitting (i.e. envelop formation and bifurcation) while spiraling in and out of a "Focal Point". The interacting party may "stay" together for a while therein to present an image of an "Antisoliton" as "Matched half solitons", possibly with a "flat top" along the real axis. Azimuthally, there will be a region of logarithmic mean flow profile over an underlying weak chaos to mark the diffusion – dispersion transition, akin to the logarithmic law of the wall in a turbulent boundary layer, (Fig. 2). Their oblique projections can appear as "horse shoes" or "cat eyes" (Fig. 3, 4), either open or closed. The ever repeating envelope formations builds up to the complicated and larger macroscopic structures; while repeating solution bifurcations pushes on the boundary of continuum images of molecular atomic particle ensembles. There, the charged Columbic forces of electrodynamics derivable from its vector potential begins to dominate the mass based gravitation, with their flux line tensions accounting for the excess dispersion (β) beyond the similitude as $\lambda \rightarrow$ o , to complete the Maxwell relations. The electronic gas, ex-atomic-nuclei, could then be "almost" but demonstrably "Ergodic". Its Bernoulli form of momentum relation degenerates in some barotropic limit to the relativistic relation. Then and there, the "closure" of mixing in Navier – Stokes Chaos of all thermodynamic properties can be effected, theoretically. For macroscopic continuum chaos, the "diffusively linear molecular chaos" may be equally satisfactory as the lower boundary of the "almost ergodic chaos".

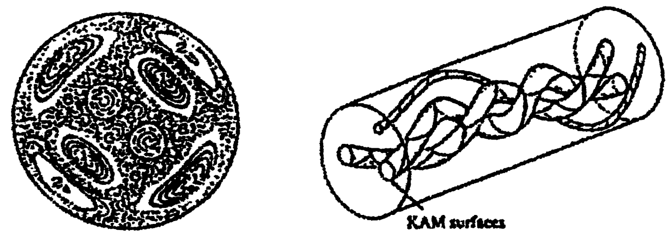

Figure 1 Mixing in a steady, spatially periodic flow. Figure (*a*) shows a typical Poincaré section, with the numbers indicating the period of the islands; figure (*b*) shows the typical three-dimensional structure of the system. Note that the fluid within the KAM tubes does not mix with the rest of the fluid. From Khakhar et al. (1987).

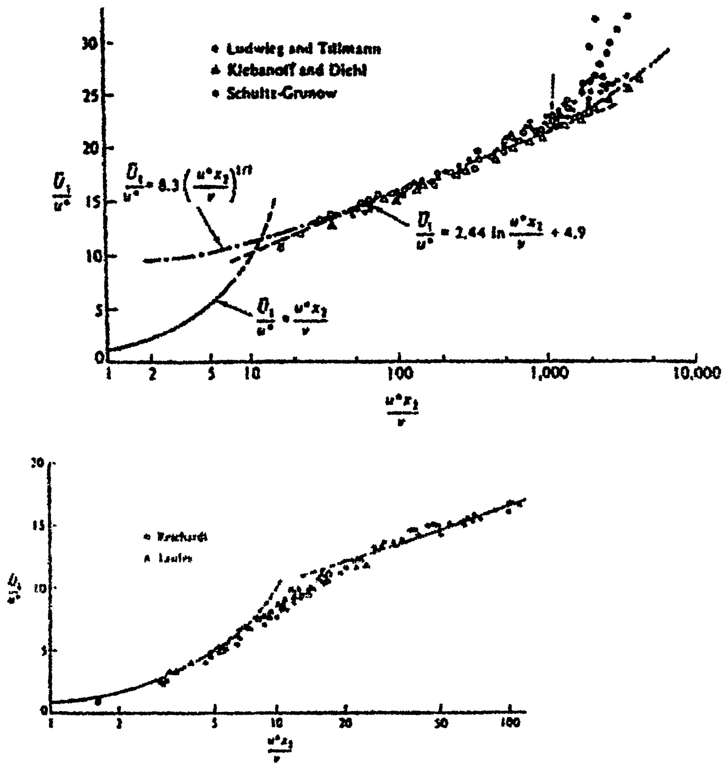

Fig 2. The mean velocity distribution near smooth walls of fully developed turbulent boundary layers and their linear sublayers

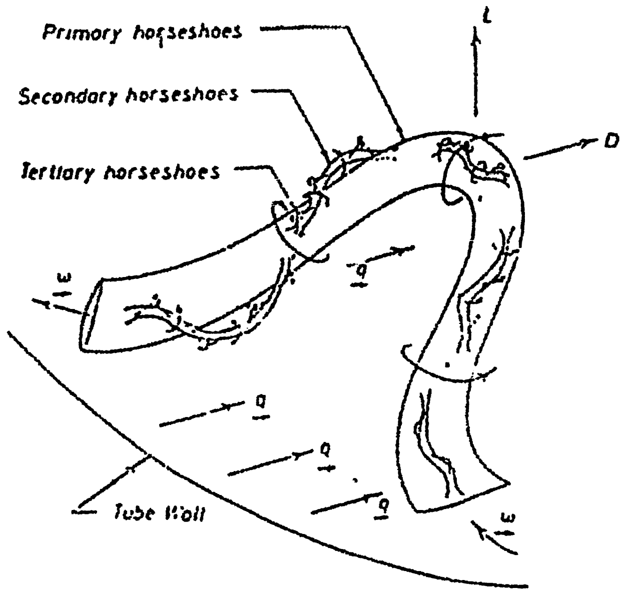

Fig. 3 Horseshoe of large Reynolds number (Theodorson 1955).

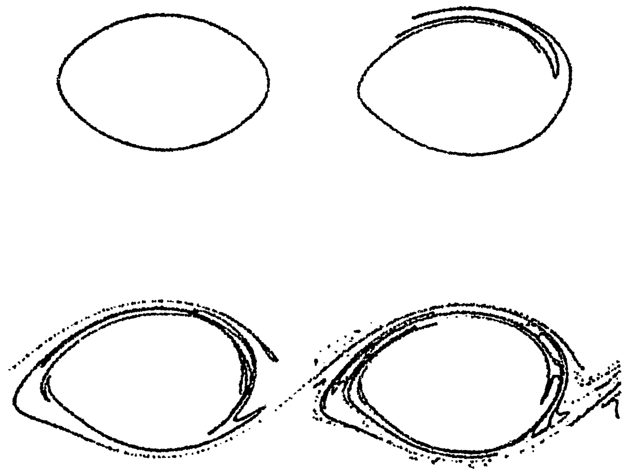

Figure 4 Evolution of a contour of isovorticity (i.e. ω_z = constant) in the perturbed Kelvin cat's-eye flow (Danielson 1989).

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