ON THE DECAY OF SHEARLESS WALL BOUNDED TURBULENCE

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ABSTRACT

The decay of turbulence in a wall bounded domain without mean velocity is investigated. Direct and Large-Eddy Simulations, as well as the Eddy Damped Quasi-Normal Markovian closure are used. The effect of the finite geometry of the domain is accounted for by introducing a low wave-number cutoff in the energy spectrum of isotropic turbulence. It is found that, once the saturation of the turbulent energy-containing length scale has occurred, the r.m.s. vorticity is decaying following a power law with a $-3/2$ exponent, in agreement with the helium superfluid experiment of Skrbek and Stalp (2000). The turbulent kinetic energy decay exponent is found to be -2, also in agreement with Skrbek and Stalp. Using scalings deduced from a simple analysis, all data can be collapsed into single curves for both the fixed scale turbulent regime and the final viscous period of decay. A spectral model for inhomogeneous turbulence is finally applied to the decay of turbulence between two plates. It is shown that the results are in agreement with the helium experiment.

INTRODUCTION

In a recent study (Skrbek and Stalp, 2000), it was experimentally shown that wall bounded turbulence, generated by towing a grid in a channel, is decaying following a t^m power law for the r.m.s. vorticity, with m=-3/2. This regime is observed after a period of classical decay of unbounded isotropic homogeneous turbulence and once the turbulent energy-containing length scale has reached the size of the experimental facility (saturation time). The experiment was performed at the cryogenic Lab. of the University of Oregon, in helium superfluid. In He_{II}, the (equivalent) Reynolds number is sufficiently high for the $-3/2$ decay regime to be observed before the final viscous decay occurs. In classical experiments, this would generally not be the case. In the same paper, it was also shown that the $-3/2$ exponent can be predicted by a simple spectral analysis, using the spectrum decay model originally proposed by Comte-Bellot and Corrsin (1966) modified by introducing a cutoff at low wave-numbers. The corresponding decay exponent for the turbulent kinetic energy was then predicted to be $n = -2$.

The idea of introducing a low wave-number cutoff (or infrared cutoff) in the spectrum to mimic the fact that in wall bounded flows, due to the presence of the geometrical boundaries, the energy-containing length scale must saturate at a size proportional to the wall distance, was suggested by Bertoglio & Jeandel (1986) and used in several related studies (Parpais, 1997, Touil *et al* , 2000). Indeed the infrared cutoff was one of the basic ingredients in the spectral approach of inhomogeneous turbulence followed by the above authors (development of the simplified closure for inhomogeneous turbulence: SCIT model). The infrared cutoff assumption was found to lead to satisfactory results for wall bounded sheared turbulence. The high Reynolds number Oregon experiment is now offering an interesting opportunity to directly test this assumption against experimental data in a simple situation where there is no mean velocity gradient.

The purpose of the present study is then to further investigate the decay of turbulence with a low wave-number spectral cutoff. The low wave-number cutoff is therefore introduced in

1. the EDQNM closure for homogeneous isotropic turbulence (Orszag, 1970).
2. DNS and LES of isotropic turbulence.
3. the SCIT model (Touil *et al* , 2000).

In the case of the EDQNM closure, the problem of the finite size geometry was recently addressed by Lesieur and Ossia (2000). They analysed the decay of turbulence at a high Reynolds number starting with an initial integral length scale one order of magnitude larger than the geometrical limit. In the present study, we use the same kind of approach to investigate the problem at different Reynolds numbers as well as to study the influence of the initial length scale ratios. In the case of numerical simulations, the problem was addressed by Borue and Orszag (1995) with an hyperviscosity and at high Reynolds number. The periodicity of the numerical box was used for taking into account the length scale limitation. Existence of a self-similar decay with a -2 exponent for the turbulent kinetic energy was shown. In the present study, we use the same procedure to mimic the finite length scale effect in DNS and in LES. Reynolds number effects are also studied.

The results of the closure and those of the simulations are compared in section IV. They are also used to check the scaling laws deduced from the analytical study presented in section II. In the last section, the SCIT model is used to address the same problem in a situation in which the presence of walls is more realistically represented.

ANALYSIS AND SCALING

A simple analysis of the problem of the decay of turbulence with a constant length scale can easily be performed. It is indeed a problem that can be proposed to students as an application of a turbulence course (see Tennekes and Lumley, 1972, p. 25, problem 1.2). In the equation for the turbulent kinetic energy

$$k_{,t} = -\varepsilon \qquad (1)$$

assuming that the dissipation ε is proportional to $k^{\frac{3}{2}}/L$ with L constant (equal to the dimension of the containing vessel d) immediately leads to a decay law with an exponent $n = -2$ for the turbulent kinetic energy. More precisely, writing

$$\varepsilon = c_2\, u'^3/d \qquad (2)$$

leads to the solution

$$k = 9d^2 c_2^{-2}\, (t - t_{vo})^{-2} \qquad (3)$$

in which t_{vo} is a virtual origin. The corresponding decay exponents for the dissipation and the *r.m.s.* vorticity are immediately found to be -3 and $-3/2$ respectively. Equation (3) corresponds to the regime in which the energy-containing length scale is saturated. A simple way to study the behaviour of the flow before saturation is to extend the spectrum analysis of Comte-Bellot and Corrsin by introducing a low wave-number cutoff at $K_{inf}=2\pi/d$. This was done by Skrbek and Stalp (2000). We reproduce here the simplest version of their approach (neglecting the influence of intermittency and of the "rounding" of the spectrum in the vicinity of its maximum). The turbulent energy spectrum is assumed to be

$$\begin{array}{lll} E(K,t) = 0 & \text{for} & K < K_{inf} \\ E(K,t) = A\ K^s & \text{for} & K_{inf} \le K \le K_e \\ E(K,t) = C\ \varepsilon^{\frac{2}{3}}(t) K^{\frac{-5}{3}} & \text{for} & K \ge K_e \end{array} \qquad (4)$$

in which A is supposed to be constant during the decay and C is the Kolmogorov constant. Expressing the continuity of the spectrum at K_e, evaluating $k(t)$ by wave-number integration of $E(K,t)$ and replacing in (1) lead to the classical decay exponent $n = -2\frac{(s+1)}{s+3}$ in the regime before saturation, that is to say before K_e has reached K_{inf}. The approach also permits to evaluate the time $t_{sat}{}^*$ at which $K_e = K_{inf}$ (saturation time). For $s = 2$, this leads to expression (24) in Skrbek and Stalp (2000), whereas for $s = 4$, it is found

$$t_{sat}{}^* = \frac{17}{7} A^{-\frac{1}{2}} C^{\frac{3}{2}} (K_{inf}^{-\frac{7}{2}} - K_e(0)^{-\frac{7}{2}}) \qquad (5)$$

The energy at saturation can also be expressed. For $s = 4$, one obtains

$$k^*_{sat} = (3/2)\ A\ K^5_{inf} \qquad (6)$$

After $t_{sat}{}^*$, saturation of scales occurs and the spectral analysis leads to recover equation (3), with $c_2 = 2\pi C^{-\frac{3}{2}}$. Turbulence then decays with the -2 exponent until the beginning of the final viscous regime. The characteristic time t^*_ν for the beginning of the viscous decay can also be estimated following Tennekes and Lumley. Assuming that during the viscous decay the dissipation is

$$\varepsilon = c_1 \nu u'^2/d^3 \qquad (7)$$

and defining t^*_ν as the cross-over time at which both (2) and (7) are valid lead to

$$t^*_\nu = t^*_{sat} - \frac{6\pi}{c_2} A^{-1/2} K_{inf}^{-7/2} + \frac{12\pi^2}{\nu c_1} K_{inf}^{-2} \qquad (8)$$

The corresponding turbulent kinetic energy is

$$k^*_\nu = \frac{3}{8\pi^2} \nu^2 (\frac{c_1}{c_2})^2 K^2_{inf} \qquad (9)$$

Note that in Skrbek and Stalp (2000), the procedure for estimating the viscous scaling is different. However the resulting expressions only slightly differ. Both approaches lead to a $1/(\nu K_{inf}^2)$ scaling for t_ν^* with a slightly different prefactor.

DNS AND LES COMPUTATIONS

To investigate the decay of turbulence in a bounded domain using LES and DNS, a simple way is to use a low wave-number cutoff in a simulation of isotropic turbulence. This can be done by taking advantage of the scale limitation introduced by the periodicity of the computational domain when using a pseudo-spectral technique in a periodic box. The wave-number cutoff is then $K_{inf} = 2\pi/d$ in which d is the size of the computational box. Indeed, such a low wave-number cutoff is present in all simulations of isotropic turbulence, but one is usually trying to avoid its effect by performing the computations in a large enough box. In decaying turbulence, since the integral length scale is increasing with time, computations are usually stopped before K_e becomes of the order of K_{inf}. Instead, we are here performing simulations in a box whose dimensions are not necessarily large compared to the integral turbulence length scale and we are not stopping the runs when the scale limitation induced by the box size begins to take place. The same procedure was used by Borue and Orszag (1995) to mimic the finite length scale effect in their hyperviscosity computations when studying the self similar decay of turbulence.

The code used for the DNS and LES computations is a classical pseudo-spectral code with second order Runge-Kutta time integration scheme. All the computations were performed at a resolution of 128^3 grid points. As initial spectrum, we use

$$E(K,0) = 2.5\ 10^{-3}(\tfrac{K}{K_{max}})^4(1+(\tfrac{K}{K_{max}})^2)^{-17/6} \quad (10)$$

with $K_{max} = 33\ m^{-1}$ (value corresponding to a fit of the spectrum measured by Comte-Bellot and Corrsin (1971) at $x/M = 42$). Note that the low wave-number cutoff is implicitly imposed at $K_{inf} = 2\pi/d$ by the numerical discretisation to this spectrum which would otherwise behave as K^4 at small K.

In the case of DNS, the Reynolds number Re_{l_0}, built on the initial integral length scale l_0 is equal to 5000. To investigate larger Reynolds numbers, LES computations were performed. The subgrid model is the Chollet and Lesieur (1981) eddy viscosity model, modified to account for finite Reynolds number effects (see Chollet (1983) and Parpais (1997)). The spectral viscosity is:

d/l_0	$Re_{l_0} \simeq 5.10^3$	$Re_{l_0} \simeq 5.10^5$	$Re_{l_0} \simeq 5.10^7$
6	DNS	LES	LES
6	EDQNM		EDQNM
9	DNS	LES	LES
9	EDQNM		EDQNM
12	Quasi-DNS	LES	LES
12	EDQNM		EDQNM
24	LES	LES	
24	EDQNM		EDQNM
48		LES	
48		EDQNM	EDQNM
240			EDQNM

Table 1: The different computed cases.

$$\nu_t(K) = 0.267\sqrt{\frac{E(K_c)}{K_c}} * f(\frac{K_c}{K_\eta})g(\frac{K}{K_c}) \quad (11)$$

in which K_c is the LES wave-number cutoff. $f(K_c/K_\eta)$ is a low Re correction:

$$f(\frac{K_c}{K_\eta}) = (1-(\frac{K_c}{K_\eta}))^{4/3}(1+\frac{1}{a}ln(\frac{1+a(\frac{K_c}{K_\eta})^{-4/3}}{1+a})) \quad (12)$$

K_η is the Kolmogorov wave-number estimated using $K_\eta = (\varepsilon/\nu^3)^{1/4}$, a is equal to $\lambda\sqrt{3C}/2$ and $g(\frac{K}{K_C})$ represents the "cusp" effect :

$$g(K/K_c) = 1 + 0.4724(K/K_c)^{0.372} \quad (13)$$

Use of (11) instead of the original Chollet and Lesieur model has the advantage of permitting a smooth transition from LES to DNS as the Reynolds number decreases. This happens to be the case during the decay of isotropic turbulence. As a matter of fact, most of the LES computations presented in the paper are indeed DNS at the end of their time evolutions. Three values of the Reynolds number were investigated using LES. The influence of the finite size of the domain were investigated by varying d. All DNS and LES computations are summarized in table 1. The initial integral length scale l_0 is used for normalization.

EDQNM COMPUTATIONS

In the case of the Eddy Damped Quasi-Normal Markovian (EDQNM) closure, computations are performed using the formulation of the model for homogeneous isotropic turbulence (Orszag, 1970). The equation for the kinetic energy spectrum is

$$E(K,t)_{,t} = -2\nu K^2 E(K,t) + T(K,t) \quad (14)$$

in which the trasfer term T is expressed using the classical EDQNM formulation for isotropic turbulence. The EDQNM characteristic time scale is given by

$$\theta_{KPQ}(t) = \frac{(1 - e^{-(\eta_K+\eta_P+\eta_Q)t})}{(\eta_K + \eta_P + \eta_Q)} \qquad (15)$$

in which the damping coefficient is expressed as

$$\eta_K = \lambda\sqrt{\int_0^K P^2 E\, dP + \nu K^2} \qquad (16)$$

as proposed in Pouquet *et al* (1975). For λ we use the classical value $\lambda = 0.355$. Note that we have here used (15), instead of its large time asymptotical form $\theta_{KPQ} = \frac{1}{\eta_K + \eta_P + \eta_Q}$ in order to permit the comparisons with the DNS which start with zero third order correlations at $t = 0$. The model is applied to wave-numbers ranging from K_{inf} (low wave-number or infrared cutoff, related to the size d of the bounded domain by $K_{min} = 2\pi/d$) to K_η (Kolmogorov wave-number). The energy-containing range is characterized by wave-number K_e. At time $t = 0$, the initial conditions are such that $K_e > K_{inf}$ and the spectrum is identical to the one used for the DNS and LES previously described. The different cases treated with the closure are summarized in table I (together with the DNS and LES computations). Compared to DNS, EDQNM has the advantage of permitting computations at higher Reynolds numbers as well as larger initial values of the ratio K_e/K_{inf}. Compared to LES, it still has the advantage of permitting to account for larger K_e/K_{inf}. The maximum value of K_e/K_{inf} in LES is limited by the fact that the filter cutoff must be in the inertial (or dissipative) range of the spectrum for existing subgrid models to be reliable.

Note that to relate the results of these EDQNM and DNS/LES computations to the experimental situation of a grid generated turbulent field in a confined geometry, it has to be assumed that, even if the flow is bounded by rigid walls, its global behavior can be accounted for by an isotropic and quasi-homogeneous description. A similar assumption was made by Skrbek and Stalp (2000) when applying the spectral analysis to their experiment.

RESULTS

In Fig. 1, the time evolution of the r.m.s. vorticity is plotted for the low Reynolds number case ($Re_{l_0} = 5000$, see table I). Three d/l_0 ratios are considered. For $d/l_0 = 6$ and 9, the simulations are fully resolved DNS. The third case, referred to as quasi-DNS, corresponds to a low Reynolds number LES that rapidly becomes a DNS as time evolves, as the subgrid viscosity defined by (11) is rapidly decreasing. At $t/t_{turn-over} = 10$, ν_t is already smaller by a factor 100 than the molecular viscosity. Before $t/t_{turn-over} = 10$, the simulation is considered a LES and therefore the r.m.s. vorticity is not plotted. In the log-log plot in Fig. 1, the existence of a $-3/2$ power law is clearly observed for the DNS runs. Before the $-3/2$ regime, a less steep decay is observed, although it is difficult to really detect a power law, saturation occurring before it could develop. It has to be pointed out that no attempts were made to adjust the virtual origin to improve the fit of the data with a power law. The corresponding

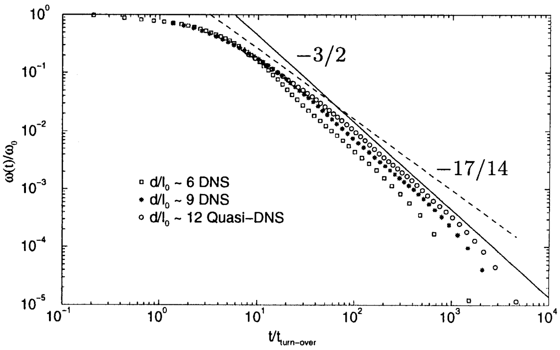

Figure 1: Decay of r.m.s. vorticity and comparison with theoritical decay laws. DNS and Quasi-DNS Re_{l_0}=5000

time evolutions of the turbulent kinetic energy are plotted in Fig. 2. Also shown in Fig. 2 are results of two LES runs. The first one corresponds to a value of d twice larger than for the quasi-DNS, at the same Reynolds number, whereas the second one corresponds to d four times larger and a higher Reynolds. In the case of the LES, only the filtered energy is plotted. After saturation has occurred, the -2 decay exponent is observed. Before saturation, a power law with an exponent close to $-10/7$ is clearly detected for the largest values of d. It can also be observed in Fig. 2 that at very large time, the power laws are no longer valid and that the viscous decay occurs. In Fig. 3, the same results are compared with the results of the EDQNM closure. The agreement is good and EDQNM appears to capture the t^{-2} regime in agreement with Lesieur and Ossia (2000). All these results can be collapsed into a single curve in the time ranges corresponding to the two turbulent decay regimes (before and after saturation) by using the scalings provided by equations (5) and (6), as appears in Fig. 4.

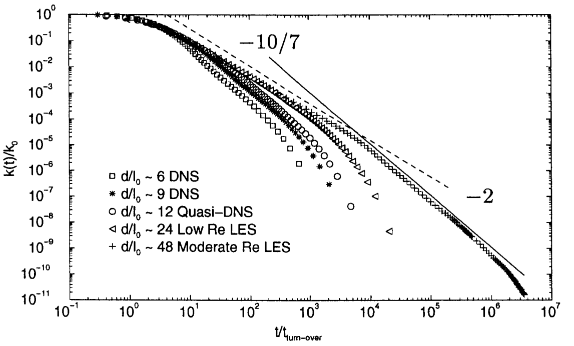

Figure 2: Decay of turbulent kinetic energy. DNS and LES results at low Re number.

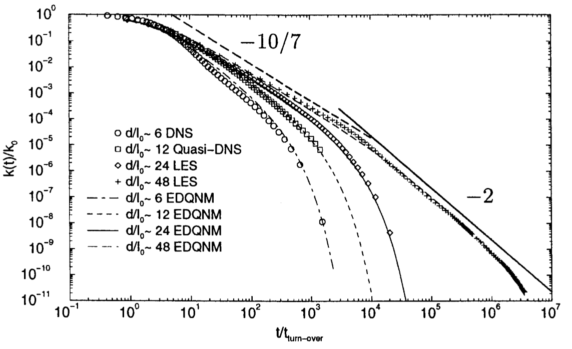

Figure 3: Same as Fig. 2, plus EDQNM results.

Also plotted in Fig. 4 are results of the closure obtained for a larger value of d ($d/l_0 = 240$ and $Re_{l_0} = 5 \times 10^7$). To collapse the data in the final viscous regime, we use the scaling provided by equations (8) and (9) (with the value for the ratio $c_1/c_2 = 10$ originally suggested in Tennekes and Lumley). The results are shown in Fig. 5. In figure 6, the turbulent kinetic en-

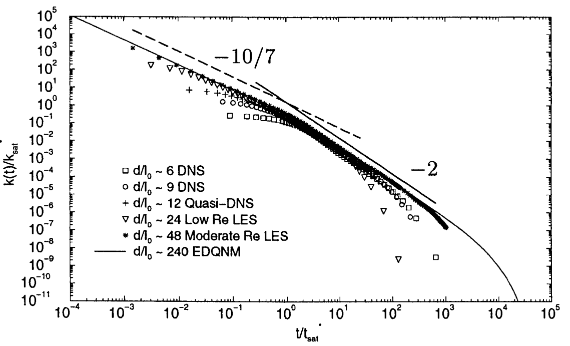

Figure 4: Decay of turbulent kinetic energy normalised by k^*_{sat} as a function of t/t^*_{sat}.

ergy spectra are plotted during the fixed scale decay regime (LES). The compensated spectra are normalised by $\varepsilon^{\frac{2}{3}}(t)$, with values of ε deduced from the LES by adding the subgrid flux to the molecular dissipation. It can be observed that the spectra collapse, which confirms the existence of a self similar regime, in agreement with what was found by Borue and Orszag (1995). A $k^{-\frac{5}{3}}$ range is clearly present,

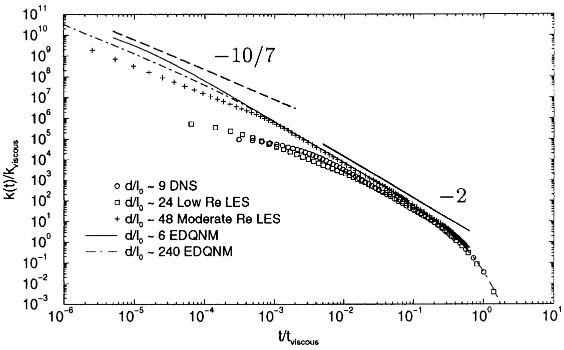

Figure 5: Decay of turbulent kinetic energy normalised by $k^*_{\nu 0}$ as a function of t/t^*_{ν}.

as was the case in Borue and Orszag, the main difference with their results being that their compensated spectra exhibited a large bump before the high wave-number cutoff due to the use of an hyperviscosity.

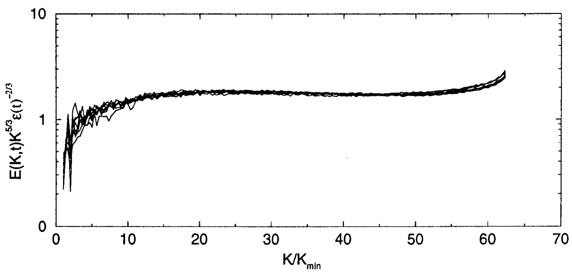

Figure 6: Normalised energy spectra between $\frac{t}{t_{turn-over}} =$ 50 and 5000, large Re LES computations.

SCIT MODEL RESULTS

As stated above to relate the results of the EDQNM and DNS/LES computations presented in the previous paragraphs to the experimental situation of a grid generated turbulent field in a confined geometry, it has to be assumed that the global behaviour of the flow can be accounted for by an isotropic and quasi-homogeneous description. In order to take into account more realistically the fact that the flow is bounded by rigid walls, we now use the SCIT model. The model proposed in Touil *et al* (2000) is applied to a flow between two plates, without mean velocity. In this case, the low wave-number cutoff is built in the model and it is no longer necessary to assume that the flow is behaving as if it was homogeneous and isotropic. Inhomogeneous transport effects as well as wall boundary conditions are accounted for in the model.

In the case of a turbulent field between two infinite plates, there is only one direction of inhomogeneity and the model equations can be reduced to a transport equation for the energy spectrum

$$E_{,t} = -2\nu K^2 E + T + D \qquad (17)$$

T is the non linear transfer term expressed using EDQNM. $D(K, x_1, t)$ is a transport term typically associated with inhomogeneous effects. It is expressed as $D = (d_{eff} E(K, x_1, t)_{,x_1})_{,x_1}$ in which d_{eff} is a turbulent diffusivity. As for the low wave-number cutoff, it is now varying locally with the distance from the wall $K_{inf} = c_{wall}/x_2$, with $c_{wall} = 0.5$ (see Parpais, 1997). The results of the model are plotted in figure 6 and illustrate that again the $-3/2$ and -2 decay exponents are found.

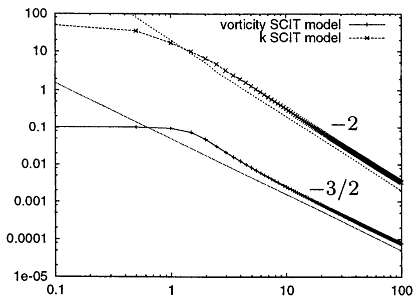

Figure 7: Decay of turbulent kinetic energy and vorticity between two plates, SCIT model.

CONCLUSION

Decaying turbulence in a wall bounded domain was investigated using DNS, LES and the EDQNM closure. The turbulent length scale growth limitation due to the bounds is reprdented by introducing an infrared spectral cutoff. The results show that once the energy-containing turbulent length scale has reached the limit, a decay regime with a -2 exponent for the turbulent kinetic energy is found. The associated exponent for the r.m.s. vorticity is -3/2, in agreement with the experiment of Skrbek and Stalp (2000). Before the length scale saturation, a classical homogeneous decay regime takes place, whereas at very large time a viscous regime appears. The three regimes and the transition between them are in good agreement with the scalings deduced from a simple analysis similar to the one proposed by Skrbek and Stalp. The fact that a low wave-number spectral cutoff assumption leads to correctly reproduce the global behaviour of the turbulent decay observed in a wall bounded experiment provides support to models for inhomogeneous turbulence relying on this assumption.

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