EVOLUTION OF CROSS-CORRELATION IN FREE DECAYING AND FORCED MHD TURBULENCE

Timofey Antonov
Laboratory of Hydrodynamics, Institute of Continuous Media Mechanics
Korolyov 1, 614061, Perm, Russia
uta@icmm.ru

Sergey Lozhkin
Laboratory of Hydrodynamics, Institute of Continuous Media Mechanics
Korolyov 1, 614061, Perm, Russia
serg@icmm.ru

Peter Frick
Laboratory of Hydrodynamics, Institute of Continuous Media Mechanics
Korolyov 1, 614061, Perm, Russia
frick@icmm.ru

Dmitry Sokoloff
Department of Physics, Moscow State University
119899, Moscow, Russia
sokoloff@dds.srcc.msu.su

ABSTRACT

The longtime evolution of free decaying and forced magnetohydrodynamic (MHD) turbulence is investigated using a shell model of turbulence. Several series of realizations with different kinds of initial conditions have been performed. In most of realizations the state with highly correlated magnetic and velocity fields arises in several dozens of turnover times. This aligned state is characterized by equipartition of magnetic and kinetic energies. However, realizations with magnetic energy exceeding the kinetic one throughout the whole period of simulation have been also observed. In both cases the energy flux is practically blocked. The evolution of the forced turbulence depends not only on the initial conditions but also on the kind of forcing. Under constant external force a long (100-200 turnover times) metastable state gives way to the aligned state and the energy flux also becomes weak causing an increase of total energy. Under the force ensuring a constant level of the kinetic energy at a given scale the system displays an oscillatory behavior.

INTRODUCTION

Three inviscid integrals of motion provide a large variety of scaling properties for MHD turbulence. The idea of constant spectral energy flux leads to the Kolmogorov spectral index "-5/3". The concept of the Alfvénic wave turbulence results in the Kraichnan-Iroshnikov spectrum "-3/2". The third possibility was suggested by Dobrovolny et al. (1980) and developed by Pouquet et al. (1986). It implies high correlation between magnetic and velocity fields. In such a flow these fields become almost parallel being subject only to molecular dissipation, because the energy cascade in this case practically vanishes. This effect is called alignment and can be quantified by a correlation coefficient $C = H_C/(E_B + E_U)$, where H_C is the cross-helicity, E_U and E_B are the kinetic and magnetic energies, respectively. Highly correlated magnetic and velocity fields yield $|C| \approx 1$. In a flow with initially weak cross-helicity, the alignment is expected to take place only at a later stage of evolution in free decaying turbulence and possibly in the forced one.

The main purpose of the present work is to follow the longtime evolution of velocity and magnetic fields in the case of MHD turbulence. Since this turbulent flow requires high kinetic and magnetic Reynolds numbers, the possibilities of the direct numerical simulations are very limited. That is why our investigations are based on a shell model of MHD turbulence.

MHD SHELL MODEL

The basic idea of any shell model of fully developed turbulence is to retain only one real or complex mode (in our case complex variables U_n and B_n correspond to velocity and magnetic field) as a representative of all modes in the shell with the wave number $k_n < |\mathbf{k}| < k_{n+1}$, $k_n = 2^n$, and to introduce a set of ODE, which mimics the original nonlinear PDE . For an introduction to shell models the readers are referred to Bohr et al. (1998).

Here we use the MHD-shell model introduced by Frick and Sokoloff (1998). For 3D turbulence it can be rewritten as

$$(d_t + Re^{-1}k_n^2)U_n = \tag{1}$$
$$= \frac{ik_n}{8}\Big\{8(U_{n+1}^* U_{n+2}^* - B_{n+1}^* B_{n+2}^*) - $$
$$-2(U_{n-1}^* U_{n+1}^* - B_{n-1}^* B_{n+1}^*) +$$
$$+(U_{n-2}^* U_{n-1}^* - B_{n-2}^* B_{n-1}^*)\Big\} + f_n.$$
$$(d_t + Rm^{-1}k_n^2)B_n = \tag{2}$$
$$= \frac{ik_n}{6}\Big\{(U_{n+1}^* B_{n+2}^* - B_{n+1}^* U_{n+2}^*) +$$
$$+(U_{n-1}^* B_{n+1}^* - B_{n-1}^* U_{n+1}^*) +$$
$$+(U_{n-2}^* B_{n-1}^* - B_{n-2}^* U_{n-1}^*)\Big\}$$

Re is the Reynolds number, $Rm = Re \cdot Pr_m$ is the magnetic Reynolds number, and Pr_m is the magnetic Prandtl number, f_n is an external force. In the free decaying case $f_n = 0\ \forall n$.

In the limit $Re, Rm \to \infty$, equations (1), (2) retain three quadratic quantities

$$E = \sum_n (|U_n|^2 + |B_n|^2)$$
$$H_C = \sum_n (U_n^* B_n + U_n B_n^*) \tag{3}$$
$$H_B = \sum_n (-1)^n k_n^{-1} |B_n|^2$$

corresponding to the three quadratic invariants of inviscid MHD flows: total energy, cross helicity and magnetic helicity. To proceed further let us note that the nonlinear terms in (1,2) identically vanish for $U_n = \pm B_n$ and the spectral energy flux is blocked as well.

FREE DECAYING MHD TURBULENCE

We performed a simulation (see Antonov et al., 2001 for details of numerical implementation) of a complete dynamo problem taking into account Lorentz force and starting from a well developed kinetic energy spectrum with a seed magnetic energy ($E_B \ll E_U$). According to Frick and Sokoloff, 1998 the time about a few dozens of turnover times is expected to be sufficient to reach the equipartition of kinetic and magnetic energies.

The simulations made for time scales of thousands turnover times show that the set of solutions with similar initial conditions displays substantially different types of behavior. In Fig.1a one can readily observe three conspicuous groups of the trajectories. In the most numerous group the module of a correlation coefficient $|C|$ quickly increases up to the unity (the group of the "right" behavior). The second group consists of the trajectories with gradual increment of $|C|$. These realizations are expected to reach a correlated state in a time. In the third group of tracks C becomes steady close to 0. A sign of alignment is usually determined by the initial conditions. If a given trajectory has a low alignment for a long time, the sign of C can reverse. However no sign reversal is observed for a state with high alignment.

In Fig.1b the time dependence of total energy for all realizations is presented in log-log coordinates. This kind of presentation is interesting from the viewpoint of power-law energy decay. It is clear that no universal time scaling is available. Most of the time tracks displays a power-law behavior at the intermediate stage of evolution. This stage seems to be shorter for states with rapidly developed alignment (1^{st} group). The slop of these tracks tends to "-0.5" (this slop is shown in Fig.1b by a thick solid line). For low-correlated states (2^{nd} group) this stage is longer and the slop tends to "-1" (thick dashed line in Fig.1b). Both slopes were observed in direct numerical simulations (Biskamp and Müller, 1999). The power-low decay changes to a state with practically stable energy (horizontal tails in Fig.1b), which indicates the establishment of alignment. It is to be noted that realizations with more or less smooth increase of $|C|$ (Fig.1a) correspond to the fastest energy decay (Fig.1b).

SUPEREQUIPARTITION

Typical evolution of the kinetic and magnetic energies in realizations of the 1^{st} and

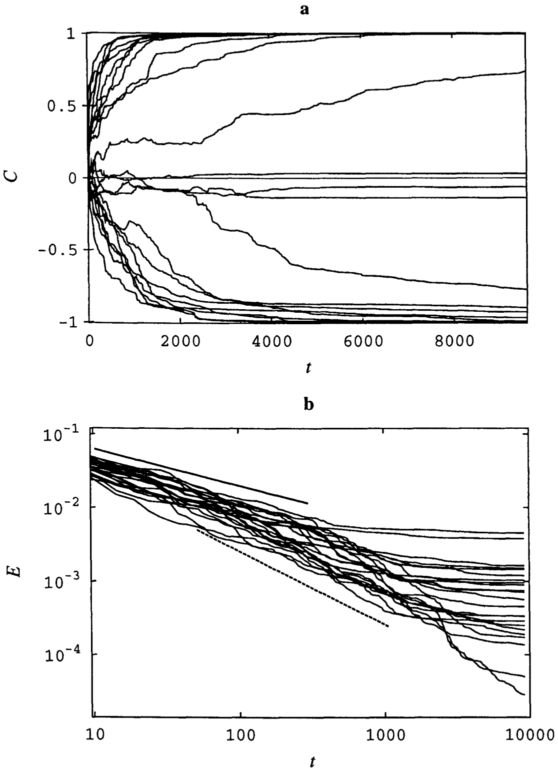

Figure 1: Time evolution of correlation coefficient C (a) and total energy E (b) with initially weak magnetic field ($E_B \ll E_U$)

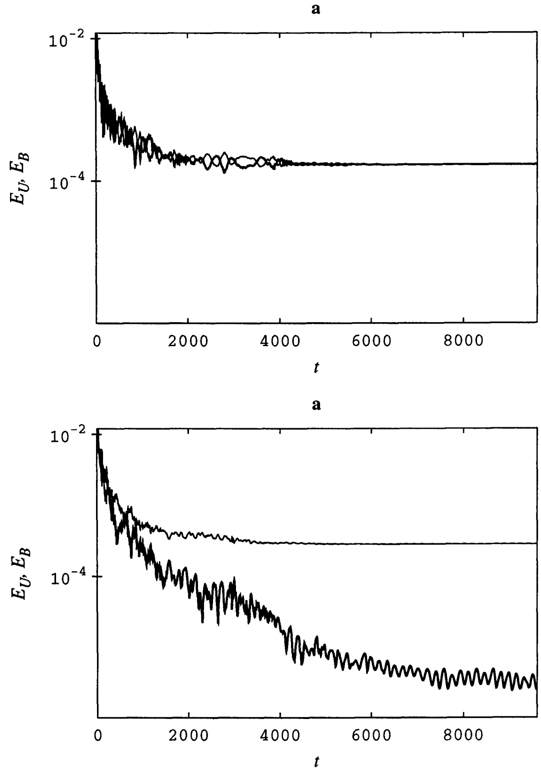

Figure 2: Typical time evolution of magnetic (thin line) and kinetic (thick line) energy in realizations of 1^{st} (a) and 3^{rd} (b) group.

3^{rd} groups is shown in Fig.2a and Fig.2b respectively. In Fig.2a tracks practically coalesce so that the figure does not show a fast growth of the magnetic field at the beginning of evolution. Realizations of the 3^{rd} group demonstrate the states with abnormally high level of magnetic energy, which exceeds the equipartition level. This strongly suggests a possibility of superequipartition in the dynamo problem. As such states are infrequent, and shell models do not contain spatial variables we may conclude that in real MHD turbulence superequipartition arises only in spatial regions that are few and far between. In these regions by some unknown reasons the action of the Lorentz force is abnormally low (non-forced configuration) and does not hamper the magnetic field growth above the equipartition level.

We have performed one more simulation with initially high magnetic energy ($E_B \gg E_U$). In this case the onset of alignment is not evident. However, in a half of realizations the magnetic energy falls down to the level of kinetic energy and statistical properties are similar to those of realizations of 1^{st} group observed in previous simulation. In the remaining half the magnetic energy exceeds the kinetic one during the whole time of simulation, and the statistical properties are identical to those obtained in realizations of the 3^{st} group.

FORCED MHD TURBULENCE

An essentially different picture is observed in the forced turbulence sustained by a constant external force acting on the largest scale only (see Frick et al., 2000 for details). Starting again with a weak magnetic field, the system at the initial evolution stage displays the same behavior as in the free decaying case, and a statistically stable state (with Kolmogorov scaling) seems to be established at a shorter time. However, longer simulations under similar forcing show that after a relatively long evolution this state is replaced by another one. In contrast to the initial stage of evolution, the magnetic field is strongly correlated (or anti-correlated) with the velocity field. The value of cross-helicity H_C is close to its maximal value (i.e. the correlation parameter $|C|$ is close to unity). From dynamical viewpoint, high correlations imply strong depletion of nonlinear terms in shell equations (1, 2) and thus a weak energy flux. As a consequence, the slope of the spectral index is expected to be very steep, as observed in numerical simulations.

As in the free decaying case, the specific be-

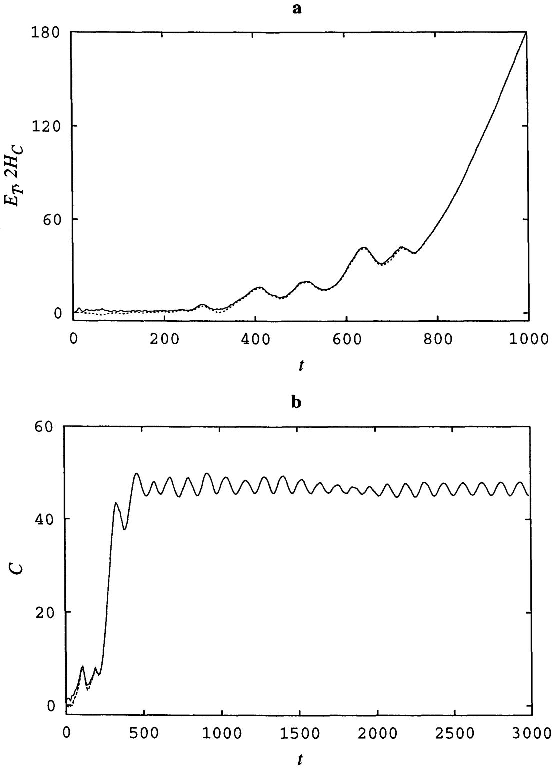

Figure 3: Time evolution of total energy $E_T = E_V + E_B$ and cross helicity H_C for the MHD shell model with constant forcing and slightly different initial conditions.

havior of a given solution of the shell model depends on the choice of initial conditions. At long times we observed either an unlimited growth of energy (Fig.3a), or a very long oscillatory behavior (Fig.3b).

The observed correlated state strongly depend on the kind of forcing. The long-term evolution of MHD shell model presented in Fig.3 demonstrates a drastic variation in the total energy of the system suggesting a strong inflow or outflow of the energy. Since our main focus is a basically isolated system, we provide conservation of the kinetic energy on the largest scale by applying a different kind of force to ensure the constancy of $|U_0|^2$ at each time step.

With this kind of forcing most of the time the total energy oscillates around the mean value with the small correlation parameter C (Fig.4). However, one can observe long stages of high correlation and small energy oscillation.

Acknowledgements. This work was partly supported by RFBR under grants 99-01-00362 and 01-01-96482.

REFERENCES

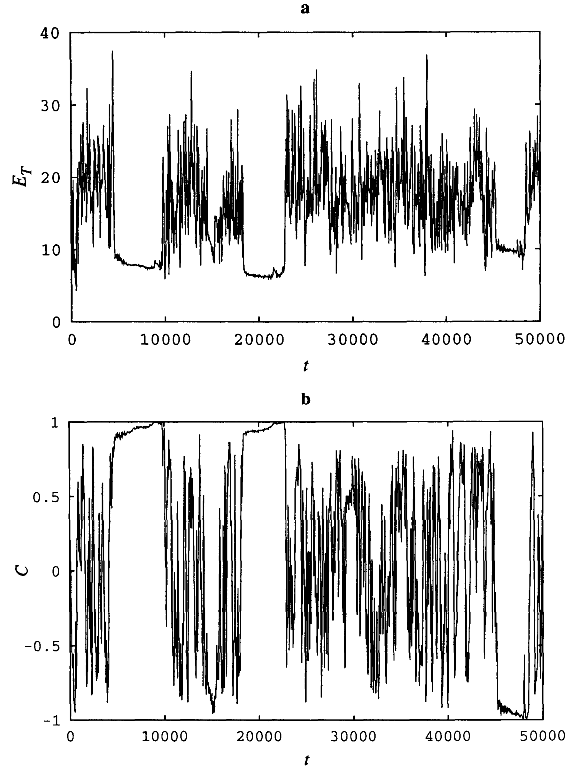

Figure 4: Time evolution of (a) total energy E_T and (b) the correlation parameter C in the "isolated" MHD shell model. The kinetic energy of the first shell $|U_0|^2$ is kept constant by rescaling the amplitude of U_0 at every time step.

Dobrowolny, M., Mangeney, A., and Veltri, P.-L., 1980, "Fully developed anisotropic hydromagnetic turbulence in interplanetary space", *Phys.Rev.Letters*, Vol. 45, No. 2, pp. 144-147.

Pouquet, A., Menneguzzi, M., and Frisch, U., 1986, "Growth of correlations in magnetohydrodynamic turbulence", *Phys. Rev. A.*, Vol. 33, No. 6, pp. 4266-4276.

Bohr, T., Jensen, M. H., Paladin, G., and Vulpiani, A., 1998, "Dynamical Systems Approach to Turbulence", Cambridge University Press, Cambridge.

Frick, P., and Sokoloff, D., 1998, "Cascade and dynamo action in a shell model of magnetohydrodynamic turbulence", *Phys. Rev. E.*, Vol. 57, No. 4, pp. 4155-4164.

Antonov, T., Frick, P., and Sokoloff, D., 2001, "Longtime evolution of free decaying MHD turbulence", *Doklady Mathematics*, Vol. 377, No. 2, in press.

Biskamp, D., and Müller, W.-C., 1999, "Decay laws for three-dimensional magnetohydrodynamic turbulence", *Phys. Rev. Letters*, Vol. 33, No. 11, pp. 2195-2198.

Frick, P., Boffetta, G., Giuliani, P., Lozhkin, S., and Sokoloff, D., 2000, "Long-time behavior of MHD shell model", *Europhys. Lett.* , Vol. 52, No. 5, pp. 359-544.