# REGENERATION CYCLE OF VORTICAL STRUCTURES AND INTERMITTENCY IN HOMOGENEOUS SHEAR FLOW


**P. Gualtieri, C.M. Casciola, R. Piva**
Department of Mechanics and Aeronautics, University of Rome *La Sapienza*
Via Eudossiana 18, 00184 Rome, *Italy*
p.gualtieri@caspur.it



## ABSTRACT

We consider a homogeneous shear flow in a confined box in order to analyze under simplified conditions the interaction of a mean velocity gradient with the turbulent fluctuations. Despite its geometrical simplicity, the evolution of this system is characterized by a cyclic behavior associated with the regeneration of the vortical structures which, in turn, by the Reynolds stresses, induce bursts in the kinetic energy. Purpose of the work is to discuss the statistical properties of the velocity fluctuations, here analyzed in terms of the extended self similarity, to understand to role of the mean shear on the scaling laws of the structure functions. When the effect of the shear is prevailing, i.e during the phases of Reynolds stress activity, the structure functions clearly manifest a double scaling regime. Theoretical considerations are discussed to link the double scaling and the increased intermittency found in shear dominated flows with respect to homogeneous isotropic turbulence.


## INTRODUCTION

Recent theoretical results for wall bounded turbulence have shown that the production of turbulent kinetic energy via the Reynolds stresses $< uv >$ leads to a radical change of the statistical properties of the flow. In particular the classical Kolmogorov-Obhukov similarity law (RKSH) (Kolmogorov 1962) fails when turbulent kinetic energy production prevails over inertial energy transfer. A new similarity law based on the second order structure function

$$< \delta V^p > \propto \frac{< \epsilon_r^{p/2} >}{< \epsilon >^{p/2}} < \delta V^2 >^{p/2} \qquad (1)$$

has been shown to hold in the near wall region of a turbulent channel flow (Benzi et. *al.* 1999). These findings have been recently confirmed by experimental data of a turbulent boundary layer over a flat plate (Jacob et. *al.* 2001) and by a DNS of a homogeneous shear flow (Gualtieri et. *al.* 2000).

There is a strong evidence that the Reynolds stresses are crucial in establishing the statistical properties of shear dominated turbulence. Under this respect, the homogeneous shear flow isolates the effect of a pure shear avoiding concurrent effects typical of wall turbulence induced by the boundary conditions. Here the mean shear can be regarded as the only feature leading to the new similarity law (1) and characterizing the dynamics of vortical structures.

Despite the restricted number of control parameters, the dynamics of the homogeneous shear flow is rather complex. Globally, the flow is characterized by a cyclic behavior associated to the regeneration of vortical structures. Large fluctuations of turbulent kinetic energy are induced by the energy transfer from the mean flow operated by the Reynolds stresses. Within a single burst, well defined phases exist where the flow is mainly controlled by the kinetic energy production mechanism which is typical of wall bounded shear flows. In the other phases the inertial energy transfer is the prevailing process, as for isotropic turbulence.

In these conditions the effect of the superposition of different statistical regimes may obscure the understanding of the relevant processes. To enhance the two different flow conditions we introduce velocity structure functions conditioned to the instantaneous value of the shear scale $L_s$. The neat separation of the scales characterized by turbulent kinetic energy production from those where energy transfer occurs is thus achieved to allow for the extraction of much more clean scaling laws than previously possible.

## REGENERATION CYCLE

We analyze the regeneration cycle of vortical structures in the homogeneous shear flow in terms of the interaction between turbulent kinetic energy and the production term $S <$

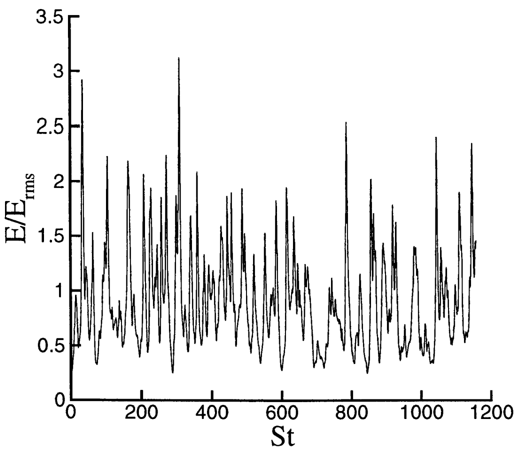


Figure 1: Time history of turbulent kinetic energy in our longest calculation up to $St = 1200$.

$uv >$.

As previously addressed by Pumir (1996) and recently confirmed by the authors (Gualtieri et. *al.* 2000) the flow reaches a statistical steady state. Actually, the energy equation

$$\frac{\partial}{\partial t}[u^2/2] + S[uv] = -\nu[\zeta^2] \qquad (2)$$

(square brackets denote spatial average) suggest a possible a balance between the production term $S[uv]$ and the dissipative one $-\nu[\zeta^2]$. In fact, since the balance is not reached for instantaneous configurations, we observe large fluctuations of the turbulent kinetic energy characterized by a pseudo-cyclic behavior, see figure 1. A time average of eq. (2) over a period of time much longer than the typical length of the bursting cycle corresponds to statistically stationary conditions.

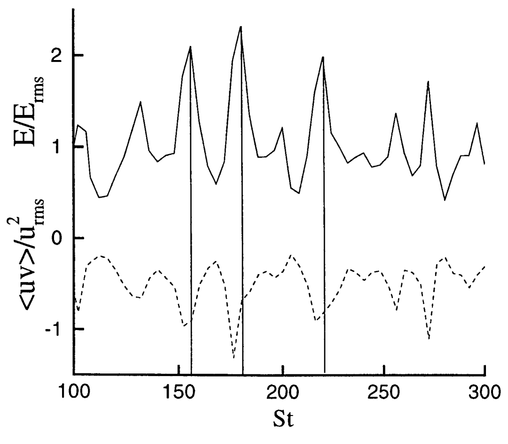


Figure 2: Time history of turbulent kinetic energy (solid line) and Reynolds stressed (dotted line)

In order to relate fluctuations of turbulent kinetic energy with the production term $S[uv]$ we have compared these two quantities in figure 2. We observe that each energy burst is clearly correlated with a corresponding (negative) burst in the production term resulting in a strong injection of energy from the mean flow. In this phase the flow is characterized by well organized streamwise vortices shown in figure 3 at $St = 210$, i.e. at the beginning of the energy burst. These structures are mainly responsible, via a lift-up mechanism, of the large values of the Reynolds stresses observed in this phase. The mechanism that leads to the large fluctuations of turbulent kinetic energy is better described by following the time evolution of the energy spectra (see figure 4). During the first phase of the burst the increase of turbulent kinetic energy may be explained in term of lift-up mechanism and related transient growth. Initially the contribution to turbulent kinetic energy is mainly due to the first spanwise mode $(0, 0, \pm 1)$ described by the linearized equations

$$\begin{cases} \dfrac{d\hat{u}}{dt} = -S\hat{v} - \nu\hat{u} \\ \\ \dfrac{d\hat{v}}{dt} = -\nu\hat{v}, \end{cases} \qquad (3)$$

that predicts a growing amplitude whenever $S[Re(\hat{u})Re(\hat{v}) + Im(\hat{u})Im(\hat{v})] < 0$. In fact, as shown by the energy spectra, energy is mainly injected through the mode $|\vec{k}| = 1$. Since the amplitude of this mode rapidly grows a non linear mechanism is required to explain the saturation of turbulent kinetic energy. Actually non linear interactions are responsible for energy transfer among adjacent Fourier modes. This is shown by the energy spectra in correspondence with the peak of the burst that is spread all over the modes of the first decade, i.e. the energy is now redistributed over a large amount of modes. The spatial configuration

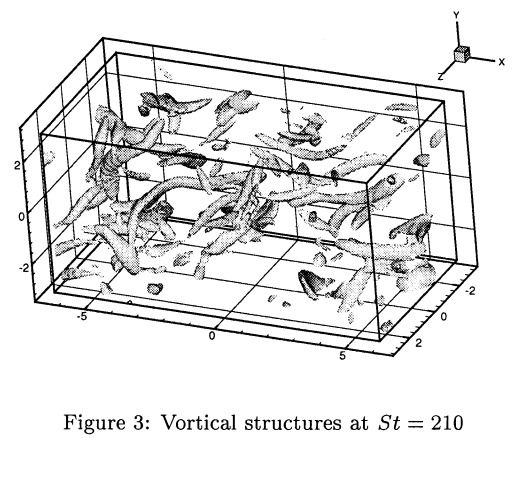


Figure 3: Vortical structures at $St = 210$

of the flow is characterized by less organized structures, (figure 5), which are unable to inject energy in the flow via the lift-up mechanism. Hence a decrease in turbulent kinetic energy is observed. A new cycle starts when the mean flow acts for a sufficiently long time to align again vorticity in the streamwise direction.

Our description of the regeneration cycle in the statistical steady state of the flow is consistent with previous results obtained by Kida and Tanaka (1994) in the early stages of the flow development suggesting that the mechanisms of interaction between the mean flow and the vorticity field are substantially identical.

We like to comment that during the bursting cycle the flow is characterized by different vortical structures associated to different mechanisms of energy production and transfer. In particular during the growing phase of turbulent kinetic energy, streamwise vortices and energy production via the Reynolds stresses are the relevant features of the flow. Afterwards, during the successive phase of energy decrease the inertial energy transfer prevails over energy production.

## THE SHEAR SCALE

Different vortical structures together with different mechanisms of energy production and transfer have been observed during a single burst in the turbulent kinetic energy.

In order to establish the range of scales where production of turbulent kinetic energy is active and transfer occurs, it is useful to introduce the shear scale $L_s$. In the homogeneous shear flow two typical velocity fluctuations may be considered: fluctuations directly induced by the mean flow $\delta U_s = Sr$ and fluctuations associated with the process of inertial energy transfer that may be evaluated following Kolmogorov phenomenology as $\delta u \propto \bar{\epsilon}^{1/3} r^{1/3}$ where $\bar{\epsilon}$ is the mean rate of energy dissipation per unit mass. The scale $r = L_s$ where $\delta U_s = \delta u$ defines the shear scale

$$L_s = \sqrt{\frac{\bar{\epsilon}}{S^3}}. \tag{4}$$

Clearly in the range of scales $L_s \ll r \ll l_d$, where $l_d = q^3/\bar{\epsilon}$ is a typical integral scale, we have $\delta U_s \gg \delta u$ and the flow is characterized by the production of turbulent kinetic energy. The effect of the mean shear over the turbulent velocity fluctuations may be characterized by the non dimensional parameter $S^* = Sq^2/\bar{\epsilon}$ (Lee et. *al.* 1990) to be interpreted as the ratio

$$S^* = \left(\frac{l_d}{L_s}\right)^{2/3}. \tag{5}$$

Hence $S^*$ measures the extension of the range of scales where the effect of the mean shear is relevant.

On the other hand in the range of scales $\eta \ll r \ll L_s$, where $\eta$ is the Kolmogorov dissipative scale, we have $\delta u \gg \delta U_s$ and the flow is dominated by the inertial energy transfer typical of isotropic turbulence. Actually the extension of this range of scales is measured by the non dimensional parameter $S_c^* = \nu(\bar{\epsilon}/S)^{1/2}$ (Saddoughi and Veeravalli 1994) to be interpreted as the ratio

$$S_c^* = \left(\frac{L_s}{\eta}\right)^{2/3}. \tag{6}$$

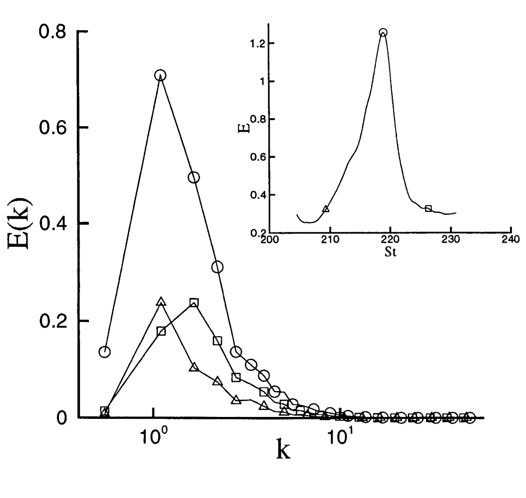


Figure 4: Time evolution for the energy spectra during the burst of the turbulent kinetic reported in the inset

## STATISTICAL ANALYSIS

A quantitative description of the statistical behavior of turbulence is achieved by analyzing the scaling behavior,if any, of the structure

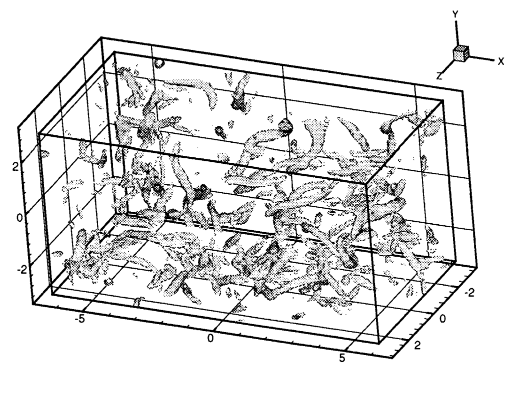

Figure 5: Vortical structures at $St = 219$

functions, i.e. the moments of longitudinal velocity increments

$$< \delta V^p >=< \{[\vec{u}(\vec{x}+\vec{r},t) - \vec{u}(\vec{x},t)] \cdot \frac{\vec{r}}{r}\}^p > \quad (7)$$

where angular brackets denote ensemble average. For homogeneous and isotropic turbulence Kolmogorov theory (K41) (Kolmogorov 1941) provides a dimensional scaling law in terms of separation. Starting from Landau objection (Frisch 1995) a revised form of similarity law has been proposed by Kolmogorov and Obhukov (K62) (Kolmogorov 1962) taking into account for the statistical properties of the energy dissipation

$$< \delta V^p > \propto < \epsilon_r^{p/3} > r^{p/3} \quad (8)$$

where $< \epsilon_r^q >$ denote the $q-th$ moment of the local energy dissipation averaged over a volume of characteristical dimension $r$. Following K62 the scaling exponents of structure functions in terms of separation are expressed as

$$\zeta(p) = \tau(p/3) + p/3 \quad (9)$$

where the correction $\tau(p/3)$ to the pure dimensional scaling $p/3$ is related to the statistical properties of the dissipation field. The anomalous scaling (9) is consistent with the intermittent nature of turbulence characterized by the non Gaussian behavior of the random variable $\delta V(r)$.

Clearly scaling laws in terms of separation can be detected only in experimental facilities at large enough Reynolds numbers. However an extension of the range where scaling is observed can be achieved by using the Extended Self Similarity (ESS) (Benzi et. *al.* 1995) considering the third order structure function as similarity variable

$$< \delta V^p > \propto < \delta V^3 >^{\zeta_p/\zeta_3} . \quad (10)$$

Experimental results, for homogeneous and isotropic turbulence, have shown how the relative scaling exponent $\zeta_p/\zeta_3$ is Reynolds independent and is consistent with the scaling exponent $\zeta_p$ directly measured in terms of separation. For this reason ESS may be used to compute scaling exponents also in the low Reynolds number flows achieved in DNS simulations (Frisch 1995) when a scaling range in terms of separation is not available.

In principle in the homogeneous shear flow the effect of the mean shear is relevant at scales $r \gg L_s$ while inertial energy transfer prevails at scales $r \ll L_s$. In figure 6 we compare the logarithmic local slope of $< \delta V^6 >$ *vs.* $< \delta V^3 >$, plotted as function of separation, with the corresponding value in homogeneous isotropic turbulence. Isotropic turbulence is characterized by a well defined scaling region while in the homogeneous shear flow the relative scaling behavior of structure functions is less pronounced and confined to a narrower range of scales. Moreover the scaling exponent substantially differs from the corresponding value in isotropic turbulence in the whole range of resolved scales. The scaling region that we observe in the homogeneous shear flow, is broken for $r/\eta \sim 20$ that corresponds approximately to the position of the shear scale $L_s/\eta \sim 15$ of our calculation.

We like to add that the shear scale, as well as the Kolmogorov scale, are fluctuating quantities. Hence the two presumed scaling regions for $r \gg L_s$ and $r \ll L_s$ are continously changing with the result of having poor scaling properties in both ranges.

In order to reduce as much as possible the fluctuations of the shear scale and the consequent overlapping of the two scaling regions a conditional statistics is required to apply. The conditioning criteria has to satisfy some prerequisites. First of all a reduction of the fluctuations of the shear scale must be achieved. Moreover it is useful to consider only flow configurations characterized by strong turbulent kinetic energy production in order to capture the scaling region, if any, dominated by the mean shear at scales $r \gg L_s$. Following the considerations related to the regeneration cycle of vortical structure, the flow configurations in the initial growing phase of the energy are characterized by turbulent kinetic energy pro-

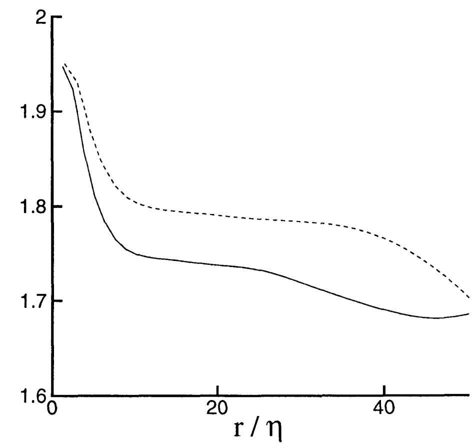


Figure 6: $d[log < \delta V^6 >]/d[log < \delta V^3 >]$ as a function of separation in homogeneous isotropic turbulence (dotted line) and in the homogeneous shear flow (solid line)

duction and by a relatively large value of the ratio $L_s/\eta \propto \Omega^{3/4}$ where $\Omega$ is the mean enstrophy. Hence the conditioning criteria may be formulated as follow

$$\begin{cases} E/E_{rms} > \sigma \\ dE/dt > 0 \end{cases} \tag{11}$$

In figure 7 we have compared the conditioned local slope obtained for different values of $\sigma$ against the local slope computed using the whole statistical sample. Two well distinct range of scales where scaling occurs can now be identified. For $r < L_s$ we recover exactly the scaling exponent of homogeneous and isotropic turbulence while for $r > L_s$ the scaling exponent differs considerably from the previous value approaching, for larger $\sigma$, the value observed in the buffer region of wall bounded turbulent flows (Toschi et. *al.* 1999). Our findings are in good agreement with experimental results by Ruiz-Chavarria et. *al.* (2000) and by Jacob et. *al.* (2001) obtained in the context of a turbulent boundary layer over a flat plate. Also in this case, in the logarithmic region, where the shear scale falls in the middle of inertial range, structure functions, when analyzed through ESS show a double scaling regime for $r < L_s$ and $r > L_s$.

In order to get a deeper insight into this kind of behavior we have checked with the present data the validity of the RKSH similarity laws. Namely we evaluate the ranges where the classical RKSH (Benzi et. *al.* 1996)

$$< \delta V^p > \propto \frac{< \epsilon_r^{p/3} >}{< \epsilon >^{p/3}} < \delta V^3 >^{p/3} \tag{12}$$

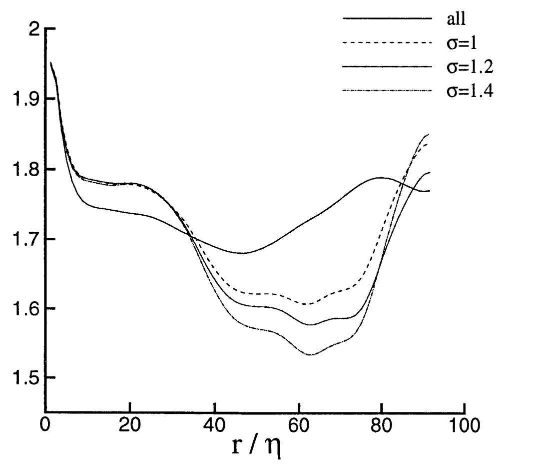


Figure 7: $d[log < \delta V^6 >]/d[log < \delta V^3 >]$ as a function of separation in homogeneous shear flow using conditioned statistics. Different curves corresponds to the values of the threshold $\sigma = 1, 1.2, 1.4$

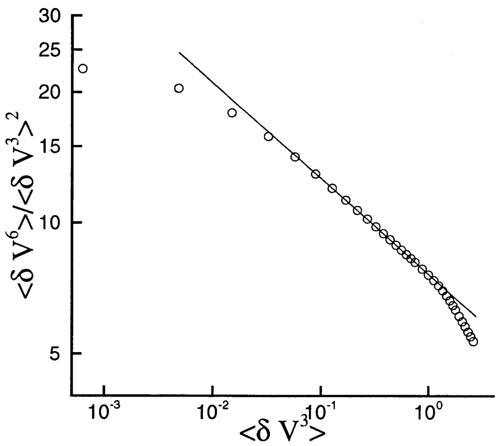


Figure 8: $< \delta V^6 > / < \delta V^3 >^2$ *vs.* $< \delta V^3 >$ for $\sigma = 1.4$ (open circles). The solid line with slope $s_2 = -0.22$ gives the fit in the range $r < L_s$.

and the new similarity law (1) hold respectively. In figure 8 we have plotted $< \delta V^6 > / < \delta V^3 >^2$ *vs.* $< \delta V^3 >$ and its fit in the range $r < L_s$ with a slope $s_2 = -0.22$. This result is consistent with eq. (12) showing how in the homogeneous shear flow at $r < L_s$ the statistical behavior of isotropic turbulence is recovered in terms of the classical RKSH. In figure 9 we have plotted $< \delta V^6 > / < \delta V^2 >^3$ *vs.* $< \delta V^3 >$ and its fit in the range $r > L_s$ with a slope $s_3 = -0.58$. This result agrees with the new form of similarity law eq. (1) that is established for $r > L_s$ where turbulent kinetic energy production prevails. The scaling exponents $s_2$ and $s_3$ reproduce the well known statistical properties of the dissipation field $< \epsilon_r^q > \propto \tau(q)$ valid for homogeneous and isotropic turbulence, e.g. She-Leveque model

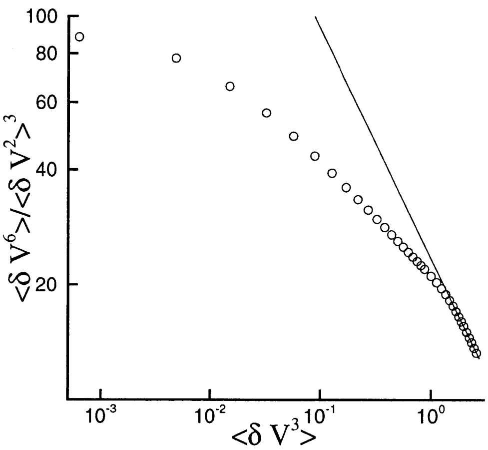


Figure 9: $< \delta V^6 > / < \delta V^2 >^3$ *vs.* $< \delta V^3 >$ for $\sigma = 1.4$ (open circles). The solid line with slope $s_3 = -0.58$ gives the fit in the range $r > L_s$.

(1994), both for $r < L_s$ and $r > L_s$. This result suggests that the statistical properties of the dissipation field are weakly dependent on the flow configuration. In other words the different nature of intermittency observed for $r > L_s$ is completely described by the new similarity law (1) without invoking any drastical change in the statistical properties of the dissipation field.

## FINAL REMARKS

We have analyzed the regeneration cycle of vortical structures in the statistical steady state of a homogeneous shear flow by relating the large fluctuations of turbulent kinetic energy to the presence of different vortical structures along the regeneration cycle. Actually the flow is characterized by phases where the production of turbulent kinetic energy via the Reynolds stresses is relevant followed by phases where energy transfer prevails. When shear dominated configurations are addressed we have preliminary evidence of a double scaling behavior of structure functions. These results are consistent with the presence of the shear scale $L_s$ that separates the range of scales where production of turbulent kinetic energy is relevant from the range of scales where energy transfer prevails. Our data are in agreement with the classical RKSH, eq. (12), at scales $r < L_s$ and with the new form of similarity law (1) that is established at scales $r > L_s$

## REFERENCES


Kolmogorov A. N.,1962, "A refinement of previous hypotheses concerning the local structure of turbulence in incompressible viscous fluid for very large Reynolds number", *J. Fluid. Mech.*, Vol. 13, pp. 82-85

Benzi R., Amati G., Casciola C.M., Toschi F., Piva R., 1999, "Intermittency and scaling laws for wall bounded turbulence", *Phys. Fluids*, Vol 11(6), pp. 1-3

Jacob B., Olivieri A., Casciola C.M., 2001, "Experimental assessment of a new form of scaling law for near wall turbulence" *Submitted to Phys. Fluids*

Gualtieri P., Casciola C.M., Benzi R., Amati G., Piva R., 2000 "Scaling laws and intermittency in homogeneous shear flow", *Submitted to Phys. Fluids*

Pumir A., 1996, "Turbulence in homogeneous shear flow", *Phys. Fluids*, Vol. 8(11), pp. 3112-3127

Kida S., Tanaka M., 1994, "Dynamics of vortical structures in homogeneous shear flow", *J. Fluid Mech.*, Vol. 274, pp. 43-68

Lee M.J., Kim J., Moin P., 1990, "Structure of turbulence at high shear rate", *J. Fluid Mech.*, Vol. 216, pp. 561-583

Saddoughi S.G., Veeravalli S.V., 1994, "Local isotropy in turbulent boundary layer at high Reynolds number", *J. Fluid Mech.*, Vol. 268, pp. 333-372

Kolmogorov A.N., 1941, "The local structure of turbulence in incompressible viscous fluid for very large Reynolds number", *Dolk. Akad. SSSR* Vol. 30, pp. 301, reprinted in 1991, *Proc. R. Soc. Lond.*, A Vol. 434, pp. 15-19

Frisch U., 1995, "Turbulence" *Cambridge university press*

Benzi R., Ciliberto S., Baudet C., Ruiz-Chavarria G., 1995, "On the scaling of three dimensional homogeneous and isotropic turbulence", *Physica D.*, Vol. 80, pp. 385-398

Toschi F., Amati G., Succi S., Benzi R., Piva R., 1999, "Intermittency and structure functions in channel flow turbulence", *Phys. Rev. Lett*, Vol. 82(25), pp. 5044-5049

Ruiz-Chavarria G., Ciliberto S., Baudet C., Leveque E., 2000, "Scaling properties of streamwise component of velocity in a turbulent boundary layer", *Physica D*, Vol. 141, pp. 183-198

Benzi R., Biferale L., Ciliberto S., Struglia M.V., Tripicccione R., 1996, "Generalized scaling in fully developed turbulence", *Physica D.*, Vol. 96, pp. 162-181

She Z.S., Leveque E., 1994, "Universal scaling laws in fully developed turbulence", *Phys. Rev. Lett.*, Vol. 70(3), pp. 336-339