THE IRREGULAR VORTEX SHEDDING REGIME FOR A SQUARE CYLINDER WAKE NEAR A WALL

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ABSTRACT

Surface pressure and wake velocity fluctuations are used to investigate the changes in vortex shedding from a square cylinder near a solid wall for Reynolds number 18,600. Time and frequency analyses show that the formation of shed vortices occurs further downstream of the cylinder as the critical gap height for suppression is approached. Permanent reattachment of the shear layer on the lower cylinder face interrupts the feedback mechanism between the wake and cylinder side face and periodic fluctuations can only be detected downstream of the cylinder. Complete suppression of vortex shedding occurs as the gap height is further reduced. The combined influence of gap blockage and reduced streamline curvature causes a reduction in the ratio of the lower to upper shear layer intensity below the critical value for the stable vortex street formation predicted from stability analysis.

INTRODUCTION

When a bluff-body is placed in the proximity of a solid wall, the structure and dynamic behaviour of the flow changes significantly as a function of the obstacle-to-wall gap height, S. For example, for two-dimensional bodies there exists a critical gap height below which vortex shedding is suppressed. However, even for a given geometry, results found in the literature differ significantly making the exact critical gap height difficult to identify. For a square cylinder of diameter D, visualization experiments and single-point frequency measurements in the wake, Durao *et al.* (1991) and Bosch *et al.* (1996) found that at high Reynolds numbers vortex shedding is suppressed below S/D = 0.35. Further, Bosch *et al.* (1996) also report that the vortex shedding occurs intermittently for 0.35 < S/D < 0.5. Also using single point frequency measurements in the wake, Wu (1999) report a critical gap height of S/D = 0.3 with intermittent vortex shedding occurring for 0.3 < S/D < 0.5. However, using frequency measurements of cylinder surface pressure, Haidn *et al.* (1999) found complete suppression occurs at a critical gap height of S/D = 0.4. Further inconsistency arrives from Taniguchi *et al.* (1983) who report a critical gap height of S/D = 0.5, using a criterion based on spurious interruptions of the shedding cycle. The primary focus of this study is thus to address the discrepancies in the literature regarding the reported critical gap height.

In this investigation the flow around a square cross-section cylinder, placed normal to the on-coming flow, is investigated as a function of S/D. Detailed characterization of the dynamic behaviour for the gap heights near suppression is provided. A mechanism is proposed that explains the discrepancies in the definition of the critical gap height and the suppression of vortex shedding.

EXPERIMENT DESCRIPTION

The experimental geometry is defined in Figure 1. Experiments were performed in a 0.45m X 0.45m suction-type wind tunnel. A smooth, flat plate, 1.0m long and 5mm thick, was placed 0.15m from a tunnel wall. The plate had a sharpened leading edge and a trailing edge flap to control the plate boundary layer and leading edge separation. A square cylinder of side dimension D = 30mm was placed 14.5D downstream of the plate leading edge. The cylinder aspect ratio was 15. The blockage ratio was 6.6%.

Experiments were conducted for Reynolds number of Re=18,600 (based on D and the free stream velocity, U_∞). The on-coming free stream turbulence intensity was 1%. The boundary layer thickness was measured without the cylinder to be 0.47D at 15D from the leading edge of the plate. The gap height was adjusted at 2mm increments from S/D = 0.1 to 1.5.

One hundred and eleven pressure taps were located along the centerline: 79 on the plate and 32 around the cylinder circumference. Tubing response was flat to 200Hz.

A hot-wire probe was located 6D downstream of the cylinder, 0.25D above the cylinder upper face. Free stream conditions were continuously measured from a pitot-static tube located 5D upstream of the cylinder and 4D away from the tunnel wall.

The pressure taps, pitot-static tube and hot-wires were sampled simultaneously at 400Hz for periods of 30s.

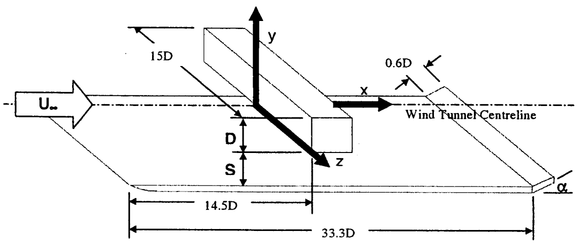


Figure 1 : *Experimental geometry and nomenclature.*

RESULTS

This study focuses on the irregular behaviour and suppression of vortex shedding occurring close to the critical gap height. Measurements of the shedding frequency and autocorrelations of wall surface pressure measurements are used to identify changes in vortex formation. The pressure distribution within the gap allows identification of the influence of the wall proximity on the lower shear layer. Velocity measurements from Wu (1999) are used to support conclusions drawn from the pressure field analysis.

Frequency Domain

The frequency of shed vortices, f_s, could be detected from spectral analysis of hot-wire anemometry signals in the wake and pressure measurements of the cylinder upper and lower faces. For gap heights S/D>0.5, the shedding frequency is nearly constant in terms of the Strouhal number: $f_s/U_\infty \approx 0.129 \pm 0.003$.

Power spectra of pressure signals from a tap located centrally on the upper face are shown in Figure 2a. At S/D = 0.47, a peak clearly identifies the existence of vortex shedding. For S/D = 0.4 a peak is difficult to distinguish which could indicate that regular vortex shedding has been suppressed. However, power spectra of the velocity fluctuations from a hot-wire placed 6D downstream of the trailing edge depicted in Figure 2b, show a shedding frequency for gap heights as low as S/D = 0.33. Immediately, it becomes apparent that when determining critical gap height the measurement location will influence results.

Time Domain

Autocorrelations provide a measure of the strength of the periodicity within a time series by returning the highest correlation at N/f_s, corresponding to N shedding periods. The autocorrelation function for a time series is defined in equation (1):

$$R(\tau) = \frac{1}{C_P'^2\,T}\int_0^T \left(C_P(t) - \overline{C_P}\right)\left(C_P(t+\tau) - \overline{C_P}\right)dt \quad (1)$$

(a)

(b)

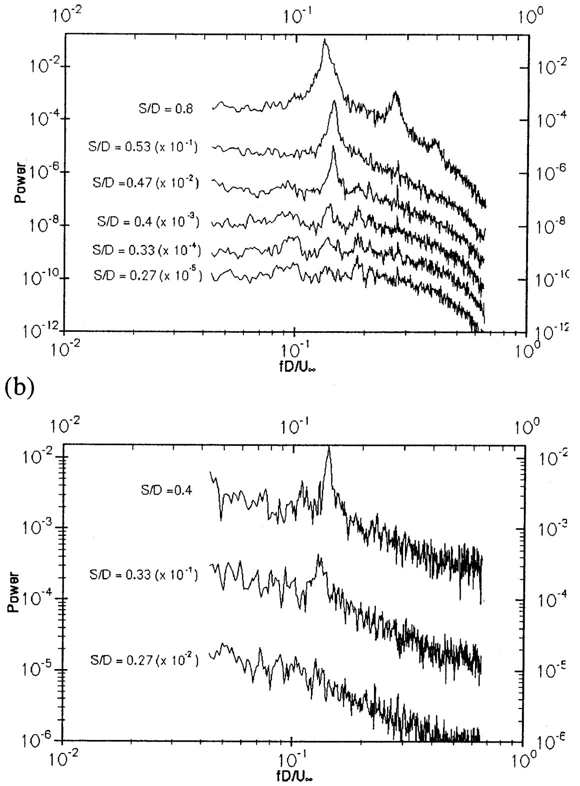


Figure 2 : *Power spectra from (a) cylinder pressure and (b) hot-wire in the cylinder wake. Power for each gap height reduced by value in brackets for clarity.*

where T is the length of the time series, $\overline{C_P}$ is the time average of the signal and C_P' is the standard deviation of the signal.

Figures 3a-d show contour plots combining autocorrelations of pressure signals acquired simultaneously at 25 downstream locations along the plate in the cylinder wake. Periodic behaviour, consistent with vortex shedding, is evident. For S/D = 0.53 (Figure 3a) the strongest shedding (highest correlation amplitude over several cycles) occurs below the cylinder trailing edge, x/D ≈ 1.0. However for S/D = 0.47 (Figure 3b) the peak location has moved downstream of the cylinder, (x/D ≈ 1.5) and for S/D = 0.4 (Figure 3c) the strongest shedding occurs at x/D ≈ 2.5 and little periodicity is evident on or below the cylinder. As the gap height is further reduced (i.e S/D = 0.27, Figure 3d) the periodic behaviour is almost nonexistent.

It is also observed that shedding was difficult to identify where the wall flow is disturbed by the end of the recirculation wake

These results suggest that the formation of the vortex street shifts to a location downstream of the cylinder as S/D is reduced below 0.5. This change in formation length serves to explain discrepancies in critical gap heights reported in the literature. Measurements made at the cylinder will find a loss of periodic behaviour below S/D = 0.4 while measurements made in the wake will identify suppression at S/D ≈ 0.3.

It is reported in the literature that shedding becomes increasingly irregular as the gap height is reduced below S/D ≈ 0.5. Bosch *et al.* (1996) and Wu (1999) report that the vortex shedding is increasingly interrupted by increasingly longer periods of random fluctuation as S/D approaches the critical gap height. This loss of coherent motion is consistent with weakening of the autocorrelation function, shown in Figure 3, as S/D is reduced.

In the following sections, it will be discussed that the intermittent shear layer reattachment on the cylinder lower face can explain at least partially the irregularity in shedding as already indicated by Okajima (1982). It will be further argued that increasing gap effects, as S/D is reduced, weaken the lower shear layer while the upper shear layer strength remains unaffected. This increase in asymmetry results in increasingly weaker shear layer coupling and eventually suppression of shedding.

Influence of the Gap

It is important to examine the pressure field within the gap to understand the influence of the wall on vortex shedding. In this section, it is argued, using a free streamline assumption, that changes in the shedding behaviour are linked to influence of the gap height on the lower shear layer curvature and reattachment and that mean pressure gradients are indicators of changes in shear layer behaviour. It is also shown that the viscous wall flow does not contribute directly to suppression of shedding.

The normal pressure gradient can be related to streamline curvature through equation (2), the Navier Stokes equation normal to a streamline (cf. Milne-Thompson, 1968).

$$\frac{1}{\rho}\frac{\partial P}{\partial n} + K_S q^2 + \frac{\partial F}{\partial n} = \upsilon\left\{2K_S\frac{\partial q}{\partial s} + 2K_n\frac{\partial q}{\partial n} + q\left(\frac{\partial K_S}{\partial s} + \frac{\partial K_n}{\partial n}\right)\right\} \qquad (2)$$

where F is the body force (which can be considered negligible), q is the velocity magnitude, s is in the direction of the streamline, n is the direction normal to the streamline, and K_s and K_n are the streamline and normal curvature terms defined by equation (3).

$$K_s = \frac{\partial \theta}{\partial s}; \qquad K_n = \frac{\partial \theta}{\partial n} \qquad (3)$$

with θ being the angle between the streamline and the x-axis.

Examining the mean velocity field of Durao *et al.* (1991) for S/D = 0.25 and 0.5, the mean wall flow never separates in the gap while the mean flow reattaches on the bottom cylinder face. Downstream of the mean reattachment point, the mean gap flow is nearly parallel to both walls. Under these conditions, an order-of-magnitude analysis shows

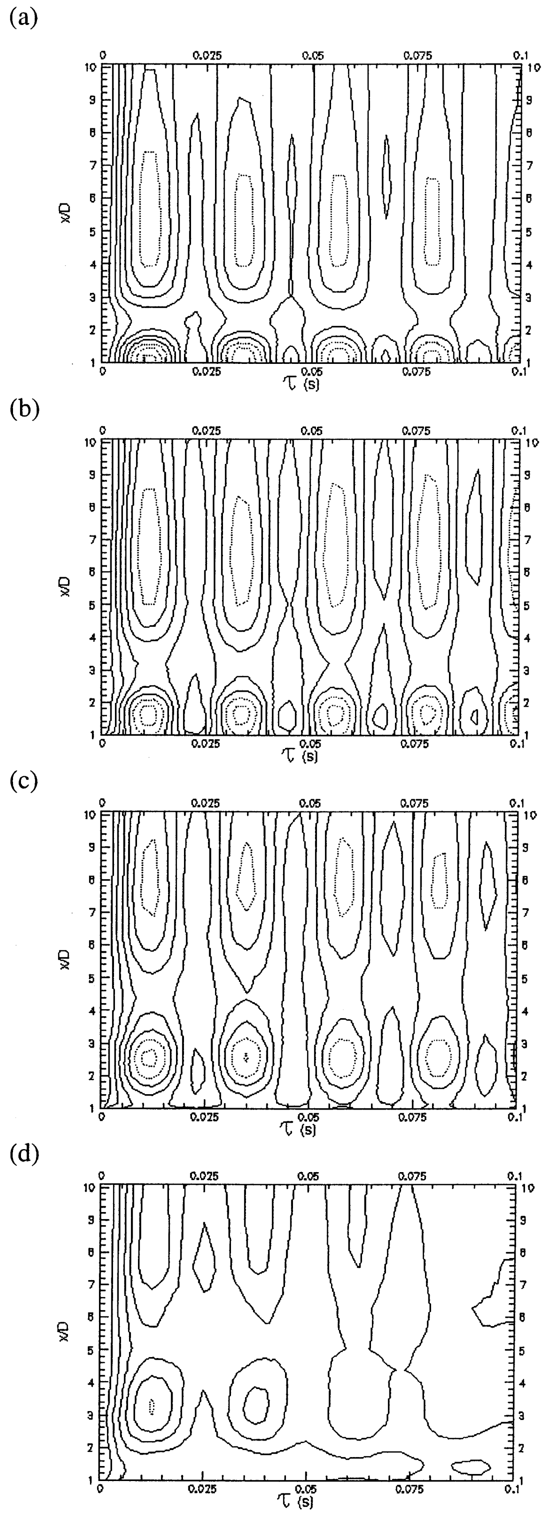


Figure 3 : *Contours of autocorrelation results downstream of cylinder for (a) S/D = 0.53, (b) S/D = 0.47, (c) S/D = 0.4 and (d) S/D = 0.27. Contour spacing is 0.2, dashed lines indicate negative contours.*

that the viscous terms on the right-hand-side of equation (2) are at least one order smaller than the inertial terms giving equation (4).

$$\frac{\partial P}{\partial n} \approx -\rho K_S q^2 \tag{4}$$

Furthermore, since the flow is nearly parallel, the vertical (y) and streamline normal (n) directions can be approximated as coincident for the downstream section of the gap (x/D ≥ 0.25). It thus appears reasonable to assume that the mean pressure differential between the wall and the bottom face, ΔC_{pwb} in equation (5), scales with the average normal pressure gradient across the gap.

$$\Delta C_{pwb} = C_{p,wall} - C_{p,cylinder} \propto \frac{\partial P}{\partial n}\Delta n \tag{5}$$

The gap pressure differential is shown as a function of the streamwise location in Figure 4. For S/D ≥ 0.6 the pressure differential is negative, suggesting negative curvature (outward normal in negative direction or towards the wall) and the flow is thus separated. The influence of the gap height is simply to reduce the relative magnitude of the pressure gradient and fluctuation levels.

For S/D ≤ 0.4, the pressure gradient is approximately zero or slightly positive downstream of the cylinder center (x/D = 0.5). In this region, the mean streamline curvature is thus opposite to that observed for S/D ≥ 0.6. This observation, together with data presented by Durao *et al.* (1991) for S/D = 0.25, suggest that the flow has permanently reattached on the bottom face of the cylinder. For this gap height range, as indicated in Figure 2a, shedding in the near wake is completely suppressed.

For 0.4 < S/D < 0.6, the differential is negative along most of the gap but reaches zero at the trailing edge, indicating that the streamline curvature is nearly zero at the trailing edge. This gap height range coincides with that for which irregular vortex shedding was observed. Reduced curvature of the lower shear layer would lead to the shear layers interacting downstream of the cylinder as observed in Figure 3.

Reattachment

Reattachment of the flow on the cylinder bottom face modifies the shedding behaviour. As the gap is reduced, the streamline curvature at the lower leading edge increases, causing the separated shear layer to reattach earlier. In this section, it will be shown that the leading edge streamline curvature, K_s, depends only on the gap geometry to approximately S/D ≈ 0.25 and that interference of the viscous wall layer is not a necessary mechanism to cause suppression of vortex shedding. Unfortunately, direct measurements of the streamline curvature are only available for a few gap heights. However, the mean pressure differential between the cylinder face

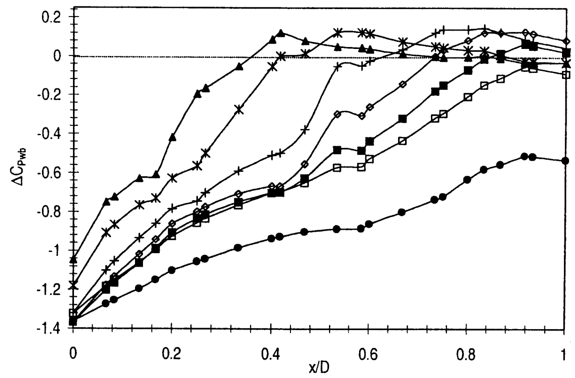


Figure 4 : *Time averaged vertical pressure gradient within gap:* ▲, S/D = 0.2; *, S/D = 0.27; +, S/D = 0.40; ◇, S/D = 0.47; ■, S/D = 0.53; □, S/D = 0.6; ●, S/D = 1.2.

and wall at the gap entrance can be used to determine the changes in streamline curvature.

The curvature of the streamlines directly upstream of the gap entrance is very large, but, in the gap, the streamlines straighten very rapidly. It is thus assumed that the rate of change of the streamline angle relative to the x-direction, θ, is much larger in the s than the n directions: $K_s >> K_n$, from which equation (6) follows:

$$-\frac{1}{\sin\theta}\frac{\partial\theta}{\partial y} \approx \frac{\partial\theta}{\partial s} \tag{6}$$

The mean streamline angle for S/D = 0.25, 0.5 and 1.0 at the gap entrance (below the leading edge) are shown in Figure 5, as a function of the normalized distance from the bottom face, $y^*=(y/D+0.5)/(S/D)$, from LDV measurements of Wu (1999). The θ distribution is similar for all three cases shown, which implies that (1/sin θ) $\partial\theta/\partial y^*$ is independent of S/D and, consequently, the streamline curvature, K_s, depends only on the gap height from equation (7).

$$\frac{K_s}{\mathrm{D}} = \frac{\partial\theta}{\partial s}\frac{1}{D} \approx -\frac{1}{\sin\theta}\frac{\partial\theta}{\partial(y/\mathrm{D})} = -\frac{1}{\mathrm{S/D}}\left(\frac{1}{\sin\theta}\frac{\partial\theta}{\partial y^*}\right) \tag{7}$$

The pressure differential ΔC_{pwb} at the leading edge can be used to verify this assumption. To approximate the normal pressure gradient in the gap, the values of q from Wu (1999) are used. From Figure 5, it is seen that the product Z in equation (8) is nearly constant across the entire gap.

$$Z = \frac{1}{\sin\theta}\frac{\partial\theta}{\partial y^*}q^2 D \tag{8}$$

It thus follows from equation (4) that the normal pressure gradient is inversely proportional to S/D given equation (9) as:

$$\frac{1}{\rho}\frac{\partial P}{\partial n} \approx -K_s q^2 \approx \frac{Z}{\mathrm{S/D}} \tag{9}$$

Subject to the assumptions of equation (6), equation (8) and (9) predict that ΔC_{Pwb} is independent of S/D, which is verified at the leading edge in Figure 6 for S/D > 0.25. Since the scaling for K_s depends only on S/D, these results imply that interference of the viscous wall layer can be neglected for S/D > 0.25 and thus justifies the assumption of equation (4) for the gap entrance. Since suppression occurs for S/D ≈ 0.3, it can also be argued that suppression is not due to vorticity generated at the wall within the gap.

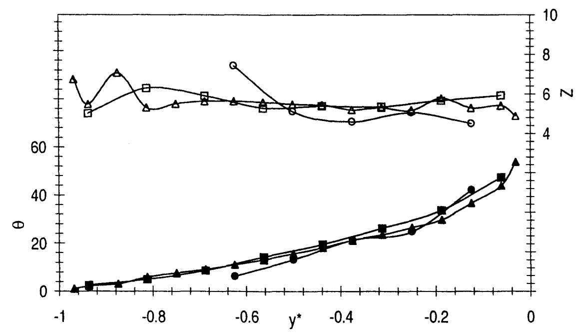


Figure 5 : *Streamline angle for* ●, S/D = 0.25; ▲, S/D = 0.5; ■, S/D = 1.0 and Z for ○, S/D = 0.25; Δ, S/D = 0.5;□, S/D = 1.0 as a function of y*.

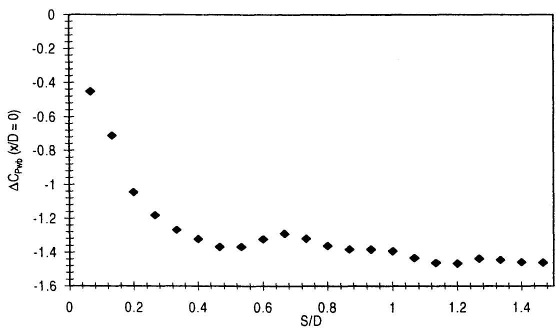


Figure 6 : ΔC_{Pwb} *at the cylinder leading edge.*

Effect of Small Gap Heights on Vortex Shedding Stability

In the original analysis of the vortex street, von Karman (1912) studied the linear stability of a double row of equally spaced point vortices to small perturbations. The vortices in the opposite rows are assumed to be counter rotating and of equal strength. However, the influence of wall proximity is to cause asymmetry in the strength of the vortices. In an analysis for vortex streets, with the circulation of the opposite rows is related by a constant ratio, γ, stable shedding was shown to exist for 0.382 < γ < 2.618 (Martinuzzi *et al.*, 2001, Haidn *et al.* ,1999).

The results of the stability analysis indicates that sufficient asymmetry in the intensity of the opposing shear layers can prevent the formation of a stable vortex street. Contributions to the bottom shear layer circulation are examined next to determine how the effect of the gap on the lower shear layer would affect the circulation ratio between the two shear layers. The lower shear layer is of particular interest since calculations performed using the LDA results of Wu (1999) show that the intensity of the top shear layer changes little with S/D.

Near the obstacle faces, the shear layers are thin and are approximated as two-dimensional vortex sheets. The circulation of a vortex sheet, Γ, is given by equation (10).

$$\Gamma = \int \kappa ds \qquad (10)$$

where κ is the vortex sheet strength and ω is the vorticity defined as in equation (11):

$$\kappa = \int \omega dn \ , \ \omega = -\frac{\partial q}{\partial n} + K_S q \qquad (11)$$

using the streamline coordinate system (Milne-Thompson, 1968).

The shear layer intensity can therefore be expressed as in equation (12).

$$\kappa = -\delta q + \int K_S q dn\Big|_{\text{across the vortex sheet}} \qquad (12)$$

The intensity of the lower shear layer can thus be modified through the velocity difference across the shear layer or through a reduction of the vortex sheet length.

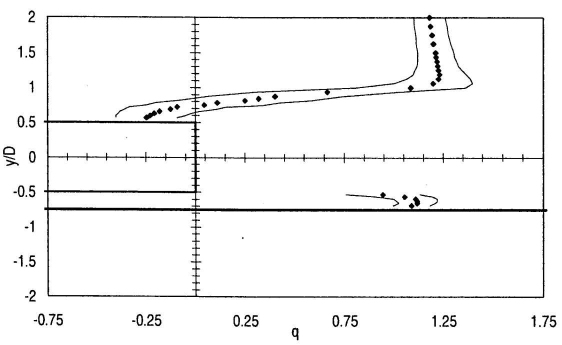
Reduction of Shear Layer Strength through Gap Blockage.

Using the LDV measurements of Wu (1999), differences in the velocity differential contribution to intensity of the upper and lower shear layers can be identified. Figure 6a shows the mean velocity magnitude, *q*, profiles at x/D = 1.0 (the cylinder trailing edge) for the permanently reattached case S/D = 0.25. The edges of the shear layer can be approximated using the maximum and minimum velocity magnitude values. Using this approximation, and using the u component of velocity to represent streamline direction, the upper shear layer maximum is $1.23U_\infty$ and the lower shear layer maximum is $1.12U_\infty$. While the maximum shear layer velocities differ a little (~10%), the minimum velocities differ significantly. Due to backflow from the formation region entering the separated region along the upper face, the minimum velocity magnitude is found to be at y/D = 0.5625 and is $-0.25U_\infty$. Due to permanent reattachment, no backflow occurs along the lower cylinder face. Hence, the lower shear layer minimum occurs at the cylinder surface and is zero (no-slip condition). Thus, based on the contribution of the velocity differential, δ*q*, the intensity κ is 30% less for the bottom than the top shear layer due to flow blockage alone.

For comparison, velocity profiles for the shedding case of S/D = 1.0 at x/D = 1.0 are shown in Figure

6b. The upper shear layer maximum velocity is $1.18U_\infty$ with a minimum of $-0.17U_\infty$. The lower shear layer maximum is $1.37U_\infty$ with the minimum again using the no-slip condition since the flow reattaches in the mean. The velocity differential δq across the top and bottom shear layers is nearly the same and, hence, so is the intensity κ, since the shear layers are also the same length.

(a)



(b)

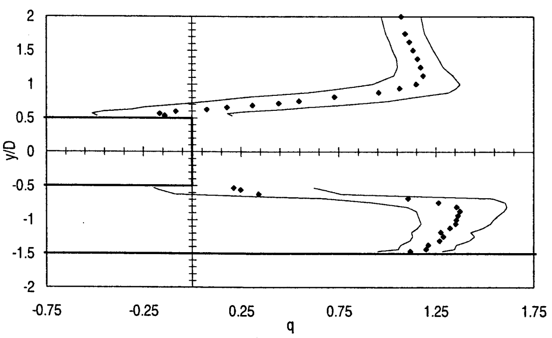


Figure 6 : *Velocity magnitude profiles at trailing edge of cylinder for* (a) S/D = 0.25 and (b) S/D = 1.0: ◆, q; —, q ± q'.

Reduction of Shear Layer Strength through Integral Length. The asymmetry in the vortex sheet strength due to the above observation may be insufficient to create $\kappa_{upper}/\kappa_{lower}$ ratios exceeding the stability criterion of 2.618. Equation (10), however, shows that another mechanism does exist to increase the asymmetry, since the circulation is cumulative over the formation length. By approximating the location of shear layer initiation as the location of separation, the upper shear layer initiates at the leading edge of the top face while the bottom shear layer, due to reattachment in the range S/D < 0.4, initiates at the trailing edge of the cylinder. Even if all other parameters are the same in both shear layers, the net vortex sheet circulation will be smaller for the bottom than the top vortex sheet due to a shorter streamwise length of integration. Conversely, if reattachment does not occur, the reduction of shear layer strength due to gap blockage will be further amplified along the length of the shear layer. Results from Figure 3 show that shear layer coupling occurs increasingly further downstream of the cylinder for S/D < 0.5, thereby increasing the length of the shear layers.

CONCLUSIONS

As the S/D is reduced below 0.5, the shear layers couple increasingly further downstream of the cylinder due to the straightening of the lower shear layer through the gap. This downstream coupling can be credited with discrepancies in critical gap height reported in the literature. Permanent reattachment of the lower shear layer occurs for S/D < 0.4 due to increased curvature of the shear layer at the leading edge of the cylinder. Intermittency in the vortex shedding is caused by a combination of intermittent reattachment of the lower shear layer and asymmetric shear layer intensity. Gap blockage reduces the lower vortex sheet strength and this effect is amplified through the length of the shear layers. When the ratio of circulation between the upper and lower vortex sheets does not satisfy the stability criterion $0.382 < \Gamma_1/\Gamma_2 < 2.618$, the formation of a regular vortex street is suppressed. Physically, the bottom shear layer is too weak to couple with instabilities in the top shear layer.

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