CURRENT AND FUTURE CHALLENGES IN TURBULENCE MODEL DEVELOPMENT

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ABSTRACT

The Reynolds-averaged Navier-Stokes approach will continue as the primary methodology used in the foreseeable future for the numerical solution of complex turbulent fluid flow problems. With this continuing demand and the continuing increase in computational capabilities, higher-order closures, such as algebraic and differential Reynolds stress models, will be more extensively utilized. Even though such closures are currently available, modeling improvements will be required in a variety of terms that appear in these formulations to satisfy the ever-increasing accuracy requirements. Beyond these approaches, as computational capabilities increase, will be the trend toward the "direct" solution of turbulent flow fields. In the interim, however, composite methods, that bridge the gap between partial and total resolution of the turbulence scales, will be required.

INTRODUCTION

Prediction and control of turbulence will continue to be an important part of many fluid flow studies of technological importance. Associated with turbulent flow prediction and control comes the need to choose a methodology that can be applied to such flows. Obviously, solving the full time-dependent Navier-Stokes equations with a sufficiently accurate numerical technique and sufficiently well-resolved grid is most desirable. In such methodologies, well known grid resolution and numerical algorithm issues arise, so it is necessary to turn to alternative methodologies that assume in each case their own set of constraints.

Nevertheless, over the last thirty years numerical codes and turbulence models developed for the solution of turbulent flow fields have reached a point of maturity where a small but vibrant industry has emerged that provides its customers with the tools needed to predict a rather complex array of flows. Unfortunately, such ease of use and availability to numerous users in a wide variety of technical disciplines has not been without its drawbacks.

In this environment, a large number of flow field predictions are reported that are too numerous and diversified to thoroughly verify. Apparently minor alterations to published models are continuously made and undocumented – with minimal regard for overall consequences, or full understanding of model origin and calibration. Factors, such as these, lead to transparent improvements to existing models, and/or proliferation of "new" models.

Demand has increased for an improved level of performance associated with the application of the various predictive techniques. This has been especially true of Reynolds averaged Navier-Stokes (RANS) models, probably because that they have been "under development" for the longest period. Two criteria that should be met for improved performance levels are the ability to *correctly replicate* the flows to be predicted, and once done, to *correctly solve* the flow problem. Unfortunately, while it is now relatively easy to get numerical answers, meeting these two criteria of doing the *right problem* and doing the *problem right* is a more challenging task.

With the need to meet these performance criteria, there have been attempts in the last few years to develop a process by which users can follow a set of guidelines in order to formulate or replicate the right problem, and then use the right tools to get the necessary answers. One such attempt initiated by NASA (see Bardina et al. 1997), involved development of a procedure for validation and testing of current state-of-the-art turbulence models. A more recent and complete endeavor on the part of ERCOFTAC (Casey and Wintergerste, 2000) has resulted in a document "intended as a practical guide giving best practice advice for achieving high-quality industrial

Computational Fluid Dynamics (CFD) simulations using the Reynolds-averaged Navier-Stokes (RANS) equations." Both of these initiatives, especially the ERCOFTAC effort, attempt to provide practical information to the technical, but non-specialist user of CFD codes.

A separate though related issue is the numerical solution of equations in a RANS formulation. In many cases, assessment of model performance has not been sufficiently separated from numerical issues, leading to model refinements predicated on purely numerical issues rather than on modeling the proper turbulent physics. The previously mentioned procedure guides will hopefully provide for improved assessments.

In the following sections, the algebraic and differential Reynolds stress levels of closure will be discussed. Within these levels, deficiencies and areas of improvement for higher-order correlation closure models will be discussed. Finally, the outlook for the development of models that provide a linkage between the traditional RANS-type models and the full Navier-Stokes simulations will be addressed.

REYNOLDS-AVERAGED NAVIER-STOKES CLOSURE SCHEMES

As noted, an optimal methodology for solving turbulent flow fields would be to efficiently and accurately solve the Navier-Stokes equations directly. Unfortunately, since the ratio of integral scales to Kolmogorov scales in a turbulent flow is $\mathcal{O}(Re_t^{3/4})$, such direct computations are not realistic, and it is necessary to formulate alternative methodologies. One alternative, introduced over three decades ago, is the large-eddy simulation (LES) approach. This methodology was intended to bridge the gap between full numerical simulations of the Navier-Stokes equations and the Reynolds-averaged Navier-Stokes approach. While its use has become widespread, ever increasing demands on performance accuracy has necessitated a more careful scrutiny of the various subgrid scale models required, as well as their interaction with the accompanying numerical algorithm. Nevertheless, as computer power and memory increases, this methodology will be an increasingly important tool in the future.

The Reynolds-averaged approach to analyzing turbulent flow fields dates back over a century to Osborne Reynolds, who assumed that velocity and pressure fields could be decomposed into a mean component and a fluctuating component. The resulting RANS equations require closure for the higher-order turbulent correlations that appear, and it is within this context that RANS modeling for turbulent flows has evolved. Figure 1 shows a hierarchy of closure models formulated to use with RANS equations for the mean flow. The focus here will be limited to single-point closures that form the basis for almost all practical flow field computations.

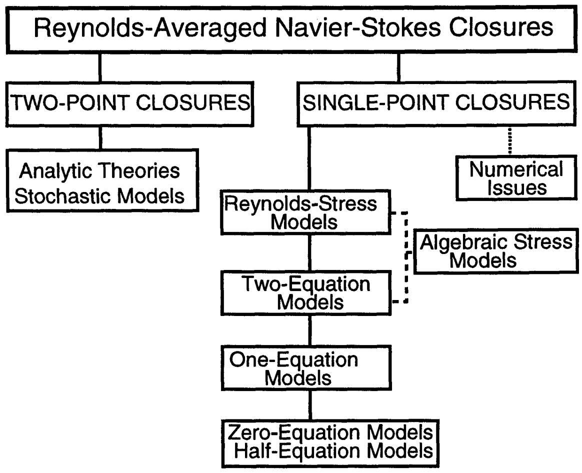

Figure 1: Hierarchy of RANS solution methods for turbulent flow problems

Inherent limitations are immediately introduced into the modeling process when restricted to single-point closures. For example, correlations involving fluctuating pressure, a nonlocal quantity, are constrained to pointwise (local) closures that may not correctly mimic the physical process. Two-point closures can account for these non-local effects, but are significantly more complex to have only been extensively studied in homogeneous flows (see the review by Cambon and Scott (1999) for more details). Nevertheless, models of such terms (and others with similar nonlocal characteristics) have been developed for single-point closures, and have been successfully used throughout the years.

Another limitation in single-point closures is the lack of eddy structure information. While two-point closures are capable of tracking such behavior, over the last decade Reynolds and Kassinos (see Kassinos and Reynolds 1997) have been developing a structure-based model in the context of single-point closures. Such models are still in the development stage but can provide information on both componentality and dimensionality of turbulence.

CURRENT HIGH-ORDER STRATEGIES

With the large number of turbulence models available, it is easily seen how practitioners of computational fluid dynamics can become increasingly confused by the available choices. The levels of closure shown in Fig. 1 divide the available models into groups sharing certain common traits. Within each level the detailed form of models may vary, but each member of a particular level will exhibit the same characteristic features. The focus here will be on the two highest levels: the Reynolds-stress models and the (explicit) algebraic stress models. Within these two closure levels, current opportunities for improved models can be identified.

Reynolds-stress models

At present, Reynolds-stress models (RSMs) are the highest level of closure, and the most common second-moment closures used in practical calculations. It is at this level where distinct differences in form exist between the incompressible and compressible transport equations, so it is advantageous to start with the compressible form of the equations and then proceed to the incompressible form. In compressible flows, the decomposition of the instantaneous variables can be in terms of either the usual Reynolds averaged variables or Favre-averaged variables

$$f = \overline{f} + f' = \tilde{f} + f''. \tag{1}$$

With this decomposition, it is straightforward, although tedious, to derive the Reynolds-averaged equation for the Reynolds stress tensor in Favre variables, $\overline{\rho}\tau_{ij}$ $(= \overline{\rho u_i'' u_j''})$. A more useful form here is the corresponding transport equation for the Reynolds stress anisotropy tensor, b_{ij},

$$b_{ij} = \frac{\tau_{ij}}{2K} - \frac{\delta_{ij}}{3} \tag{2}$$

given by

$$\begin{aligned}\frac{Db_{ij}}{Dt} - \frac{1}{2K}\left(\mathcal{D}_{ij} - \frac{2}{3}\mathcal{D}\delta_{ij}\right) = &-\frac{b_{ij}}{g\tau} - \frac{d_{ij}}{\tau} \\ &-\frac{2}{3}S_{ij} + (b_{ik}W_{kj} - W_{ik}b_{kj}) \\ &- \left(b_{ik}S_{kj} + S_{ik}b_{kj} - \frac{2}{3}b_{mn}S_{mn}\delta_{ij}\right) \\ &+ \frac{1}{2K}\left(\Pi_{ij}^{d} - \frac{2}{3}\Pi^{dl}\delta_{ij}\right) \\ &+ \frac{1}{2K}\left(M_{ij} + \frac{2}{3}\mathcal{M}\delta_{ij}\right),\end{aligned} \tag{3}$$

where $\overline{\rho}K$ is the turbulent kinetic energy, τ $(= K/\epsilon)$ is a turbulent time scale, S_{ij} is the (traceless) strain rate tensor, and W_{ij} is the rotation rate tensor $(S_{ij} + W_{ij} = \partial\tilde{u}_i/\partial x_j)$. Higher-order correlations also appear in Eq. (3), including the turbulent transport and viscous diffusion term

$$\begin{aligned}\overline{\rho}\mathcal{D}_{ij} = -\frac{\partial}{\partial x_k}\Bigg[&\underbrace{\overline{\rho u_i'' u_j'' u_k''} + \overline{p'(u_i'\delta_{jk} + u_j'\delta_{ik})}}_{\text{turbulent transport}} \\ &- \underbrace{(\overline{\sigma_{ik}' u_j' + \sigma_{jk}' u_i'})}_{\text{viscous diffusion}}\Bigg],\end{aligned} \tag{4a}$$

where $\mathcal{D} = D_{ii}/2$; the pressure-strain rate correlation

$$\overline{\rho}\Pi_{ij} = \overline{\rho}\Pi_{ij}^{dl} + \overline{\rho}\Pi_{ij}^{d} = \overline{p'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)}, \tag{4b}$$

where the correlation has been partitioned into a deviatoric part and a dilatational part,

$$\overline{\rho}\Pi_{ij}^{dl} = \frac{2}{3}\overline{p'\frac{\partial u_k'}{\partial x_k}}\delta_{ij} = \frac{2}{3}\Pi^{dl}\delta_{ij}\ ; \tag{4c}$$

the turbulent dissipation rate

$$\overline{\rho}\epsilon_{ij} = \frac{2}{3}\overline{\rho}\epsilon\delta_{ij} + 2\overline{\rho}\epsilon d_{ij} = \overline{\sigma_{ik}'\frac{\partial u_j'}{\partial x_k}} + \overline{\sigma_{jk}'\frac{\partial u_i'}{\partial x_k}}, \tag{4d}$$

where the isotropic part is assumed partitioned into a solenoidal part and a dilatational part (the deviatoric components d_{ij} are associated with the solenoidal part); and the contribution due to the mass flux given by

$$\begin{aligned}\overline{\rho}M_{ij} = &\overline{\rho u_i''}\left(\frac{\partial\overline{\sigma}_{jk}}{\partial x_k} - \frac{\partial\overline{p}}{\partial x_j}\right) \\ &+ \overline{\rho u_j''}\left(\frac{\partial\overline{\sigma}_{ik}}{\partial x_k} - \frac{\partial\overline{p}}{\partial x_i}\right),\end{aligned} \tag{4e}$$

where $\mathcal{M} = M_{ii}/2$. Finally, the coefficient g is given by

$$g = \left(\frac{\mathcal{P}}{\epsilon} - 1 + \frac{\mathcal{D}}{\varepsilon} + \frac{\Pi^{dl}}{\varepsilon} + \frac{\mathcal{M}}{\varepsilon}\right)^{-1} \tag{5}$$

where $\mathcal{P} = -2Kb_{ik}S_{ki}$ (recall S_{ki} is traceless).

Once expressed in Favre variables, it is readily seen which terms are directly related to compressibility effects, and which terms are amenable to variable mean density extensions of their incompressible counterparts. From Eq. (3), three terms can be identified that are directly affected by compressibility. These are the pressure-dilatation Π_{ij}^{dl}, Eq. (4c), the turbulent energy dissipation rate ϵ, Eq. (4d), and the mass flux $\overline{\rho u_i''}$, Eq. (4e). In addition to these three terms, the turbulent heat flux is also required since it appears in the conservation equation for the mean total energy as well as (indirectly) in the pressure-velocity correlation term (see Eq. (10) below). Currently, the remaining terms in the equation are handled through variable mean density extensions of their incompressible forms. As noted, dissipation rate ϵ has been partitioned into a solenoidal and dilatational part with effects of compressibility isolated in the dilatational part, and the solenoidal part ε is treated as in the incompressible case. For constant mean density fields, these compressible terms are neglected and the transport equation for the Reynolds stress tensor takes on its incompressible form.

Before assessing the current models for some of these unknown correlations, a method for obtaining explicit algebraic stress models using tensor representations will be outlined. This methodology uses Eq. (3) with Db_{ij}/Dt, and an assumed form for turbulent transport and viscous diffusion. This allows for the development of explicit models without introducing any new closure coefficients other than those that appear in the full differential form.

Explicit tensor representations

In the current environment, where development of improved turbulence closure models is driven more from the practical engineering side rather than the academic side, cost/benefit ratios become important criteria for deciding which closure methodology to use. When coupled with the reality that only a small subset of government or industrial production codes have fully implemented second-moment closures, it becomes more apparent why the Reynolds stress models are rather infrequently used in an applications oriented environment. In the future, this should change as the user community becomes more acclimated to increasingly complex closure models.

With this background, the focus in this subsection will be on a level of closure that retains a key aspect of second-moment closures – the ability to predict Reynolds stress anisotropies – but also retains the same type of numerical robustness associated with the two-equation, linear eddy viscosity model (LEVM) level of closure. This level of nonlinear eddy viscosity models (NLEVMs) has existed for some time and is adapted to accounting for turbulent stress anisotropies that may arise in the flow.

In both NLEVMs and explicit algebraic stress models (EASMs), the proper choice for tensor representation is dependent on the functional dependencies associated with the Reynolds stress $\overline{\rho}\tau_{ij}$, or the corresponding anisotropy tensor b_{ij}. For example, it is common to assume that the functional dependency of the anisotropy tensor is given by

$$b_{ij} = b_{ij}(S_{kl}, W_{kl}, \tau) = \mathbf{b}(\mathbf{S}, \mathbf{W}, \tau). \quad (6)$$

A polynomial expansion is then assumed that is a subset of a general set of basis tensors (Pope 1975, Gatski and Jongen 2000) deduced from the functional dependencies of the Reynolds stress anisotropy tensor. What distinguishes the two approaches is the method by which the expansion coefficients are determined. For the NLEVMs, expansion coefficients are determined based on calibrations with experimental or numerical data, and on some physical consistency constraints. For the EASMs, the expansion coefficients are derived consistent with a full differential Reynolds stress equation such as that given in Eq. (3) or its incompressible counterpart. In both cases an explicit tensor representation for $\mathbf{b}$ is obtained in terms of tensor functions of $\mathbf{S}$ and $\mathbf{W}$; however, substantial differences exist that find their origin precisely in the way the expansion coefficients are obtained. In light of this rather broad approach to coefficient calibration, which can be taken in developing any particular NLEVM, it is not possible to detail these models here. The interested reader is referred to Gatski and Jongen (2000) and Gatski and Rumsey (2001) for details as well as additional references.

For the development of EASMs, a formalism has been developed that, when rigorously applied to the second-moment transport equation given in Eq. (3), yields an explicit tensor representation for the Reynolds stress anisotropy tensor with closure coefficients directly extracted from the full differential form. A linear relation of the form,

$$\mathbf{b} = \sum_{n=1}^{\mathrm{N}} \alpha_n \mathbf{T}^{(n)}, \quad (7)$$

can then be obtained between the tensor $\mathbf{b}$ and a finite number N of other tensors $\mathbf{T}^{(1)}$, $\mathbf{T}^{(2)}$,..., $\mathbf{T}^{(\mathrm{N})}$ that are formed from elements of $\mathbf{S}$ and $\mathbf{W}$. Scalar coefficients in this linear relation are invariants of the independent tensors and of $\mathbf{b}$. Since $\mathbf{b}$ is required to be an isotropic function of **S** and **W**, the form of Eq. (7) must also be unaltered by simultaneous orthogonal transformations of the matrices **b**, **S** and **W**. Thus, it is possible to obtain an expression for the expansion coefficients that are independent of the coordinate system used, and are algebraic functions of invariants formed with **b** and the basis tensors. A simple way to obtain this expression is to form the trace of the N matrix equations (Gatski and Jongen 2000) from Eq. (7),

$$\sum_{n=1}^{N} \alpha_n \{\mathbf{T}^{(n)}\mathbf{T}^{(m)}\} = \{\mathbf{b}\mathbf{T}^{(m)}\}, \qquad (8)$$

where $m = 1, 2, ..., N$. These N scalar equations in the coefficients α_n may be solved to obtain the desired expressions for α_n as functions of the invariants.

This same approach can be generalized to the implicit algebraic equation given by the right-hand-side of Eq. (3). This implicit equation is the usual starting point for development of EASMs, and under the formalism just described each term in the equation can be projected onto the function space spanned by the tensors $\mathbf{T}^{(N)}$ to yield an optimal representation for **b**.

Even though the EASMs are now being used in many applications with generally good success, it is important to recognize the inherent deficiencies in the formulation. First is the weak equilibrium assumption ($Db_{ij}/Dt = 0$) and second is the assumed form of the turbulent transport and viscous diffusion model. This means that effects such as relaxation of the individual stress components are not included, and turbulent transport is not fully accounted for in the formulation, however, these deficiencies can be addressed in a rigorous manner in the EASM formulation. For example, the weak equilibrium assumption can be shown to be related to the frame-invariance properties of the algebraic constitutive equation. As such, modifications can be introduced that yield an algebraic stress model retaining the same invariance properties as the full differential stress model (Gatski and Jongen 2000).

The EASM level of closure remedies some deficiencies inherent in the two-equation level of closure (as well as other linear eddy viscosity models). The most obvious deficiency is the isotropic eddy viscosity assumption that is a consequence of the Boussinesq approximation of assuming a direct proportionality between $\boldsymbol{\tau}$ and **S**. Another deficiency is that the models are materially-frame indifferent, a consequence of the models' sole dependence on the objective strain rate tensor **S** through the Boussinesq assumption. This is in contrast with the EASMs (and RSMs), which are not frame-indifferent, since they also display a functional dependency on the non-objective rotation rate tensor **W**. It can be shown that turbulent closure models do not have to be materially frame-indifferent even though the Reynolds stress tensor is (Speziale 1998). This deficiency means there is an insensitivity to non-inertial (as well as curvature) effects.

MODELING CHALLENGES

The transport equation for the compressible, Reynolds stress anisotropy tensor in Eq. (3) shows the set of higher-order correlations that require closure. Since formal development of explicit algebraic stress models simply uses unaltered closure models for these higher-order correlations, it is only necessary to assess the various terms which appear in (3).

As alluded to previously, the deviatoric part of the pressure-strain rate correlation Π_{ij}^d is commonly treated in the same way for both incompressible and compressible flows. High-Reynolds number models for this correlation (Launder et al. 1975, Speziale et al. 1991) have been under development for over twenty-five years and have probably reached a point of maturity where further improvements will be difficult to justify in thin shear flows or predominantly two-dimensional flows. In addition, databases of sufficient accuracy or completeness for fully three-dimensional flows are difficult to find, yet necessary in order to establish the reliability of current models in more complex flows.

Other unknown correlations that appear in Eq. (3) are not as well established (even for their high-Reynolds number forms) as Π_{ij}^d and should be considered further.

Compressibility effects

Compressible formulations rely mainly on variable mean density extensions of incompressible models and to a lesser extent on the modeling of terms which appear solely in the compressible transport equations. Even with variable mean density extensions, care must be taken or log-layer balances can be adversely affected (Huang et al. 1994).

There appears to be no "complete" compressible model where all terms that represent the compressibility effects have been included.

Nevertheless, models do exist, but have not been evaluated in total. Brief comments follow on each of these terms.

Heat and mass flux. The heat flux term plays a prominent role in the mean (total) energy equation, and also affects the pressure-velocity correlation $\overline{p'u_i'}$. Gradient-diffusion models have been the most popular and simplest closures for turbulent heat fluxes, although more recent attempts take into account variable Prandtl number effects. These require solution of transport equations for the temperature variance and dissipation of temperature variance. Both algebraic and differential scalar flux models have been proposed but, for the most part, their calibration has not been based on compressible flows.

The average fluctuating velocity $\overline{u_i''}$ is related to the mass flux $\overline{\rho' u_i'}$ through

$$\overline{u_i''} \equiv -\frac{\overline{\rho' u_i'}}{\overline{\rho}} \ . \tag{9}$$

This term is important to flow dynamics in the vicinity of shocks and in reacting flows. Many of the models for this term were developed about a decade ago (e.g. Rubesin 1990) and have only received limited attention (Huang et al. 1995).

The level of sophistication in each of these closures varies and new approaches may be necessary. While a modeled, full differential form is an alternative, such an approach would certainly over-burden any general purpose numerical solver. One attractive alternative is to use the representation theory discussed in the previous section. While the discussion here focuses on tensor representations, vector representations of the heat and mass fluxes may be a way to further enhance current models. The current form of such scalar flux closures lies more in the category of nonlinear eddy diffusivity models since they have a proper functional behavior, but the expansion coefficients are determined from imposed physical consistency constraints.

Dilatation terms. Both the dilatation dissipation and the pressure-dilatation models currently available originated about a decade ago (Zeman 1990 and 1993, Sarkar et al. 1991, Sarkar 1992, see also Wilcox 1998). Dilatation dissipation and the pressure-dilatation were modeled and calibrated based on early DNS results of homogeneous flows. They were also applied to inhomogeneous flows, such as mixing layers (dilatation dissipation) and boundary layers (pressure-dilatation). In a compressible mixing-layer, the dilatation dissipation can have a significantly favorable effect on the spreading rate; whereas, in a flat-plate boundary-layer, the log-law can be adversely affected (Zeman 1993).

These early models attempted to account for the experimental observation that a decrease of turbulent fluctuations corresponded with an increase in turbulent Mach number, and led to the conclusion that turbulent anisotropies had a relatively constant behavior over a convective Mach number range. Dilatational dissipation models were able to account for turbulence reduction and brought calculations into agreement with experiments. Thus, the results strongly suggested that an increase in the compressible dissipation and the resultant decrease in turbulence were the dynamic reasons for the reduction in spreading rate. Recent DNS studies (e.g. Pantano and Sarkar 1999), however, have shown that differences in the level and distribution of pressure fluctuations are central to the reduction in turbulence. This means that the previous rational for modeling dilatational dissipation was not justified even though it yielded the correct observed spreading rate behavior for the mixing layer.

At present, the correct modeling approach to dilatational terms remains an open question. This provides an opportunity for reassessment and consideration of alternative forms for such models.

Scale equation

Associated with the closure of the second-moment transport equation described by Eq. (3) is the need to find a model for the turbulent, tensor dissipation rate. The high-Reynolds number form of such models assumes isotropy and generally involves a transport equation for the solenoidal part. The most popular alternative to the dissipation rate is the specific dissipation rate ω (see Wilcox 1998).

One deficiency of single-point closures is that the models used for higher-order correlations (high-Re form) are characterized by a single time scale (say K/ε). Various unknown correlations that appear in the turbulent transport equation (3) represent turbulent interactions associated with different parts of the turbulent spectrum. For this reason multiple-time scale modeling was introduced over two decades ago (see Schiestel 1987). Such concepts have not been able to replace the usual single-scale modeling to date; although, with the ever-increasing availability of direct simulation, interest in this approach is being renewed.

Rigorous modeling of a tensor dissipation rate transport equation has long been restricted by lack of detailed data; however, more details about the balance of terms in the "exact" transport equation for ε have emerged from direct simulations of (simple) inhomogeneous flows. This information has not been extensively used in developing a revamped model of either the tensor dissipation rate or isotropic dissipation rate equation. The recent attempts at deriving such transport equations, and utilizing some simulation databases, have been from either an approach analogous to the development of Reynolds stress model (Speziale and Gatski 1997) or development of a length scale equation extracted from a two-point correlation tensor equation (Oberlack 1997).

Wall proximity effects

Complex wall-bounded turbulent flows represent a large portion of the application areas of current interest. Unfortunately, as the complexity of the flow field increases and the corresponding need for higher-order closures increases, the robustness and efficiency of solution procedure decreases. Presence of the wall and a corresponding decrease in local Reynolds number implies that additional characteristic turbulent scales need to be introduced into the modeling process. This has been most commonly done in the dissipation rate (or scale) equation through the introduction of damping functions, which effectively switch between high-Reynolds number turbulent scales such as those characterized by K and ε alone, and the Kolmogorov scales. Higher-order closures such as Reynolds stress models impose the additional constraint that individual components of the Reynolds stress tensor need to be "damped" in a manner consistent with both the turbulent kinetic energy and dissipation rate equations.

Recent approaches focus on incorporating dynamic characteristics into near-wall closure schemes in order to minimize or eliminate any dependence on geometry dependent parameters. Examples of the incorporated characteristics are the two-component limiting behavior of the turbulent stress anisotropy and normalized length-scale gradients (e.g. Craft et al. 1998). Still other approaches have introduced an elliptic relaxation of components of a velocity-pressure gradient correlation and tensor dissipation rate. This relaxation approach was proposed almost a decade ago (Durbin 1993) for a Reynolds stress model, but has been successfully applied by a number of authors in a subset two-equation $\overline{v^2} - f$ formulation.

Other complicating factors, such as anisotropies in the (solenoidal) dissipation rate, arise as walls are approached. Clearly, several effects arise in proximity to walls, but up to this point an overall consistent approach to developing a near-wall model has not been fully achieved.

In order to highlight this point, consider the turbulent transport term. Direct simulation results have shown that this term is important in the near-wall region. Several models have been proposed over the last three decades. In the following subsection, an analysis of various models within a common framework will show that while the models differ in detail, they belong to the same general level of closure, and as such should not be expected to give significantly different results.

Turbulent transport. Turbulent transport plays a pivotal role outside regions dominated by either production or redistribution effects. It can be an important contributor to turbulence dynamics in close proximity to walls, near the centerline of free-shear flows, and outer edges of such flows. The focus is on the triple-velocity correlation term. With incompressible flows, the pressure-velocity correlation term is either neglected or assumed modeled with the triple-correlation. In compressible flows, the pressure-velocity correlation is given by

$$\overline{p'u_i'} = R\overline{\rho u_i'' T''} + R\tilde{T}\overline{\rho' u_i'} \qquad (10)$$

where $\tilde{T}$ is the Favre-averaged mean temperature and R is the gas constant. Thus, both heat flux and mass flux models need to be developed that are calibrated in a manner consistent with observed behavior in both the individual terms and the pressure-velocity correlation.

In regions of weak shear, spatial gradients of the time–averaged triple velocity product represent the rate at which Reynolds stresses are transported by turbulent fluctuations. Conventional models of these correlations have been formulated with this physical role in mind, and have been confined to gradient transport forms (proportional to the spatial gradients of the Reynolds stresses) with varying degrees of complexity. It has not been possible to calibrate these alternatives over a range of flows – even in the incompressible case. For the purpose of this discussion, a variable density extension of an incompressible triple-velocity correlation model will be considered, that is,

$$\overline{\rho u_i'' u_j'' u_k''} \approx \overline{\rho} \mathcal{T}_{ijk}, \qquad (11)$$

where $\mathcal{T}_{ijk}$ for the compressible case is a Favre-averaged triple-velocity correlation.

Several models have been proposed over the last thirty years; however, none yield significantly improved predictions. This suggests that a more rational model needs to be developed for the triple-velocity correlation.

One way to proceed is to use tensor representation theory to construct a suitable tensor polynomial expansion consistent with the functional relation,

$$\mathcal{T}_{ijk} = \mathcal{T}_{ijk}\left(\tau_{lm}, \varepsilon, A_{lmn}, S_{mn}, W_{mn}\right) \qquad (12)$$

where $A_{lmn} \equiv \partial \tau_{lm}/\partial x_n$.

Equation (12) is the most general algebraic representation for the symmetric third–order tensor $\mathcal{T}_{ijk}$ in terms of both second– and third–order tensors. Such a representation was developed by G. F. Smith (see Younis et al. 2000) and can be divided into four groups.

Group I: Terms that are linear in $\tau_{ij,k}$ (3 linearly independent terms)

Group II: Terms that are bilinear in $\tau_{ij,k}$ and τ_{ij} (11 linearly independent terms)

Group III: Terms that are bilinear in $\tau_{ij,k}$ and S_{ij} (11 linearly independent terms)

Group IV: Terms that are bilinear in $\tau_{ij,k}$ and W_{ij} (5 linearly independent terms)

These thirty terms produce a general representation of the form

$$\mathcal{T}_{ijk} = \sum_{n=1}^{30} c_n \Phi_{ijk}^{(n)} \qquad (13)$$

where $\Phi_{ijk}^{(n)}$ $(n = 1, \ldots, 30)$ are third-order symmetric tensor-valued isotropic functions, and the c_n scalar expansion coefficients can (in general) be functions of invariants formed from the independent tensors of $\mathcal{T}_{ijk}$.

As noted, the literature contains a diverse assortment of closure models for the triple-velocity correlation. It can be shown (Younis et al. 2000), however, that these fall into two categories that are either linear or bilinear in the tensor dependencies given by

$$\mathcal{T}_{ijk} = \mathcal{T}'_{ijk}\left(\tau_{lm}, \varepsilon, \frac{\partial \tau_{lm}}{\partial x_n}\right) . \qquad (14)$$

Table I summarizes some of the models proposed and the corresponding basis terms involved. It is clear from the table that the most of these models are bilinear in the Reynolds stresses and their gradients, and that no models contain the dependence on mean velocity gradients required by the exact equation. It is not surprising, therefore, that there is little real difference in their performance across a wide range of benchmark turbulent shear flows (Cormack et al. 1978).

Clearly, it is by no means a simple matter to develop a rational model for the triple-velocity correlation. A complicating factor is that one would suspect the dynamics described by the transport equation for triple-moments would differ between a wall-bounded flow and a free-shear flow. Nevertheless, with demands on accuracy increasing, the inability to accurately predict such behavior will become a liability.

FUTURE STRATEGIES

One might expect that the current modeling challenges just outlined will not disappear from the modeling arena in the near future. More complex flow fields, which may include effects such as heat transfer or buoyancy, will challenge the existing forms and require even more accurate models. As computational resources increase, inevitably that there will be more emphasis placed on direct computation of some flow field regions, and in some cases on entire flow fields themselves. Nevertheless, as emphasized at the outset, it will not be possible to handle direct flow field computations of practical geometries for many decades. Composite approaches that utilize aspects of suitably averaged and filtered transport equations for higher-order turbulent correlations will surely be required.

Composite approaches of this type are now being formulated, for example, under headings such as Detached Eddy Simulation (DES) (Spalart 1999) and Flow Simulation Methodology (FSM) (Zhang et al. 2000). In the former approach, the one-equation Spalart-Allmaras turbulence model is altered to account for subgrid-scale effects through a modification of the destruction term. In the latter approach, a defined turbulent stress tensor is delimited between zero (implies DNS) and the Reynolds stress tensor $\overline{\rho}\tau_{ij}$.

In the RANS limit of such approaches, those used have been closure models developed and calibrated for equilibrium conditions of homogeneous flows, equilibrium regions of inho-

Table 1: Summary of $\mathcal{T}_{ijk}$ models and corresponding tensor bases

Basis Tensor	Model Hanjalic & Launder (1972)	Mellor & Herring (1973)	Lumley (1978)	Cormack et al. (1978)	Magnaudet (1993)
$\sum A_{ijk}$		0.073		0.069	
$\sum A_{ipp}\delta_{jk}$				−0.136	
$\sum A_{ppi}\delta_{jk}$				−0.632	
$\sum A_{ipp}\tau_{jk}$				0.102	0.16
$\sum A_{ppi}\tau_{jk}$				−0.068	
$\sum A_{ijp}\tau_{pk}$	0.11		0.098		0.125
$\sum A_{ipq}\tau_{pq}\delta_{jk}$			0.013		
$\sum A_{ppq}\tau_{qi}\delta_{jk}$				0.192	

mogeneous flows, or modeling of statistically steady flows. The adaptation of these models outside the RANS limit has not inherently altered their functional form, but simply blended this RANS behavior in smooth fashion to either a new subgrid scale model (DES), or a direct simulation (FSM). In general, nonequilibrium effects should be the norm in such evolving flow fields, and as such, new models for some already established higher-order correlations will be needed. This is especially true if the hierarchy of closure models established for the RANS approach is carried over to these new composite approaches. Another effect that can have a significant impact on these new models is the single-scale concept currently used. This point was addressed in a previous section and is further amplified in the context of composite schemes due to increased importance of a broader range of turbulent scales. Such questions of functional compatibility should and will be addressed as the variety of flows studied increases, and as demands on performance increase.

CONCLUDING REMARKS

Contrary to some of the more pessimistic forecasts about turbulence modeling and its viability for predicting turbulent flows, there exists a wide variety of uses and challenging problems that can be successfully solved using RANS-type closures. However, based on recent trends, as the maturity level and practical needs of the prediction increase, demands on the performance levels of the models will also increase.

In response to these performance demands, hopefully there will not be a surge of new models that simply reshuffle existing formulations. Specific correlation models (some of which have been outlined here) that possess the correct functional dependencies and can be calibrated unambiguously are needed. Uniquely identifying deficiencies in individual correlation closure models is difficult, especially in complex flow fields. Some of these difficulties may be due to issues associated with the numerical modeling of the physical flow problem rather than modeling of the higher-order turbulence correlations.

While the current challenges outlined focused primarily on traditional RANS solution approaches, it will become increasingly important to formulate enhanced composite approaches that can be easily merged into more direct simulations of turbulent flows. In both cases, these challenges should be viewed as wide ranging opportunities for turbulence model developers and users.

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