DIRECTIONAL SCALE DEPENDENCY ON FORCE COUPLING FOR DISPERSED TWO-PHASE TURBULENT FLOWS

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The mechanism of energy transport by particles in a turbulent channel flow was investigated by large eddy simulation considering directional scale dependency on force coupling between particles and turbulence. The present coupling method introduced the weighted function to distribute the momentum to grid points in a wider region considering the energy backscatter enhanced by particles when the inter-particle spacing was comparable to half of the energy-containing eddy scale, which was ensured by the experiments by Sato *et al.* (2001). The coupling method using both the gaussian and wavelet-like functions were examined and it was found that a distribution scale has a significant influence on modification to fluid turbulence. The expansion of the momentum-distribution scale in the streamwise direction in the order of half of the energy-containing eddy scale realized the directional scale-dependent structure of turbulence modification by particles.

INTRODUCTION

Knowledge of the behavior of discrete particles in turbulent flows has attracted increased interest, as turbulent flows laden with solid particles are a common occurrence in both nature and technology. Prediction of these flows is of importance because turbulent flows laden with particles occur in many technologically important areas and nature. It should be remembered that adding particles to a single-phase flow dramatically complicates characterization of the flow and renders the traditional empirical methodology for single-phase flow far from complete. Previous efforts by experiments to understand turbulence modification by particles have revealed that the addition of small particles suppresses turbulent kinetic energy, while large particles increase turbulence (Tsuji and Morikawa 1982, Tsuji *et al.* 1984, Fleckhause *et al.* 1987, Rogers and Eaton 1991, Kulick *et al.* 1994). While the works using direct numerical simulation (DNS) by Squires and Eaton (1990), Elghobashi and Truesdell (1993), and Boivin *et al.* (1998) have advanced our understanding, the effect of particles on turbulence modification is not fully resolved up to this day.

The *exact* DNS by Takiguchi *et al.* (2000) has become a promising and powerful supplement to experimental investigations of turbulence augmentation by particles. They simulated fluid flows around each particle in a turbulent channel flow without a point-force approximation and showed a good agreement with the experiment using a particle image velocimetry (PIV) by Sato *et al.* (1995). From the viewpoint of industrial application, however, the *exact* DNS remains restricted to relatively low-Reynolds-number turbulent flows with simple geometry. An alternative approach is large eddy simulation (LES) which has been used to date to investigate particle dispersion (Deutsch and Simonin 1991, Simonin *et al.* 1995, Wang and Squires 1995) and particle/turbulence interactions (Boivin *et al.* 2000). None of the studies, unfortunately, proposed a subgrid-scale (SGS) model considering interactions between particles and turbulence, therefore the existing models are forced to be owing to the SGS models for single-phase flow and the point-force approximation.

The objective of the present study is to propose a new type of coupling scheme, what is called *directional scale dependency on force coupling*, in order to overcome the significant problem using the point-force approximation in the LES. The present study concentrates on the two-way-coupling simulation, i.e., particle volumetric fraction is less than 5.0×10^{-4}, which means that the inter-particle collisions are neglected. Sato *et al.* (2001) first reported the directional interactions between particles and fluid turbulence in a turbulent channel flow using a PIV, in which particles enhanced the energy backscatter especially when particles aligned with the transverse direction. The present study focuses on the directional scale-dependent structure revealed in the experiment in order to give further insight into universal understanding for energy transport by particles in dispersed two-phase turbulent flows.

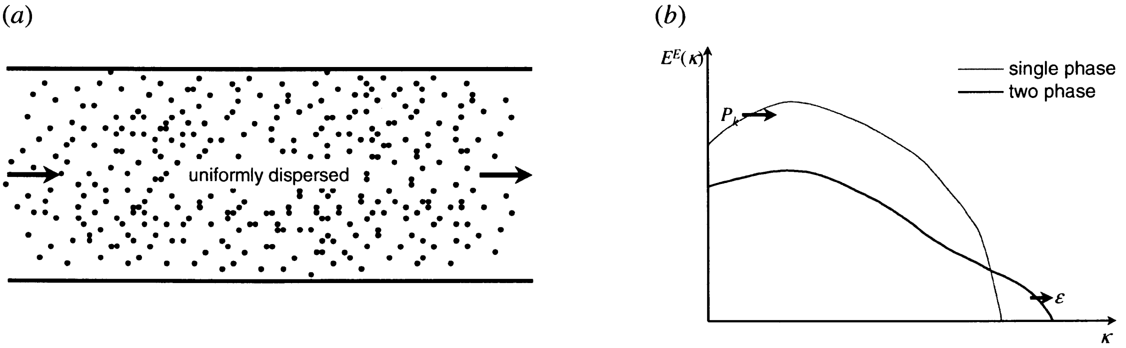

Figure 1. Schematic of turbulence attenuation by small particles. (*a*) Particles are dispersed uniformly in a turbulent flow, (*b*) yielding a decrease in turbulence energy in the low-wave-number region, while an increase in the high-wave-number region.

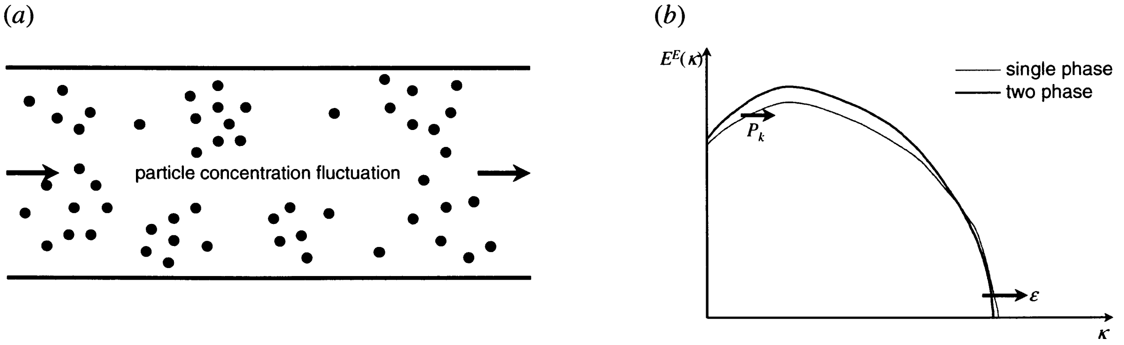

Figure 2. Schematic of turbulence augmentation by large particles. (*a*) Particles are clustered in a turbulent flow, (*b*) yielding an increase in turbulence energy in the overall wave-number region.

THEORETICAL BACKGROUND FOR FORCE COUPLING

Turbulence modification of the continuous phase typically occurs when particles are present in large enough concentrations such that the momentum loss or gain to turbulence provided by particles is no longer negligible. The particle density is assumed to be several orders of magnitude greater than the fluid density so that particles may carry a significant fraction of the momentum of flow even at small volumetric fractions, especially solid particles in air, i.e., turbulence attenuation by particles. Particles dispersed uniformly in a fluid flow, yielding a decrease in turbulence energy in the low-wave-number region, while an increase in the high-wave-number region as illustrated in figure 1 (see Rogers and Eaton 1991, Kulick *et al.* 1994). The conventional point-force-approximation method can predict turbulence attenuation by small particles reasonably well. The conventional method distributes the momentum based on the particle counterforce to only eight grid points around a particle, which is identical to the fact that particles always dissipate turbulence energy.

On the other hand turbulence is augmented by particles which are comparable to or slightly greater than the Kolmogorov micro length scale of fluid flow, which results in increasing turbulence energy in the overall wave-number region. Sato *et al.* (1995) and Sato and Hishida (1996) concluded that particle concentration fluctuations induced turbulence augmentation as shown in figure 2, which was reflected in the multiple-scale concept used in the time-averaged turbulence model (Hishida and Sato 1998). When particles enhance turbulence energy, the directional interactions were found amongst particles in the experiments by Sato *et al.* (2000), moreover recent experimental efforts by Sato *et al.* (2001) have revealed that strong energy-backscatter regions were dominant when the inter-particle spacing was in the order of half of the energy-containing eddy scale as displayed in figure 3. It means that the momentum should be distributed to grid points in a wider region, taking into account scale dependency.

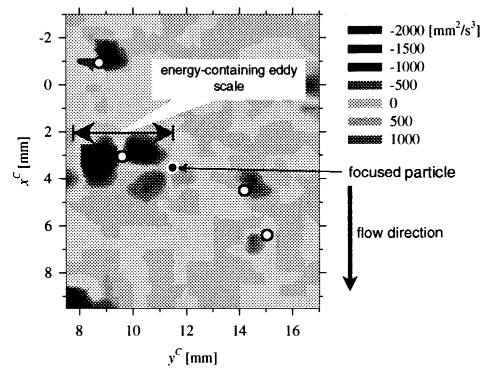

Figure 3. Contour plot of the energy flux to subgrid scale *seen by particle* obtained from experiments by Sato *et al.* (2001). The experimental evidence revealed that the inter-particle spacing comparable to half of the energy-containing eddy scale is a key parameter for energy transport by particles.

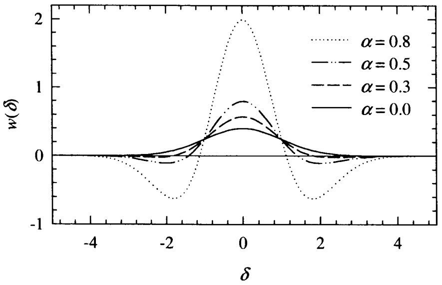

Figure 4. Profiles of weighted function with various values of α.

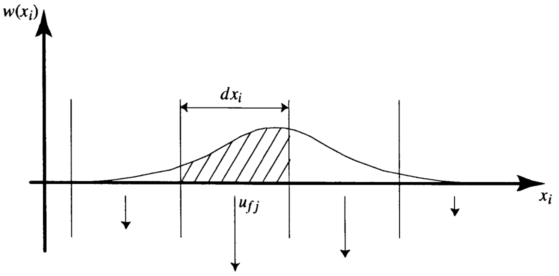

Figure 5. Schematic of momentum distribution to grid points using weighted function for force coupling.

In the conventional point-force-approximation method, i.e., the momentum is distributed to only eight grid points around a particle, a spatial energy spectrum of fluid around particle(s) is governed by only a grid width in a computational domain. From a physical point of view, considering the experimental evidence by Sato *et al.* (2001), the present study introduces the point-force-approximation method using a weighted function, that is,

$$w(\delta) = w_1(\delta_1)\cdot w_2(\delta_2)\cdot w_3(\delta_3)\,. \tag{1}$$

A function $w_i(\delta_i)$ is defined as,

$$w_i(\delta_i) = \frac{1-\alpha\delta_i^2}{(1-\alpha)\sqrt{2\pi}\sigma}e^{-\frac{\delta_i^2}{2\sigma^2}}\,, \tag{2}$$

where δ_i is the ith-component of nondimensional distance between a grid point and a particle, σ is the standard deviation, and α is a factor which defines a function shape with a range of $0 \le \alpha < 1$. If a value of α equals to 0, $w_i(\delta_i)$ becomes the gaussian function, while if α takes a value close to unity, $w_i(\delta_i)$ takes asymptotically the shape of wavelet function. The function $w_i(\delta_i)$ takes on the form,

$$\int_{-\infty}^{\infty} w_i(\delta_i)d\delta_i = 1\,, \tag{3}$$

which is independent of a value of α. Figure 4 shows various shapes of $w_i(\delta_i)$ calculated using α with a range from 0.0 to 0.8. It is obvious that as α takes a value close to unity, a peak value of α at $\delta_i = 0$ becomes larger and $w_i(\delta_i)$ has negative loops. During a process of force coupling, the momentum is distributed to grid points in proportional to the integral of weighted function between grid points, which is illustrated in figure 5.

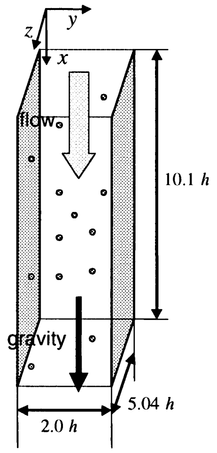

Figure 6. Schematic of computational domain for LES of turbulent channel flow.

OVERVIEW OF THE SIMULATION

LES for turbulent channel flow

The present simulations solved the following forms of the filtered continuity and Navier-Stokes equations:

$$\frac{\partial \bar{u}_{f_i}}{\partial x_i} = 0\,, \tag{4}$$

$$\frac{D\bar{u}_{f_i}}{Dt} = -\frac{1}{\rho_f}\frac{\partial \bar{p}}{\partial x_i} + \nu\frac{\partial^2 \bar{u}_{f_i}}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} - \frac{1}{\rho_f}\bar{f}_{D_i}\,, \tag{5}$$

where $\bar{u}_{f_i}$ and $\bar{p}$ are resolved, i.e., grid scale, velocity and pressure, and i = 1, 2, 3 denote streamwise (x), transverse (y) and spanwise (z) directions respectively. The last term is the extra term in the presence of particles calculated by the drag term in the equation of particle motion (equation (10)).

The right-hand side of equation (5) contains the SGS stresses which represent the effect of the residual velocity field on the resolved scales,

$$\tau_{ij} = \overline{u_i u_j} - \overline{u_i}\,\overline{u_j}\,, \tag{6}$$

and are modeled using the Smagorinsky model, a closure based on the eddy viscosity assumption, using a box filter,

$$\tau_{ij} = -2\nu_S \bar{S}_{ij}\,, \tag{7}$$

$$\nu_S = \left(c_S f_w \bar{\Delta}\right)^2 \sqrt{2\bar{S}_{ij}\bar{S}_{ij}}\,, \tag{8}$$

where the Smagorinsky constant, c_S, takes a value of 0.1 and a filter width, $\bar{\Delta}$, is equal to $\sqrt[3]{\Delta x \Delta y \Delta z}$. The wall function,

$$f_w = 1 - \exp\left(-\frac{y^+}{25}\right), \tag{9}$$

was employed for dumping the SGS stresses near the wall.

The time-stepping scheme employed the second-order Adams-Bashforth method using the SMAC formulation. Time-stepping errors are small

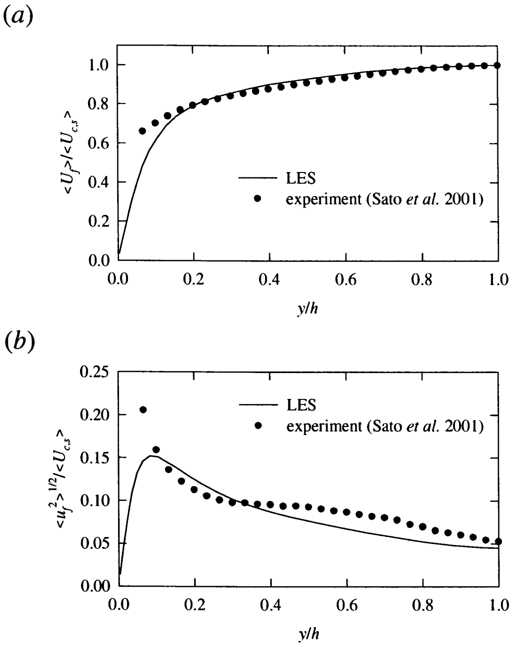

Figure 7. Profiles of (*a*) mean and (*b*) fluctuating streamwise velocities of fluid flow in the presence of particles.

as long as the Courant number is up to 0.15. The spatial-derivative terms were computed by the second-order central finite-difference method. The solution was obtained with 63x48x63 grid points, i.e., uniform grid points in both the streamwise and spanwise directions, while nonuniform in the transverse direction. Figure 6 depicts a schematic illustration of computational domain, in which the periodic boundary condition was applied to both the streamwise and spanwise directions, and the no-slip boundary condition was used at the wall. The readers should refer to the subsection of "Experimental facility" in the conference paper by Sato *et al.* (2001) for further information of fluid flow parameters of a channel flow. The pressure gradient was subtracted from the equation (5) in only the streamwise direction as a driving force of fluid flow, because due to the gravity force acting on particles the pressure gradient is reduced, therefore this operation kept the momentum flux at the wall as a constant. Figure 7 shows profiles of mean and fluctuating streamwise velocities of fluid flow in single phase. The present simulation shows a good agreement with experimental results by Sato *et al.* (2001) except the near-wall region.

Particle tracking method

The equation of particle motion used in the present study is,

$$\frac{d\tilde{u}_{p_i}}{dt} = \underbrace{-\frac{3}{4}\frac{C_D}{d_p}\frac{\rho_f}{\rho_p}\tilde{u}_{r_i}|\tilde{u}_r|}_{\text{drag}} + \underbrace{\left(1-\frac{\rho_f}{\rho_p}\right)g_i}_{\text{gravity}} \underbrace{-\frac{1}{2}\frac{\rho_f}{\rho_p}\frac{d\tilde{u}_{r_i}}{dt}}_{\text{added mass}}$$

$$\underbrace{+1.62\mu d_p^2\sqrt{\nu\left|\frac{\partial \tilde{u}_{r_i}}{\partial x_i}\right|}\,\frac{\partial \tilde{u}_{r_i}}{\partial x_i}\Big/\left|\frac{\partial \tilde{u}_{r_i}}{\partial x_i}\right|\tilde{u}_{r_i}}_{\text{Saffman lift force}}, \qquad (10)$$

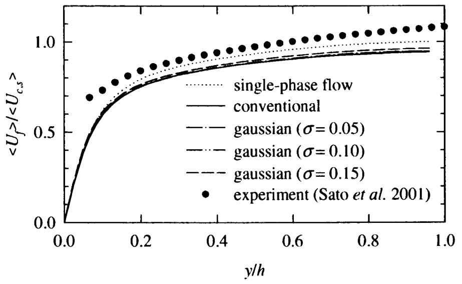

Figure 8. Mean streamwise velocity profiles of fluid flow in the presence of particles using the gaussian function.

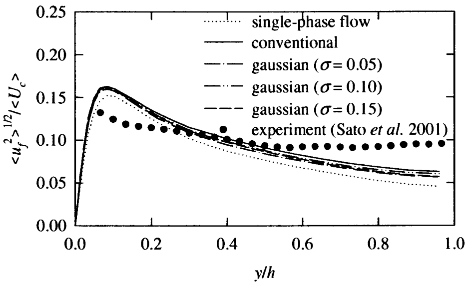

Figure 9. Streamwise velocity fluctuations profiles of fluid flow in the presence of particles using the gaussian function.

where d_p is particle diameter, C_D is the coefficient of drag and $\tilde{u}_{r_i} = \tilde{u}_{p_i} - \bar{u}_{f@p_i}$ is the relative velocity between particle and fluid along the particle path. Interpolation is required to obtain fluid information at a particle point. In the present study the fourth-order accurate Lagrange polynominals were used to interpolate velocities. The Saffman lift force was considered in only the streamwise direction.

Equation (10) was time advanced using the second-order Adams-Bashforth method. Particles were uniformly injected within the computational domain and the inter-particle collisions were neglected. The periodic boundary condition was applied to both the streamwise and spanwise directions, and the elastic collisions were considered at the wall. The readers should refer to table 1 in the conference paper by Sato *et al.* (2001) for further information of particle properties.

RESULTS AND DISCUSSION

A Cartesian coordinate system (x,y,z) at the center of the channel is used for all data presentation. In most of the plots in this section, these coordinates are normalized by the channel half-width, h. The velocity axis of figures presented in this section has been nondimensionalized values using a centerline mean velocity of single-phase flow, $<U_{c,s}>$. The grid sizes in the present simulation are $\Delta x = 0.16$, $\Delta y = 0.008$–0.08 and $\Delta z = 0.08$ and the particle diameter corresponds to $d_p = 0.025$. Particle volumetric fraction takes a value of $\phi_{\mathrm{vol}} = 3.3\times10^{-4}$.

Force coupling using gaussian function

The force coupling method investigated in the present study can be classified into two categories. The method using the gaussian function, i.e., $\alpha = 0$,

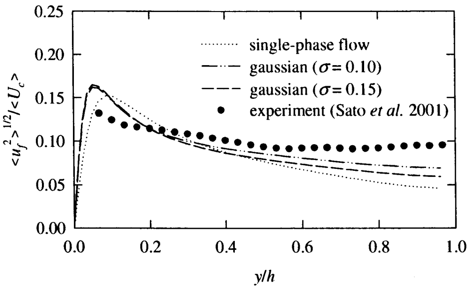

Figure 10. Streamwise velocity fluctuations profiles of fluid flow in the presence of particles using the wavelet-like function.

is examined in this subsection with values of $\sigma =$ 0.05, 0.10 and 0.15.

Figure 8 shows mean streamwise velocity profiles of fluid flow in the presence of particles, in which numerical results with the present method are compared with the conventional point-force-approximation method. It is observed that both of the methods decelerated the mean flow, which is the opposite trend of the experiments by Sato *et al.* (2001).

Streamwise fluid velocity fluctuations are displayed in figure 9. The conventional method augments turbulence rather than that using the gaussian function at the channel centerline. When one concentrates on only the results using the weighted function, significant augmentation was obtained using a value of $\sigma = 0.10$.

Force coupling using wavelet-like function

Another method is investigated using the wavelet-like function, i.e., using a value of $\alpha = 0.8$. In the previous subsection all the results considered subtracting the pressure gradient from the equation (5), which induced the mean-velocity deceleration, therefore this subsection neglected this operation.

Figure 10 depicts fluid velocity fluctuations in the streamwise direction, which exhibited an increase in turbulence intensity at the channel centerline with a value of $\sigma = 0.10$. The numerical result shows a similar trend of that of the experiments by Sato *et al.* (2001), which means that the momentum-distribution method has a significant influence on modification to turbulence. Moreover a distribution scale induced by a value of $\sigma = 0.10$ corresponds to one-third of the energy-containing eddy scale, which means that the force coupling is sensitive to a selection of scale, i.e., the scale-dependent structure of turbulence modification by particles is ensured by the present numerical simulation.

Effect of directional scale effect on turbulence modification by particles

Sato *et al.* (2001) found the directional interactions between particles and turbulence in their experiments, therefore this subsection examines whether the present technique has an ability to realize the scale-dependent structure or not. Figure 11 shows profiles of streamwise turbulence intensity taking into account the directional scale dependency on the momentum distribution. Turbulence intensity is augmented when the distribution scale in only the streamwise direction was expanded to a value of $\sigma = 0.15$ while others take $\sigma = 0.10$, i.e., circles in figure, more than when those in other directions are expanded. On the other hand a slight difference is observed in the transverse and spanwise directions as shown in figures 12 and 13 respectively. It can be summarized that the directional effect on modification to turbulence is predicted by the present force-coupling method.

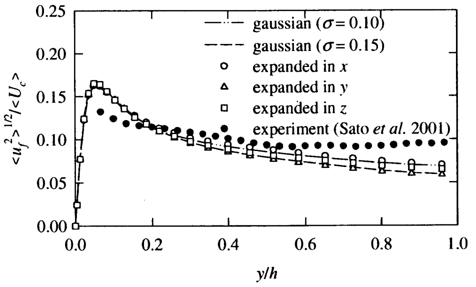

Figure 11. Streamwise velocity fluctuations profiles of fluid flow in the presence of particles using the wavelet-like function considering scale expansion.

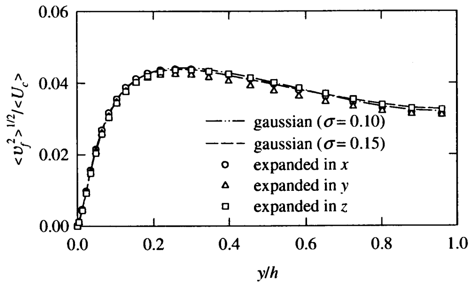

Figure 12. Transverse velocity fluctuations profiles of fluid flow in the presence of particles using the wavelet-like function considering scale expansion.

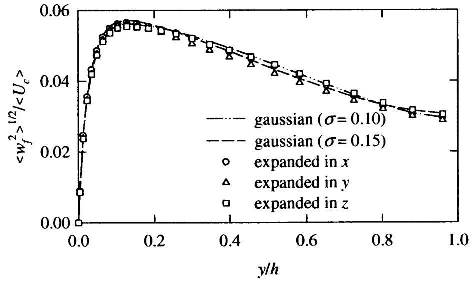

Figure 13. Spanwise velocity fluctuations profiles of fluid flow in the presence of particles using the wavelet-like function considering scale expansion.

For further insight into the directional scale-dependent structure is provided by the flow power spectra in the streamwise and transverse directions as shown in figure 14. An increase in streamwise power spectra in the low-wave-number region was obtained using the expanded distribution scale in only the streamwise direction. While a slight directional effect can be seen in the transverse power spectra. It is concluded that the directional scale dependency on turbulence augmentation by particles is realized when the momentum-distribution scale corresponds to half of the energy-containing eddy scale. The experimental efforts by Sato *et al.* (2001) suggested the same idea, which is consistent to conclusions in the

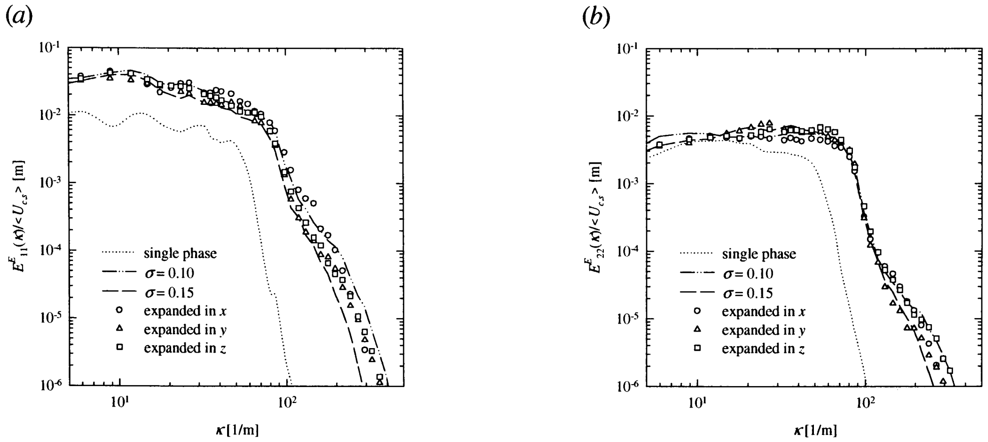

Figure 14. Profiles of velocity power spectra of at centerline of the channel in the presence of particles in the (*a*) streamwise and (*b*) transverse directions using the wavelet-like function considering scale expansion.

present study.

CONCLUSIONS

The force coupling method in large eddy simulation has been developed using the weighted function in order to represent turbulence augmentation by particles. The important conclusions obtained from this work are summarized below.

(1) The momentum distribution to grid points in a wider region yielded enhancement of turbulence energy in the presence of particles, in order to overcome the significant problem using the point-force approximation.

(2) The present LES suggested that half of the energy-containing scale is a key parameter for the scale-dependent structure of modification to turbulence.

(3) The directional effect of momentum distribution on turbulence augmentation by particles was ensured by the present force-coupling technique, which was consistent to the experiments by Sato *et al.* (2001).

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