EQUILIBRIUM PUFFS DO INDEED EXIST: ACCURATE DESCRIPTION BY MEANS OF LONG-WAVE NAVIER-STOKES SOLUTIONS

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ABSTRACT

The possibility of intermittent turbulent shear flows description by means of nonstationary long-wave 3D Navier-Stokes solutions is shown in principle. The classical problem of viscous incompressible fluid flows in a circular pipe at transitional Reynolds numbers $1800 \leq Re \leq 4000$ is taken as a model one. By means of direct numerical simulation we have obtained statistically stationary Navier-Stokes solutions which describe turbulent (at $2500 \leq Re \leq 4000$), intermittent (at $Re = 2200, 2350$) and laminar ($Re \leq 2000$) flow regimes. Numerical solutions at $Re = 2200, 2350$ describe equilibrium self-sustained flow regimes in which turbulent structures surrounded by almost laminar flow propagate downstream while preserving their length. Thus, theoretical confirmation of the existence of particular transitional flow regimes – so-called equilibrium puffs – is obtained.

INTRODUCTION

We consider the classical problem of laminar-turbulent transition in a circular pipe. It is well established that far from entry and exit sections of the pipe two kinds of equilibrium flow states may be observed: stationary (Poiseuille) and nonstationary (turbulent) ones, both being statistically homogeneous along the pipe axis. Such flow regimes are well described by stationary and statistically stationary solutions of 3D incompressible Navier-Stokes equations (Priymak, 1991; Eggels et al., 1994). At the same time there are experimental evidences (Wygnanski et al., 1973 and 1975) that at Reynolds numbers around $Re = U_b 2R/\nu = 2200$, where U_b, R and ν are the bulk velocity, pipe radius and kinematic viscosity, one more equilibrium self-sustained flow regime exists in which turbulent structures (so-called puffs) surrounded by almost laminar flow propagate downstream while preserving their length. Nevertheless, until now it was far from being accepted that equilibrium puffs really exist. The reasons come mainly from the facts that laboratory pipes are limited in length and that the leading edge of the puff does not have a clearly defined interface. Fortunately an equilibrium puff is a good candidate for direct numerical simulation with periodic boundary conditions for velocity along the pipe axis provided that the streamwise period X is large enough. Two attempts of such simulation were carried by Leonard and Reynolds (1985) and by Shan et al. (1999) at $Re = 2200$ with streamwise periods $X = 36R$ and $32\pi R$ and specially chosen initial conditions consisting of some localized structures in the computational domain. Although the results of these calculations are very suggestive of a turbulent puff rather a short total integration time $T \approx 120R/U_b$ in (Leonard and Reynolds, 1985) didn't allowed to answer the question whether an equilibrium puff indeed exists. As to the work (Shan et al., 1999), its authors report different values $U_{LE} = 1.56U_b$ and $U_{TE} = 0.73U_b$ for the convective speeds of the leading and trailing edges of a puff. The latter means that either the computed puff turned out to be not in equilibrium or the simulation time ($T \approx 60R/U_b$) is also insufficiently long.

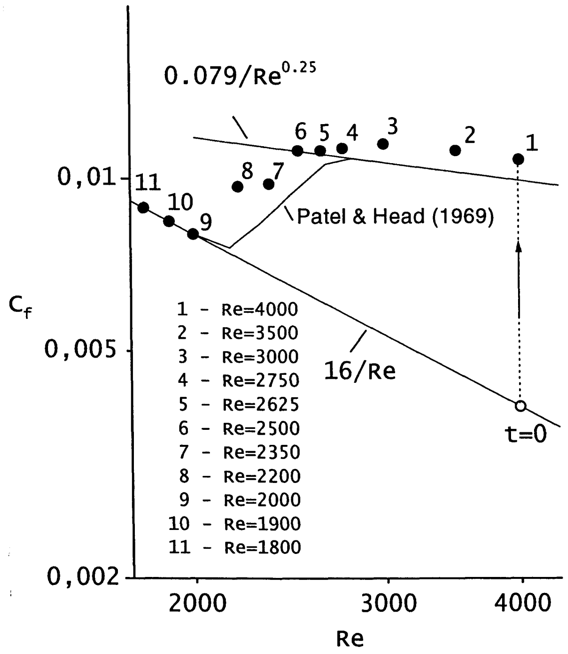

Figure 1: Skin-friction coefficient versus Reynolds number. Solid circles from 1 through 6 correspond to turbulent flow states; flow regimes 7 and 8 are intermittent and contain equilibrium puffs; 9–11 correspond to laminar Poiseuille flows.

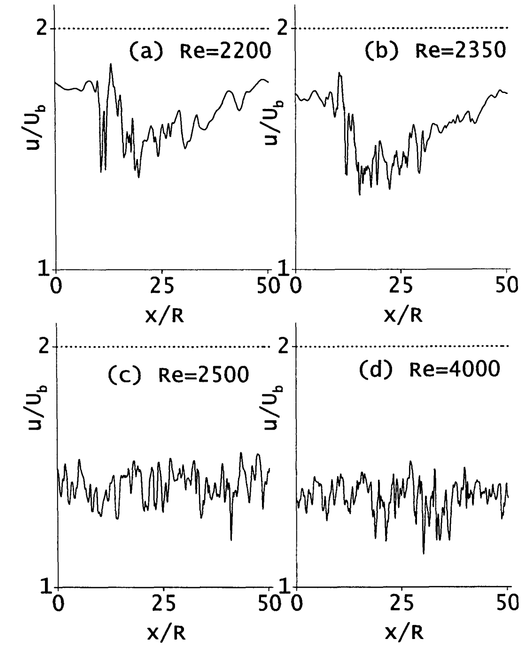

Figure 2: Typical axial velocity distributions along the pipe center.

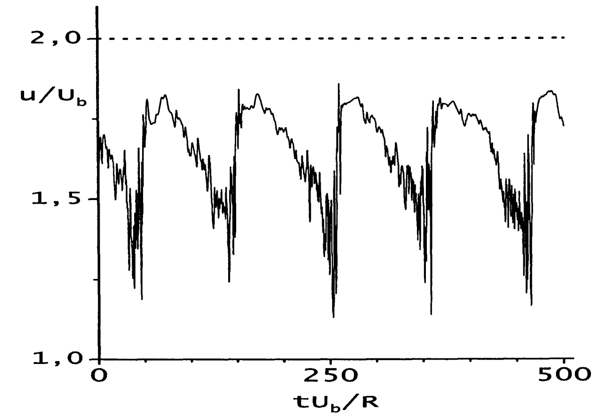

Figure 3: Typical time trace of the streamwise velocity at the pipe center ($Re = 2200$).

LONG-WAVE NAVIER-STOKES SOLUTIONS

To give clear answer to the question whether equilibrium puff exists or not and what is it's space-time structure we have performed a series of long-term computer runs at a range of transitional Reynolds numbers $1800 \leq Re \leq 4000$. We used accurate and efficient numerical method (Priymak and Miyazaki, 1998) which made it possible to carry out calculations for very long time intervals of about $(2000-4000)R/U_b$ at every value of Re. Time consuming computations were carried out on the newest Hitachi SR8000 parallel supercomputer at the University of Tokyo. Such extremely long-term calculations allow one to be fully confident in equilibrium self-sustained character of computed flows. In addition, our computations show that the stabilization of flow characteristics (after changing, e.g., the value of Re)is rather a slow process at transitional Reynolds numbers taking of about $(500-1000)R/U_b$ which is from 5 to 10 times larger than total integration times in (Leonard and Reynolds, 1985) and (Shan et al., 1999). All our computer runs were performed in the so-called long-wave limit: we looked for numerical Navier-Stokes solutions with rather a large streamwise period $X = 16\pi R$ for the velocity. As it was shown in (Priymak and Miyazaki, 1994) $16\pi R$ is close to the minimal admissible value of the streamwise period, i.e. to the value which provides an adequate description of long-wave motions typical to puff-like structures. It should be stressed also that we didn't utilized any initial conditions *a priori* close to the desired puff-like structure.

Computations were carried out with $(Q+1)\times(2N+1)\times(2M+1) = 33\times 41\times 321$

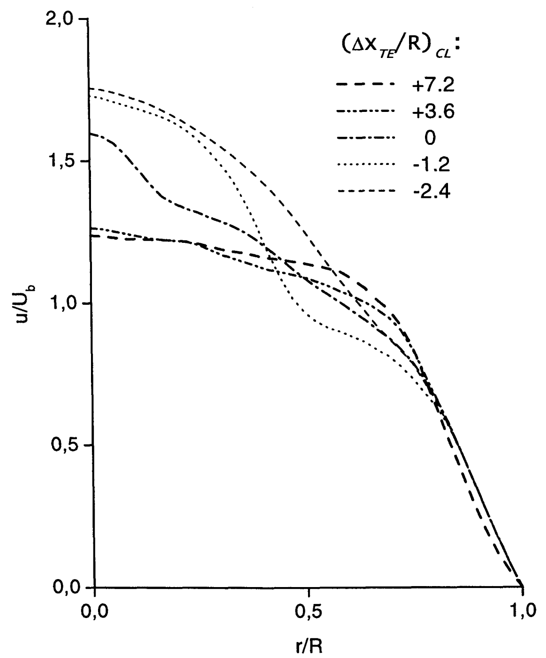

Figure 4: Experimental data from Wygnanski et al. (1975): radial distributions of velocity at different distances $(\Delta x_{TE}/R)_{CL}$ from the trailing interface of the puff normalized by the bulk velocity

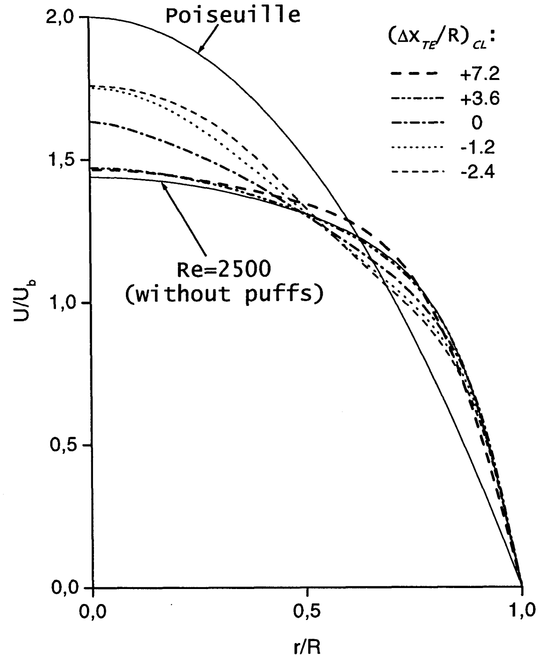

Figure 5: Radial distributions of velocity at different distances from the trailing interface of the puff normalized by the bulk velocity

basis functions in r, φ, x – radial, azimuth and streamwise coordinates respectively. For the computer run at $Re = 2200$, for example, the value of $Re_\tau = u_\tau 2R/\nu$, u_τ being the wall-shear velocity, is about 153; streamwise period $X^+ = Xu_\tau/\nu \approx 3846$; the minimal resolved wavelengths in φ (at $r = R$, along the pipe surface) and x are: $\lambda_\varphi^+ = \pi 2Ru_\tau/N\nu \approx 24$, $\lambda_x^+ = Xu_\tau/M\nu \approx 24$; maximum and minimum grid spacings in r are $\Delta r_{max}^+ = \Delta r_{max}u_\tau/\nu \approx 3.75$ and $\Delta r_{min}^+ = \Delta r_{min}u_\tau/\nu \approx 0.09$; the total integration time $T \approx 2405R/U_b \approx 167R/u_\tau$; skin-friction coefficient $C_f = - < \nabla p >_{r\varphi xt} D/2\rho U_b{}^2 \approx 9.68 \times 10^{-3}$ ($D = 2R$, $< \nabla p >_{r\varphi xt}$ – space- and time-averaged pressure gradient, ρ – the constant density of the fluid).

Numerical simulations at eleven values of Reynolds number were performed (see figure 1). As an initial condition for the first computer run at $Re = 4000$ we took the Poiseuille flow disturbed by the superposition of axisymmetric and nonaxisymmetric least stable vector eigenfunctions corresponding to certain streamwise and azimuth Fourier modes. An arrow (figure 1) connects initial and final states of the transition process: point " ∘ " ($t = 0$) corresponds to the disturbed laminar flow, and point "• 1" to the established turbulent flow regime at $Re = 4000$. Turbulent, intermittent and laminar flow states at lower Reynolds numbers (solid circles from 2 through 11) were computed successively choosing the final statistically stationary stage of computer run i as an initial condition for run $i+1$, $i = 1, 2, \ldots$.

EQUILIBRIUM PUFFS AT $RE = 2200$ AND $RE = 2350$

The calculated flow regimes 1–6 may be considered to be truly turbulent: their skin-friction coefficients C_f agree well with Blasius friction law $C_f = 0.079Re^{-1/4}$ and these regimes do not contain laminar inclusions. The latter is illustrated by figure 2 where we show typical axial velocity distributions along the pipe center for four Reynolds numbers: $Re = 2200$(a), 2350(b), 2500(c) and 4000(d). Note that for $Re = 4000$ and $Re = 2500$ there is no any deficit in streamwise velocity along the pipe axis. On the contrary, at $Re = 2200$ and 2350 (figures 2a, 2b) the deficit in streamwise velocity – a feature of all turbulent puffs – is clearly visible and resembles experimental one. Dotted lines correspond here to laminar Poiseuille flows. In addition in figure 3 we

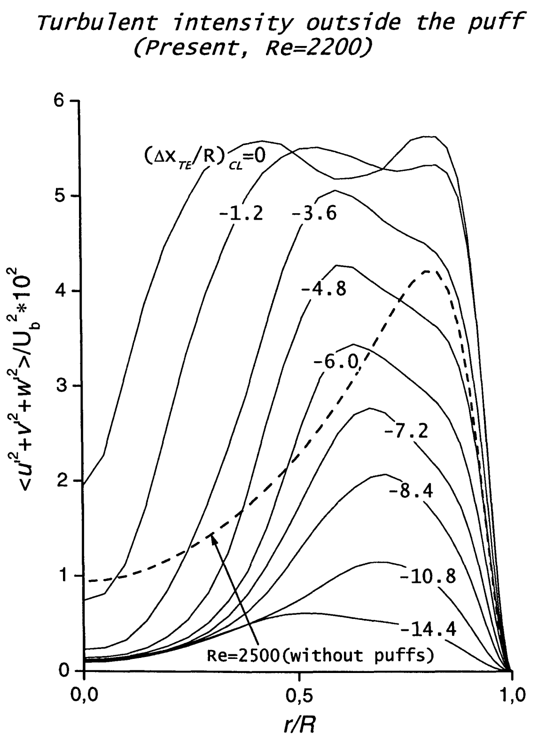

Figure 6: The total turbulent intensity profiles at different upstream distances from the trailing interface.

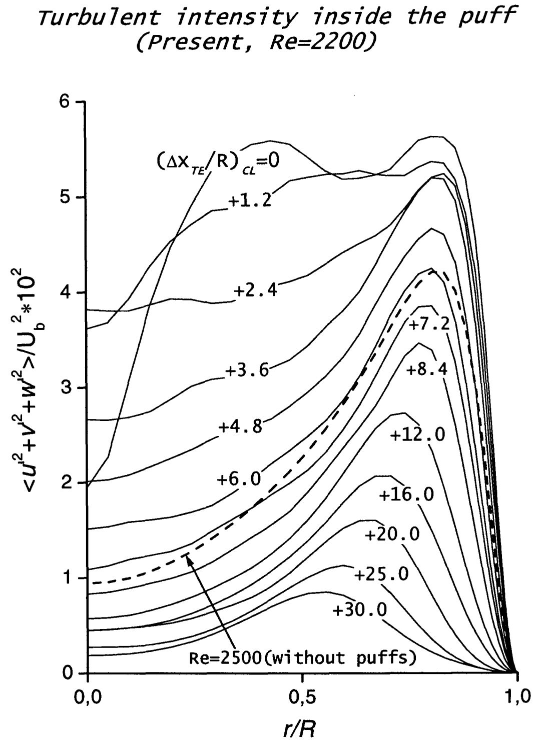

Figure 7: The total turbulent intensity profiles at different downstream distances from the trailing interface.

show the fragment of a typical time trace of the streamwise velocity at the pipe center at $Re = 2200$. The total integration time was much longer: after reaching the statistically steady state computations were carried out further in time for about $2000R/U_b$. This extremely long computation allowed us to make a conclusion that the computed localized puff-like structure is an equilibrium self-sustained one.

TURBULENCE STATISTICS INSIDE AND OUTSIDE THE PUFF

The radial distributions of velocity at different distances from the trailing interface of the puff at $Re = 2200$ are shown in figure 5. Here the computed velocity profiles are φ (azimuth coordinate) and time-averages; $(\Delta x_{TE}/R)_{CL}$ is the distance from the trailing interface (TE) measured along the centre-line (CL) of the pipe (in downstream direction, i.e. inside the puff, $(\Delta x_{TE}/R)_{CL} > 0$). In figure 5 we also show the Poiseuille parabolic profile and the velocity profile corresponding to $Re = 2500$ (flow regime 6, figure 1) at which there are no puffs in the computed fully turbulent flow field. The computed velocity profiles can be compared with those of experimental data from Wignanski et al. (1975) shown in figure 4. As it is clearly seen from figure 5 the velocity profiles computed in the core of turbulent region ($(\Delta x_{TE}/R)_{CL} = 3.6$ and 7.2) are very close to the velocity distribution corresponding to $Re = 2500$. The same is correct for the total turbulent intensity radial distributions at $(\Delta x_{TE}/R)_{CL} \approx 6 - 7$ (see figure 7). Thus, in the heart of the puff flow resembles the fully turbulent pipe flow at low Re.

On the other hand for the flow region outside the puff ($(\Delta x_{TE}/R)_{CL} = -1.2$ and -2.4; $0 \leq r/R < 0.5$) the radial velocity distributions are more typical of laminar flows. The overall shape of the computed velocity profiles and the mean values of velocity at the pipe axis are in good agreement with that of Wignanski et al. (1975) (see figures 4 and 5). At the same time the computed flow outside the turbulent region is not a truly laminar one (compare, e.g., Poiseuille profile and velocity distributions at $(\Delta x_{TE}/R)_{CL} = -1.2$ and -2.4; figure 5). The latter is in agreement with experimental evidences (Patel and Head, 1969; Wignanski et al., 1975) and is verified by the fact that the computed turbulent intensities outside the core of turbulent zone are far from being negligible (figure 6).

It is important to note that the slopes of the computed velocity profiles near the wall (see figure 5) are almost the same inside and outside the turbulent region and are very close to the corresponding value at $Re = 2500$ when we observe the fully turbulent pipe flow without puffs. This is not the case for experimental profiles obtained by Wignanski et al. (1975) (figure 4) where the slope of velocity distributions at $r = R$ (and hence the skin friction at the wall) is "hardly affected by the presence or absence of turbulence". Besides, Wignanski et al. (1975) assume that their measurements are not precise enough near the wall.

CONCLUSIONS

The major conclusions and results of the paper are the following: (a) equilibrium puffs do indeed exist at $2200 \leq Re \leq 2350$; (b)we can describe them by means of numerical long-wave Navier-Stokes solutions; (c) as far as we know this is the first direct simulation at the whole range of transitional Reynolds numbers including equilibrium self-sustained laminar ($Re = 1800, 1900, 2000$), intermittent ($Re = 2200, 2350$) and established turbulent ($2500 \leq Re \leq 4000$) flows in a circular pipe.

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