

ASSESSMENT OF THE NEWLY DEVELOPED SUBGRID-SCALE EDDY VISCOSITY FOR THE DYNAMIC PROCEDURE USING FDM ON CHANNEL FLOW AND RAYLEIGH-BÉNARD CONVECTION

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ABSTRACT

The newly developed Subgrid-scale(SGS) eddy viscosity representation (Tsubokura, 2001) for the dynamic SGS model (Germano et al., 1991) using finite difference method (FDM) was further studied and the related SGS flux model was proposed to apply the method to a flow with passive scalar transport and a buoyancy-driven flow. Our main concern here is to see the dependence of the proposed models' results on the discretized filtering operation. The proposed models were tested in the plane channel flow with passive scalar transport and the Rayleigh-Bénard convection. Proposed models were found to predict better statistics than the dynamic Smagorinsky's and contrarily to the Smagorinsky's, showed insensitivity to the explicit *test* filtering operation in both flows

INTRODUCTION

The validity of the dynamic subgrid-scale (SGS) eddy viscosity model proposed by Germano *et al.*(1991) in Large Eddy Simulation (LES) has been established in various turbulent flows. But when we apply finite difference method (FDM) to this procedure, serious problems arise. One is the insufficient optimization of the model coefficient. The typical case of this problem is the overestimation of the streamwise mean velocity in simple turbulent plane channel flow (e.g., Cabot and Moin, 1993). The other problem is that additional parameter for explicit *test* filtering operation must be determined *a priori*. The previous report says that dependence of the results on this explicit filtering is not small(Tsubokura *et al.*, 1997). Recently Tsubokura (2001) developed a new eddy viscosity representation which is suitable for the dynamic procedure using FDM. He developed the model in the sense that the consistency of the numerical error due to FDM between L_{ij} and M_{ij} (see eqs. (14) and (16)) should be maintained. The proposed model was tested in the plane channel flow and found to be less sensitive to the discretized *test* filtering parameter as well as predicting better statistics than Smagorinsky based dynamic procedure.

The objective of this study is to investigate further the applicability of the proposed SGS stress models to the flow with passive scalar or the buoyancy-

driven flow. Accordingly new SGS flux models are proposed based on the analogy of the previous SGS stress models. We will also mention the anisotropic effect of the SGS flux models. A comparison of the proposed SGS stress and flux models with the dynamic Smagorinsky's model is made on the plane channel flow with passive scalar transport and the Rayleigh-Bénard convection. We especially focus on the proposed models' sensitivity to the discretized filtering operation.

NUMERICAL SCHEME

Governing equations

In this study, incompressible flows with passive scalar and thermal convection are considered. The governing equations of LES for such flow fields consist of the continuity equation, the momentum equations with or without buoyancy term and the transport equation of heat or scalar. The normalized governing equations are given by following equations.

$$\frac{\partial u_i^g}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial u_i^g}{\partial t} + \frac{\partial u_i^g u_j^g}{\partial x_j} = -\frac{\partial p^g}{\partial x_i} + \frac{1}{\text{Re}} \frac{\partial^2 u_i^g}{\partial x_j \partial x_j} + \text{Ri} \theta^g - \frac{\partial}{\partial x_j} \tau_{ij} \quad (2)$$

$$\frac{\partial \theta^g}{\partial t} + \frac{\partial \theta^g u_j^g}{\partial x_j} = \frac{1}{\text{Re Pr}} \frac{\partial^2 \theta^g}{\partial x_j \partial x_j} + \Theta_0 - \frac{\partial}{\partial x_j} q_j \quad (3)$$

Here *superscript* of "g" represents *grid* filtering operation. In eq. (3), Θ_0 is the volumetric source term of scalar or heat. The last terms of eqs. (2) and (3) are the SGS stress and flux which must be modeled,

$$\tau_{ij} = (u_i u_j)^g - u_i^g u_j^g \quad (4)$$

$$q_j = (\theta u_j)^g - \theta^g u_j^g \quad (5)$$

Discretization of the governing equations

Governing equations given by eqs. (1), (2) and (3) are discretized based on FDM. The newly developed 4th order accurate finite difference schemes proposed by Morinishi *et al.*(1998) is adopted in which both momentum and kinetic energy are conserved at the 4th order accuracy in the discretized sense in a staggered grid system. A semi-implicit time marching algorithm is adopted in which normal wall direction of the diffusion terms

are only implicitly solved with the Crank-Nicolson scheme while the third order Runge-Kutta scheme is used for the other terms.

Discretization of the filtering operation

The spatial filtering operation can be expressed as

$$f^F(x, t) = \int_{-\infty}^{+\infty} G(x - x'; \Delta^F) f(x, t) dx' \quad (6)$$

in which G is a filtering function to eliminate the small-scale components of turbulence with a characteristic length Δ^F . When we apply the Taylor expansion to the filtered value, we can obtain the differential form of the filtering operation at the fourth order of accuracy (Leonard, 1974) which is expressed as

$$f^F = f + \frac{(\Delta_x^F)^2}{24} \frac{\partial^2 f}{\partial x \partial x} + O\left\{(\Delta_x^F)^4\right\}. \quad (7)$$

Filtering is conducted only for one direction for simplicity. We should note that at the 4th order of magnitude for the filter width, both Gaussian and top-hat filtering function give the same formulae. When we apply the second order central FDM to the second term of the right-side of eq. (7), we can obtain the following discretized filtering operation,

$$f_i^F = f_i + \frac{(\Delta_x^F)^2}{24} \frac{f_{i-1} - 2f_i + f_{i+1}}{h_x^2}. \quad (8)$$

Here only for this equation, *subscript* denotes the stencil of a grid for FDM and h_x is the width of the grid. *It should be noted here that the ratio of the characteristic filter width Δ_x^F and the grid width h_x is the parameter of the discretized filtering operation which must be determined prior to the numerical simulation.*

SGS MODELING

Dynamic procedure

In the dynamic procedure of Germano *et al.* (1991), *test* filtering operation is adopted which is indicated by *superscript* of "T" in this study. Accordingly subtest-scale (STS) stress and flux also must be modeled which are given as

$$T_{ij} = (u_i u_j)^{gT} - u_i^{gT} u_j^{gT} \quad (9)$$

$$Q_j = (\theta u_j)^{gT} - \theta^{gT} u_j^{gT}. \quad (10)$$

In the original procedure, Germano *et al.* proposed to use Smagorinsky's representation for the SGS and STS stress modeling.

$$\tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{kk} = -2C(\Delta^g)^2 |S^g| S_{ij}^g \quad (11)$$

$$T_{ij} - \frac{1}{3} \delta_{ij} T_{kk} = -2C(\Delta^{gT})^2 |S^{gT}| S_{ij}^{gT} \quad (12)$$

where

$$S_{ij}^g = \frac{1}{2} \left(\frac{\partial u_i^g}{\partial x_j} + \frac{\partial u_j^g}{\partial x_i} \right), \quad |S^g| = \sqrt{2S_{ij}^g S_{ij}^g}. \quad (13)$$

Throughout the history of SGS modeling, the determination of the model coefficient C in the eddy viscosity representation has been one of the main problems. Germano *et al.* found following identity between the SGS and STS stress,

$$T_{ij} - \tau_{ij}^T = (u_i^g u_j^g)^T - u_i^{gT} u_j^{gT} = L_{ij}, \quad (14)$$

in which right-hand side consists of resolvable value even though left-hand side is made up of the unknown SGS and STS stresses. When we substitute modeled SGS and STS stresses of eqs. (11) and (12) into eq. (14), the error between L_{ij} and $T_{ij} - \tau_{ij}^T$ is given as,

$$e_{ij} = L_{ij} + 2CM_{ij} \quad (15)$$

$$M_{ij} = (\Delta^{gT})^2 |S^{gT}| S_{ij}^{gT} - \left\{ (\Delta^g)^2 |S^g| S_{ij}^g \right\}^T \quad (16)$$

in which model parameter C is supposed to be constant during the *test* filtering operation. Lilly (1992) modified the original procedure and obtained the model parameter using least square method to minimize the above error tensor of eq. (15) as

$$C = -\frac{1}{2} \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle}. \quad (17)$$

In eq. (17) an *angle bracket* means an averaging operation to avoid negative coefficient which causes numerical instability. In this study, averaging is conducted for the statistically homogeneous direction, e.g., streamwise and spanwise directions in case of the channel flow.

Following the same manner as the SGS and STS stress, model parameter included in the SGS flux also can be obtained using the dynamic procedure (Cabot and Moin, 1993). When we adopt an analogous eddy diffusivity models with Smagorinsky's model to the SGS and STS flux, these two flux can be expressed as

$$q_j = -C_\theta (\Delta^g)^2 |S^g| \frac{\partial \theta^g}{\partial x_j} \quad (18)$$

$$Q_j = -C_\theta (\Delta^{gT})^2 |S^{gT}| \frac{\partial \theta^{gT}}{\partial x_j}. \quad (19)$$

For the SGS and STS flux, we can obtain the relation

$$Q_j - q_j^T = (\theta^g u_j^g)^T - \theta^{gT} u_j^{gT} = F_j. \quad (20)$$

Conclusively the model parameter can be obtained by

$$C_\theta = -\frac{\langle F_j H_j \rangle}{\langle H_j H_j \rangle} \quad (21)$$

$$H_j = (\Delta^{gT})^2 |S^{gT}| \frac{\partial \theta^{gT}}{\partial x_j} - \left\{ (\Delta^g)^2 |S^g| \frac{\partial \theta^g}{\partial x_j} \right\}^T. \quad (22)$$

Problems of the original dynamic procedure

Two problems arise from the original dynamic procedure.

Firstly the ratio of the STS and SGS length scale Δ^{gT} / Δ^g must be determined prior to the numerical

simulation. Previous report showed strong dependence of the predicted statistics on the value (Cabot and Moin, 1993). In this study, we adopt the value $\Delta^{gT} / \Delta^g = 2^{2/3}$ considering the following three facts: $\Delta_a^{gT} / \Delta_a^g = 2$ is proposed for each filtering direction in Spectral method (Germano et al., 1991), no filtering is considered for normal wall direction, and the length scale in eqs. (11) and (12) is given by $\Delta^g = \sqrt{\Delta_x^g \Delta_y^g \Delta_z^g}$ and $\Delta^{gT} = \sqrt{\Delta_x^{gT} \Delta_y^{gT} \Delta_z^{gT}}$ (no filtering for y direction).

Secondly Tsubokura et al. (1997) pointed out that dynamic Smagorinsky's model showed strong dependence on the filtering parameter Δ^T / h required for the explicit test filtering operation discretized by FDM (see eq. (8)). That is to say, the original procedure using FDM has two important parameters of Δ^{gT} / Δ^g and Δ^T / h which sensitively affect the predicted statistics. It is not easy to optimize these two values at various flows. In fact, it is well known that when we apply this procedure to the simple turbulent channel flow using lower order FDM, excessively large streamwise mean velocity and turbulence intensity are predicted even though the same model parameter as the spectral method is adopted for Δ^{gT} / Δ^g (Cabot and Moin, 1993, Tsubokura et al., 1997).

SGS stress modeling

Tsubokura (2001) proposed to use following isotropic eddy viscosity representation instead of Smagorinsky's model for the dynamic procedure using FDM in the sense that numerical error between L_{ij} and M_{ij} is more consistent than that of Smagorinsky's,

$$\tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{kk} = -2C \left\{ (u_k^g u_k^g)^g - u_k^{ggs} u_k^{ggs} \right\} S_{ij}^g / |S^g| \quad (23)$$

$$T_{ij} - \frac{1}{3} \delta_{ij} T_{kk} = -2C \left\{ (u_k^g u_k^g)^{gT} - u_k^{ggsT} u_k^{ggsT} \right\} S_{ij}^{gT} / |S^{gT}| \quad (24)$$

The notable features of these representation are as follows.

Firstly explicit SGS or STS length scale of Δ^g or Δ^{gT} included in the Smagorinsky's is not contained in these models. Accordingly we do not have to determine the filter ratio of Δ^{gT} / Δ^g prior to the simulation.

Secondly strain rate tensor is divided by its magnitude, the effect of numerical error caused by the strain rate tensor on the representation of M_{ij} may be less sensitive than that of Smagorinsky's. That is to say, numerical error of M_{ij} is mainly determined by the representation of the velocity scale of eddy viscosity. Considering this opinion, eqs. (23) and (24) can be finally obtained (for the detail, see Tsubokura, 2001).

According to the numerical tests of simple channel flows at the Reynolds number of up to 590, these representation was found to eliminate the overestimation of mean velocity profiles at the log law region observed in the original dynamic Smagorinsky's model.

The most notable feature of the proposed model is its insensitivity to the test filtering parameter, Δ^T / h . In other words, even though the representation of eqs. (23) and (24) includes both explicit grid and test filtering operations, results are only sensitive to the grid filtering parameter of Δ^g / h .

SGS flux modeling

Analogous eddy diffusivity models to the proposed eddy viscosity models given by eqs. (23) and (24) can be easily obtained as

$$q_j = -C_\theta \left\{ (u_k^g u_k^g)^g - u_k^{ggs} u_k^{ggs} \right\} / |S^g| \frac{\partial \theta^g}{\partial x_j} \quad (25)$$

$$Q_j = -C_\theta \left\{ (u_k^g u_k^g)^{gT} - u_k^{ggsT} u_k^{ggsT} \right\} / |S^{gT}| \frac{\partial \theta^{gT}}{\partial x_j} \quad (26)$$

These are the typical gradient-diffusion models in which SGS or STS flux is supposed to be proportional to the gradient of the scalar. As the first step to predict an anisotropic effect of SGS flux, following expressions are also possible,

$$q_j = -C_\theta / |S^g| \left[\left\{ (u_k^g u_k^g)^g - u_k^{ggs} u_k^{ggs} \right\} \frac{\partial \theta^g}{\partial x_j} + \left\{ (u_i^g \theta^g)^g - u_i^{ggs} \theta^{ggs} \right\} \frac{\partial u_j^g}{\partial x_i} \right] \quad (27)$$

$$Q_j = -C_\theta / |S^{gT}| \left[\left\{ (u_k^g u_k^g)^{gT} - u_k^{ggsT} u_k^{ggsT} \right\} \frac{\partial \theta^{gT}}{\partial x_j} + \left\{ (u_i^g \theta^g)^{gT} - u_i^{ggsT} \theta^{ggsT} \right\} \frac{\partial u_j^{gT}}{\partial x_i} \right] \quad (28)$$

Eq. (27) can be obtained by following the method proposed by Yoshizawa et al. (1996). They derived the SGS stress model as the product of the production term of its transport equation and turbulent time scale under the assumption that in SGS field, production of the stress is dominant among the others. The production term of the SGS flux transport equation is given by

$$P_j = -\tau_{ji} \frac{\partial \theta^{gT}}{\partial x_i} - q_i \frac{\partial u_j^{gT}}{\partial x_i} \quad (29)$$

Here when we consider the isotropic representation for the SGS stress appearing in the first term of the right-side of eq. (29) to obtain the gradient-diffusion term, anisotropic SGS flux model can be obtained by just multiplying the time scale T to the modified production term,

$$q_j = -T \left\{ \frac{\tau_{kki} \partial \theta^{gT}}{3 \partial x_j} + q_i \frac{\partial u_j^{gT}}{\partial x_i} \right\} \quad (30)$$

When the time scale, SGS stress and SGS flux included in the right-side of eq. (30) are modeled considering the consistency of the numerical error between F_j and H_j in eq. (21), eqs. (27) and (28) can be derived.

RESULTS

The proposed SGS flux models given by eqs. (25) and (26) or eqs. (27) and (28) is tested in actual numerical simulations. For the SGS stress models, we adopt the previously proposed representations given by eqs. (23) and (24). It should be noted here that these models contain both explicit *grid* and *test* filtering operations. But as is mentioned before, the remarkable feature of the SGS stress models is that they are less sensitive to the *test* filtering parameter. Therefore the major objective of the actual numerical simulations here is to investigate how such insensitivity of the SGS stress models is inherited to the SGS flux models.

Discretized filtering parameter

To study the sensitivity of the SGS stress and flux models, three values for both *grid* and *test* filtering parameters are considered which are summarized in Table 1. The parameter is denoted by capital letter for the *test* filtering while *grid* filtering is given by small letter. For example the discretized filtering operation of (Ab) indicates $(\Delta^T/h)^2 = 4$ and $(\Delta^s/h)^2 = 4/3$.

Channel flow (passive scalar case)

The simple turbulent plane channel flow with passive scalar transport is solved here to see the fundamental property of the proposed SGS scalar models. A flow itself is driven by constant pressure gradient for streamwise direction. Periodic boundary condition is adopted for streamwise and spanwise directions while no-slip condition is considered on the wall. Reynolds number normalized by friction velocity (u_τ) and channel half width (δ) is set to 180 in this case. Computational domain size is set to $4\pi\delta$ and $2\pi\delta$ for streamwise and spanwise direction. Total grid number is 32, 64, 32 for streamwise(x), normal-wall(y), and spanwise(z) direction respectively. The molecular Prandtl number is set to 0.7 (air) in this study. Two types of wall boundary conditions are considered for the passive scalar. Type I is the condition in which the scalar is uniformly produced within the fluid and removed at both walls.

$$\text{Type I} \begin{cases} \theta^s = 0 & \text{on the both walls} \\ \Theta_0 = 1 & \text{in the fluid} \end{cases} \quad (32)$$

The other is the condition in which the scalar was input to the flow at one wall and removed at the other side.

$$\text{Type II} \begin{cases} \theta^s = 1 & \text{on the lower wall} \\ \theta^s = 0 & \text{on the upper wall} \\ \Theta_0 = 0 & \text{in the fluid} \end{cases} \quad (33)$$

Table 1: Parameters for discretized *grid* and *test* filtering operations

	a	b	c	A	B	C
$(\Delta^s/h)^2$	1	4/3	2			
$(\Delta^T/h)^2$				4	6	8

Figs. 1 and 2 shows mean velocity and scalar profiles of Type I condition predicted by Smagorinsky's and proposed models at various filtering parameters. DNS results were obtained by Horiuti(1992). For the Smagorinsky's models, SGS stress models of eqs. (11) and (12) with SGS scalar models of eqs. (18) and (19) are used. For the proposed models, SGS stress models are given by eqs. (23) and (24) while SGS flux models are eqs. (25) and (26). We can clearly see the overestimation of the Smagorinsky's models in both velocity and scalar profiles while proposed models mitigate such tendency. Another important feature of these two figures is the sensitivity of Smagorinsky to the discretized *test* filtering parameter. As has already been reported(Tsubokura, 2001) but the proposed SGS stress models is less sensitive to the discretized *test* filtering parameter which is indicated in Fig. 1 (note that results of (Ab) and (Cb) are almost identical in the figure). Such insensitivity of the proposed models is willingly inherited to the SGS flux models which is shown in Fig. 2. We can observe that mean scalar predicted by proposed models shows almost identical profiles between (Ab) and (Cb) cases. While Smagorinsky's model extremely overestimate the scalar profiles with the dependance on filtering parameters.

To further study the dependence of the proposed SGS flux models on the discretized filtering parameter, predicted turbulent flux is indicated in Fig. 3. We should notice that statistics of passive scalar are affected by flow field. Therefore filtering parameter is fixed to (Ab) for SGS stress models and only the parameter for SGS flux is changed for Fig. 3. In both types of conditions, proposed SGS flux models favorably show almost identical results between (Ab) and (Cb) cases which definitely say the insensitivity of the proposed SGS flux models on *test* filtering parameter. But important results of Type I is that (Bc) predicts better correlation with DNS than (Ab) and (Cb). According to the previous study, optimized *grid* filtering parameter for the proposed SGS *stress* models in turbulent channel flow was (b). Therefore there is a possibility that optimized value is different between flow field(or SGS *stress*) and scalar field(or SGS *flux*). These difference might come from the difference of molecular viscosity and diffusivity (Prandtl number is 0.7 in this study), but further work is remained in future.

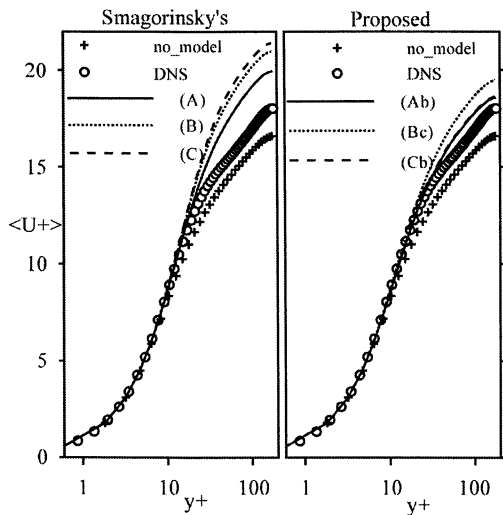


Fig. 1: Mean velocity profiles (Type I)
(Smagorinsky vs Proposed at various filtering parameters)

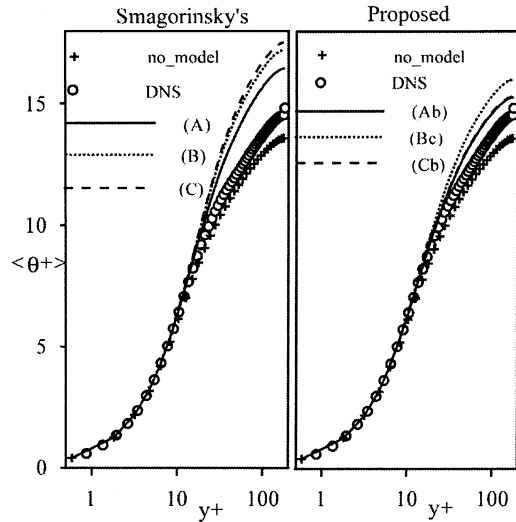


Fig. 2: Mean scalar profiles (Type I)
(Smagorinsky vs Proposed at various filtering parameters)

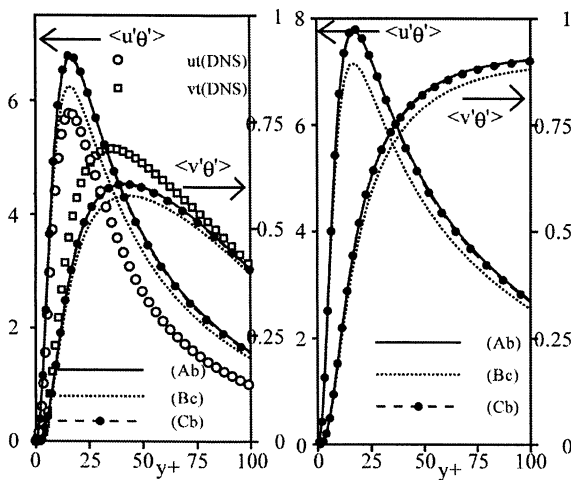


Fig. 3: GS turbulent flux (left:Type I, right:Type II)
(Dependence of the proposed SGS flux model on the filtering parameter when the parameter for the SGS stress models is fixed to (Ab).)

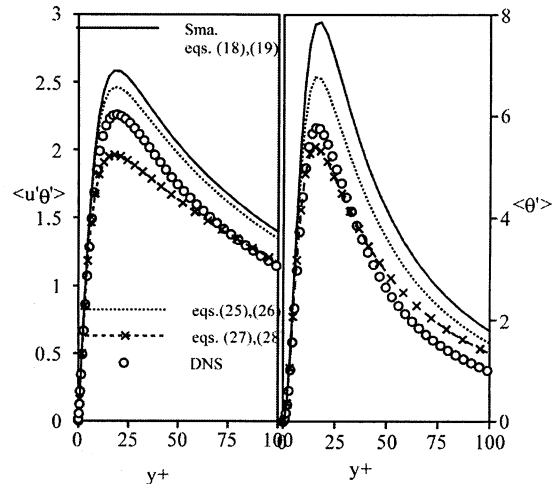


Fig. 4: GS turbulent flux and scalar rms.(Type I)
Isotropic vs Anisotropic SGS flux models
(Filtering parameter is (Ab))

Finally we would like to see the anisotropic effect on SGS flux. Fig. 4 indicates turbulent scalar flux and scalar r.m.s. predicted by eqs. (25), (26) (Isotropic) and eqs. (27), (28) (Anisotropic). We can identify that the overestimation of the isotropic models are clearly improved by the anisotropic models.

Rayleigh-Bénard convection (buoyancy-driven case)

The proposed models (eqs. (23)(24)(25)(26)) are applied to the Rayleigh-Bénard convection. Here, thermal convection between the square plates ($7.92H \times 7.92H$) is solved where H is the distance of each plate. Periodic boundary condition is adopted for horizontal direction while no-slip condition is supposed on the wall. In this study, Rayleigh number and Prandtl number is set to $Ra = H^3 \beta g \Delta T / \kappa \nu = 3.8 \times 10^5$, $Pr = 0.7$ which is

the same as the DNS obtained by Grötzbach. When representative velocity is given as $u_o = \kappa / H$, Reynolds and Richardson number in eqs. (2) and (3) are given as $Re = 1 / Pr$ and $Ri = Ra \cdot Pr$. Total grid number used is 32×32 for horizontal direction while 64 grid is allocated for normal wall direction otherwise stated.

Figs. 5 and 6 indicate GS scalar r.m.s. and turbulent intensity for normal-wall direction predicted by Smagorinsky's and proposed models at various filtering parameters. At this low Rayleigh number flow, both models predict acceptable statistics at the resolution adopted here. But important difference between them is that proposed models was found to be less sensitive to the *test* filtering operation ((Ab) and (Cb) correlate very well) while Smagorinsky's show certain difference depending on the *test* filtering parameter. Another problem of

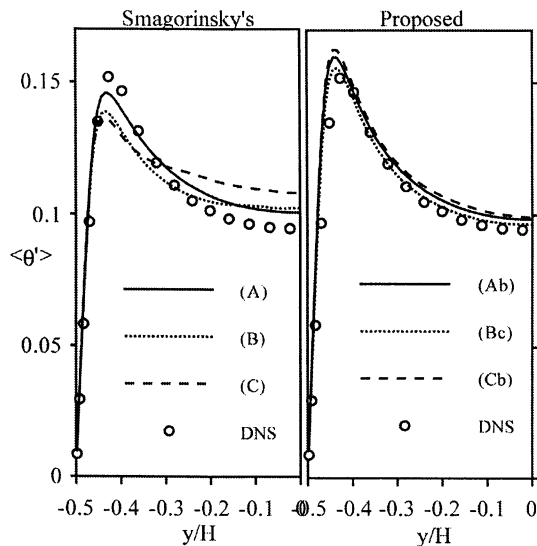


Fig. 5 : GS scalar rms. (Rayleigh-Bénard) Smagorinsky's vs Proposed (Dependence on the filtering parameter)

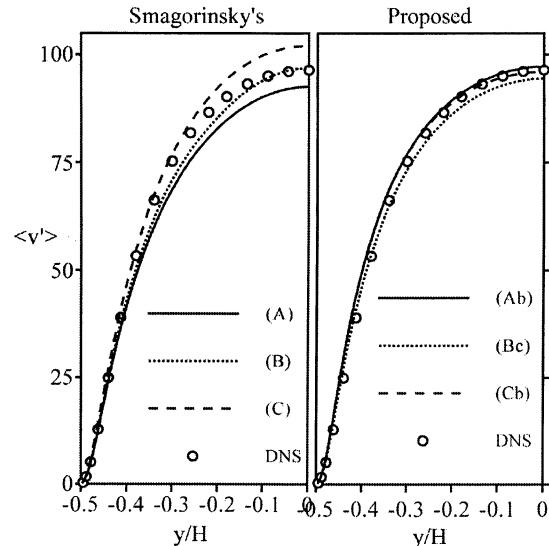


Fig. 6 : GS turbulent intensity (Rayleigh-Bénard) Smagorinsky's vs Proposed (Dependence on the filtering parameter)

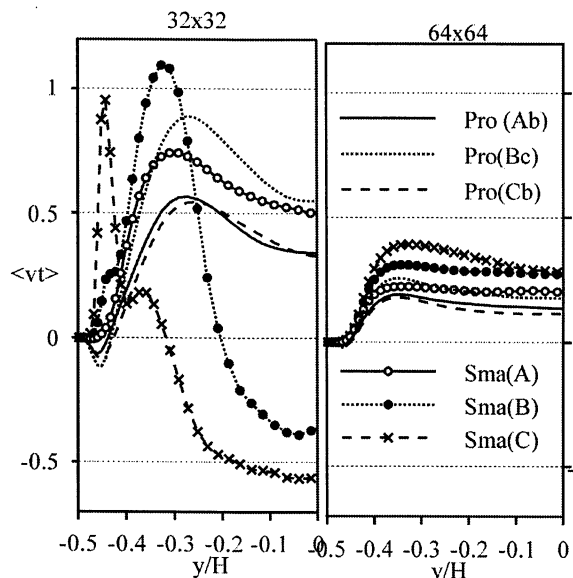


Fig. 7 : Eddy viscosity (Rayleigh-Bénard) (left: 32x32 mesh, right: 64x64 mesh) Smagorinsky's vs Proposed (Dependence on the filtering parameter)

Smagorinsky is that negative eddy viscosity is estimated at the center region between the plate in case coarse mesh (32x32 for horizontal) when the filter parameter (B) or (C) is adopted which is indicated in Fig.7(left). (In the simulation, negative eddy viscosity was set to zero to avoid numerical instability). While such an ill-condition of the dynamic procedure is eliminated by using finer mesh (64x64) (Fig.7, right), this is definitely the another problem of the Smagorinsky's dynamic procedure using FDM. Contrarily to the Smagorinsky, proposed model does not predict the ill-condition at the center region while slight negative value is estimated at the vicinity of the wall. The

Smagorinsky's models also show the ill-condition near the wall even though its absolute value is smaller than the proposed models. We may say that this ill-condition of both models at this region indicates the limitation of the isotropic eddy viscosity and diffusion models to such a region or the fundamental problems of the dynamic procedure adopted here (such as the plane averaging in eqs. (17) and (23)).

RESULTS

SGS flux models were proposed for the dynamic procedure using FDM based on the method previously proposed for the SGS stress models. Proposed SGS stress and flux models predict better statistics on the turbulent channel flow with passive scalar and Rayleigh-Bénard convection than Smagorinsky's. The advantage of the previously proposed SGS stress models such as the insensitivity to the explicit test filtering operation was found to be satisfactory inherited to the SGS flux models.

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