MODELLING ROTATIONAL EFFECTS IN INDUSTRIAL CLOSURE MODELS

B. Anders Pettersson Reif
Norwegian Defence Research Establishment, FFI
NO-2027 Kjeller, Norway
bre@ffi.no

Per E. H. Abrahamsen
Hamworthy KSE as
NO-3401 Lier, Norway

Helge I. Andersson and Stian Solbakken
Division of Applied Mechanics
Norwegian University of Science and Technology, NTNU
NO-7491 Trondheim, Norway

ABSTRACT

This study explores the possibility to sensitise the standard $k-\varepsilon$ model, used in conjunction with the wall function approach, to rotational effects by a modification of the eddy-viscosity coefficent. This modification has been derived through a phenomenological method based on equilibrium and bifurcation analysis of homogeneous turbulence. It has been proved to be succesful in conjuntion with the novel near-wall $v2f$ model, and the objective of the present study has therefore been to investigate if this modification also works well for industrial closure models. The results obtained so far are encouraging and further testing are therefore being conducted.

INTRODUCTION

Computational fluid dynamic (CFD) predictions play an increasingly important role in the design process in modern industry. The increased computer speed has made it possible to set up and solve fluid flow problems characterised by an increasing range of complex flow phenomena. Effects of body forces associated with frame or flow rotation constitute frequently occurring examples. More sophisticated turbulence closure models than traditionally employed are therefore needed to faithfully predict many of these intricate phenomena that arise in industrial fluid flow problems. However, for a numerical flow analysis to be of any significant value in the design process, a very short flow-through-time is required. That is, the time it takes from setting up the problem to the final results are obtained must be kept to a minimum. This means in practise that a numerically very efficient and robust computer code is required. This requirement has subsequently led to that full blown second-moment closure (SMC) computations only occasionally are conducted. This is unfortunate since SMC's represent one of the most physically appealing levels of single point closure modelling within the framework of Reynolds averaged computations (RANS). *Industrial* closure models therefore allude to models based on an algebraic constitutive relationship between turbulent stresses and the mean flow field. The standard two-equation $k-\varepsilon$ formulation is probably still the most commonly used closure model despite its many inherent and well known shortcomings.

Many improvements of the standard $k-\varepsilon$ formulation have been proposed over the years. Probably the most frequently referred advancement is the development of non-linear eddy-viscosity models. While the non-linearity itself might be of minor importance, it is most likely the notion of a variable coefficient eddy-viscosity formulation that has justified this more elaborate constitutive relation in practise. The terminology non-linear alludes to tensoral non-linearity as opposed to products of scalars. General tensor analysis is oftentimes used to *postulate* a non-linear consitutive model. This method plays an important role to ensure a mathematically consistent model. However, general tensor calculus does not ensure a physically plausible formulation *per se*.

After its first appearence 25 years ago (Pope, 1975) structural equilibrium and bifurcation analysis of SMC models have re–emerged as a powerful tool for improved algebraic modelling. This analysis does not only serve as a basis for systematic derivations of non-linear eddy-viscosity models, so-called explicit algebraic stress models (EASM); cf. e.g. Gatski and Speziale (1993) and Wallin and Johansson (2000), it also provides insights into how closure schemes respond to imposed forcing such as accelerations by system rotation or streamline curvature (Durbin and Pettersson Reif, 1999). Non-linear eddy-viscosity models systematically derived by imposing structural equilibrium on SMC's therefore constitute an invaluable link between the simpler linear $k-\varepsilon$ type models and full-blown SMC models. A particularly attractive feature of this analysis is that the resulting algebraic stress model retains elements of some of the more physically appealing characteristics of full SMC models, in particular the ability to respond to imposed forcing. This ability originates solely from a proper functional form of the *linear* eddy-viscosity coefficient; it should be expressed in terms of the turbulent-to-mean flow times scale ratio such that it satisfies the bifurcation criterion (Durbin and Pettersson Reif 1999).

The connection between the EASM and the standard $k - \varepsilon$ formulation is not direct. The latter can be viewed as a special case of the EASM provided the flow is parallel and that a clear-cut seperation of turbulent and mean flow time scales exists[1]. The former requirement implies that the non-linear EASM reduces to a linear eddy-viscosity model whereas the latter implies that the variable coefficient $C_\mu^* \to$ *const*; the result is thus a constant coefficient linear eddy-viscosity model, i.e. the 'standard' formulation. This is the formal connection between an SMC and Boussinesq's linear eddy-viscosity hypothesis in the limit of homogeneous and equilibrium turbulence. However, since a clear cut seperation of scales do not exist in boundary layer-type flows it is simply not a matter of just replacing the constant coefficient C_μ by its 'exact' counterpart C_μ^* from the equilibrium analysis. The reason is that $C_\mu^* \neq const = C_\mu$ which results in significantly less accuarate boundary layer predictions. It should be recalled that the standard model was design to perform well in precisely this case because of its relevance in many industrial applications.

Accomodating effects of imposed forcing has usually been based on *ad hoc* modifications of the dissipation rate transport equation, e.g. Howard *et al.* (1980). This is obviously not consistent with SMC models. This fact motivated Pettersson Reif *et al.* (1999) to develop a new methodology to incorporate effects of rotation into already existing eddy-viscosity models. This methodology suggests how to construct a variable coefficient C_μ^* such that the model mimics the respons of an SMC in the limit of homogeneous and equilibrium turbulence, at the same time as it ensures that $C_\mu^* = C_\mu = const$ in parallel shear flow *irrespectively* of scale separation. Any eddy-viscosity model could therefore, in principle, be modified in order to improve predictions in flows where rotational effects are important, and without significantly deteriorating the predictions in nearly parallel shear flows in inertial frames of reference.

The objective of the present study is to investigate whether the new approach suggested by Pettersson Reif *et al.* (1999) to sensitise eddy-viscosity models to rotational effects can be used within the framework of industrial closure models. In other words if the proposed methodology can be used to improve the predictive capability of the standard $k - \varepsilon$ model used in conjunction with the wall-function approach.

THEORETICAL BACKGROUND

Consider two-dimensional, homogeneous turbulent flow subjected to steady frame rotation rotation Ω_k^F about the x_k axis. The mean velocity gradient can be written as

$$\left[\frac{\partial U_i}{\partial x_j}\right] = \left[\begin{array}{cc} \mathcal{S} & \omega \\ -\omega & -\mathcal{S} \end{array}\right] \tag{1}$$

in principal axes. The evolution equation for the mean flow to turbulent time-scale ratio can then be written as

$$\frac{d}{d\tau}\left(\frac{\varepsilon}{\mathcal{S}k}\right) = \left[\mathcal{P} - \frac{C_{\varepsilon 2} - 1}{C_{\varepsilon 1} - 1}\right]\left(\frac{\varepsilon}{\mathcal{S}k}\right)^2 (C_{\varepsilon 1} - 1) \tag{2}$$

where

$$\mathcal{P} \equiv \frac{P}{\varepsilon} = -\frac{\overline{u_i u_j}}{\varepsilon}\frac{\partial U_i}{\partial x_j} = 2C_\mu^*\left(\frac{\mathcal{S}k}{\varepsilon}\right)^2 \tag{3}$$

[1]A clear-cut separation of scales implies that the turbulent-to-mean flow time scale ratio (η) is very small, i.e. $\eta << 1$.

is the production to dissipation rate ratio and $\tau = \mathcal{S}t$ is a nondimensional time. The standard form of the dissipation rate equation has been used to derive (2). The last equality in (3) is obtained by invoking the linear eddy-viscosity formulation

$$\overline{u_i u_j} = \frac{2}{3}k\delta_{ij} - 2\underbrace{C_\mu^* \frac{k^2}{\varepsilon}}_{=\nu_T} S_{ij} \tag{4}$$

where

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{5}$$

is the mean rate of strain tensor. ν_T is the eddy-viscosity.

Structural equilibrium

True structural equilibrium (Durbin and Pettersson Reif, 1999) is defined by constant values of the Reynolds stress anisotropy tensor;

$$d_t b_{ij} = d_t(\overline{u_i u_j}/k) = 0 \tag{6}$$

and

$$d_t(S_{ij}k/\varepsilon) = 0 = d_t(\Omega_{ij}^A k/\varepsilon) \tag{7}$$

where

$$\Omega_{ij}^A = \underbrace{\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right)}_{=\omega_{ij}} + \epsilon_{ijk}\Omega_k^F \tag{8}$$

is the absolute mean vorticity tensor in a non-inertial frame of reference. ω_{ij} is the local mean vorticity tensor.

In homogeneous flow, the mean velocity gradient $\partial U_i/\partial x_j = S_{ij} + \omega_{ij}$ must be uniform in space and true equilibrium also requires that $\mathbf{S}$ and ω be independent of time. Durbin and Pettersson Reif (1999) showed that the latter constraint has an important implication for equilibrium analysis; it imposes an additional requirement on $S_{ij} + \omega_{ij}$, namely that the determinant must vanish, i.e.

$$\det|S_{ij} + \omega_{ij}| = \frac{1}{3}\underbrace{S_{ij}S_{jk}S_{ki}}_{=III_S} + \underbrace{\omega_{ij}\omega_{jk}S_{ki}}_{=IV_S} = 0. \tag{9}$$

in order to be a true equilibrium. This result is a direct consequence of the mean vorticity equation provided that $2\Omega_k^F \neq -\epsilon_{kjm}\partial U_m/\partial x_j$. The terms on the right hand side of (9) is commonly referred to as the third (III_S) and fourth (IV_S) mean invariant, respectively. Steady mean flow in general and thus also true equilibrium requires that $III_S = 0 = IV_S$. Hence, *true equilibrium exludes three-dimensional mean flows.* If a three-dimensional homogeneous flow is subjected to rotation the dynamical equations will cause it to evolve to a state of true equilibrium, i.e. such that $III_S = 0 = IV_S$.

Bifurcation of equilibria

Equation (2) has two equilibria:

$$\text{(i)} \quad \frac{\varepsilon}{\mathcal{S}k} = 0 \tag{10}$$

$$\text{(ii)} \quad \mathcal{P} = \frac{C_{\varepsilon 2} - 1}{C_{\varepsilon 1} - 1} \tag{11}$$

SMC models are able to bifurcate between these two solutions, this is what makes them able to predict relaminarisation of turbulence affected by external forcing. Standard $k - \varepsilon$ models, on the other hand, are only able to represent the non-trivial solution (ii) where $\varepsilon/\mathcal{S}k \neq 0$ since $C_\mu^* = C_\mu = const$, cf. (2) and (3). The trivial solution (i) is of most interest here. Physically it signifies that the turbulent time scale is much larger than the mean flow time scale; the flow is about to relaminarise. This solution branch is associated with a power law solution for the turbulent kinetic energy k:

$$k \sim t^\lambda; \qquad \lambda = \frac{\mathcal{P} - 1}{C_{\varepsilon 2} - 1 - \mathcal{P}(C_{\varepsilon 1} - 1)} \tag{12}$$

where $\mathcal{P}$ depends on the model;

$$\mathcal{P} = \lim_{\eta_1 \to \infty} 2C_\mu \left(\frac{\mathcal{S}k}{\varepsilon}\right)^2 \tag{13}$$

Relaminarisation thus occurs if the exponent $\lambda < 0$, i.e. when $\mathcal{P} < 1$. The point $\mathcal{P} = 1$ is referred to as the point of stabilisation whereas $\varepsilon/\mathcal{S}k \to 0$ signifies the point of bifurcation. The point of stabilisation can thus be written as

$$\lim_{\eta_1 \to \infty} 2C_\mu^* \left(\frac{\mathcal{S}k}{\varepsilon}\right)^2 = 1 \tag{14}$$

This criterion only depends on one model coefficient; C_μ^*.

A model that stabilises due to imposed rotation

The present model was originally developed to sensitise the $v2f$ model (Durbin, 1991) to rotational effects (Pettersson Reif *et al.*, 1999, and Ooi *et al.* 2000). The modification consists of a new formulation of the (linear) eddy-viscosity coefficient (C_μ^*) which replaces the original coefficient C_μ used in the generalised Boussinesq hypothesis (4):

$$C_\mu^* = C_\mu \frac{\mathcal{F}_1}{\mathcal{F}_2} \tag{15}$$

where

$$\mathcal{F}_1 = \frac{1 + \alpha_2|\eta_3| + \alpha_3\eta_3}{1 + \alpha_4|\eta_3|} \tag{16}$$

$$\mathcal{F}_2 = \sqrt{\frac{1 + \alpha_5\eta_1}{1 + \alpha_5\eta_2}} + \alpha_1\sqrt{\eta_2}\sqrt{|\eta_3| - \eta_3} \tag{17}$$

, cf. Pettersson Reif *et al.* (1999). The non-dimensional velocity gradient invariants are given by

$$\eta_1 = S_{ij}^* S_{ji}^*; \ \eta_2 = W_{ij}^* W_{ji}^*; \ \eta_3 = \eta_1 - \eta_2 \tag{18}$$

where

$$S_{ij}^* = S_{ij} k/\varepsilon \tag{19}$$

and

$$W_{ij}^* = (\Omega_{ij}^A + 1.25\epsilon_{ijm}\Omega_m^F) k/\varepsilon \tag{20}$$

The model coefficients α_i are determined such that the model stabilises close to the EASM:

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = (\frac{1}{18}, \frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{40}) \tag{21}$$

The point of stabilisation is given by a fourth order algebraic equation:

$$\mathcal{R}^2(\mathcal{R}^2 - 1) = 2\left[\frac{C_\mu(\alpha_2 - \alpha_3}{\alpha_1\alpha_4}\right]^2 \tag{22}$$

where $\mathcal{R}^2 \equiv \eta_2/\eta_1$. For a given C_μ, the model coefficients α_i can be altered to match the stabilisation points any SMC. The present choice gives approxiamtely the same stabilisation points as the $v2f$ model for which (15) originally was developed for.

A remedy of the stagnation point anomaly

A well known deficiency of the $k - \varepsilon$ model is the excessive production of turbulent energy that occurs for large normal strains. The reason is associated with quadratic dependence of $\mathcal{S}$ in (3), i.e. it is a consequence of the eddy-viscosity hypothesis. This sensitivity to normal strains is somewhat increased by the modification (15); it is especially designed to exhibit C_μ^*(plane strain) > C_μ^*(pure shear) in homogeneous turbulence. This is fully consistent with the behaviour of an SMC.

To remedy this problem, the constraint

$$\nu_T = \min(C_\mu^* k^2/\varepsilon, k/(\sqrt{6S_{ij}S_{ji}})) \tag{23}$$

proposed by Durbin (1996) has been invoked in the present modification of the standard $k - \varepsilon$ model.

RESULTS

The modification (15), together with (23), has been implemented into the commercial CFD solver Fluent, in order to sensitise the standard $k - \varepsilon$ model to rotational effects. The wall-function approach has been used throughout this study. All computations have been conducted on a Cartesian grid, with a relatively coarse grid resolution suited for the wall function approach. Two cases have been considered in this study: (i) spanwise rotating channel flow and (ii) the flow over a backward facing step in orthogonal mode rotation. The first case is one-dimensional whereas the latter has been treated as a two-dimensional flow. The computational domain extended 8 stepheights upstream the step and 30 step heights downstream in order to ensure small effects of inflow and outflow boundary conditions.

Spanwise rotating channel flow

This case constitutes a standard benchmark test for turbulence closures developed to predict rotational effects on turbulent flows. A salient feature of this flow is that the imposed rotation only directly affects the turbulence; the mean flow is only indirectly affected through changes in the turbulence. The true performance of the closure model can therefore be revealed [2]. The most accurate reference data for this case is low Reynolds number (Re) direct numerical simulations (DNS) data. However, the low Re render these data not particular suitable as a reference for model predictions utilising the wall function approach; these models are primarily intended for considerable higher Re. The experimental measurements reported by Johnston *et al.* (1972) have therefore been primarily used to validate the model predictions. Figures 1 and 2 display the distribution of mean velocity across the channel at $Re \equiv 2hU_b/\nu \approx 11500$ at two different rotation numbers $Ro \equiv 2\Omega_3^F h/U_b$. $2h$ denotes the channel height whereas U_b is the bulk velocity. It is especially encouraging that the almost irrotational region of the flow (i.e. $2\Omega \approx dU/dy$) at the center of the channel is nicely predicted by the model. Figure 3 displays the varia-

[2] There is however an open question about the three-dimensionality of rotating channel flows, i.e. rotational induced secondary mean flow.

tion of wall friction velocity ($u_* \equiv \sqrt{\tau_w/\rho}$) as a function of rotation number. The predictions (at $Re \approx 11500$) compares favourable with the experimental data as well as with the low Re DNS data of Kristoffersen and Andersson (1993). Full blown SMC predictions are also displayed in the firgure.

Rotating backstep

This case has been considered to test the model in a significantly more complex flow. The direct impact of the imposed rotation on the mean velocity field is much weaker than the indirect effect through changes in the turbulence intensity. The rotating backstep therefore constitutes an interesting test case.

The flow downstream a backward facing step is characterised by the presence of streamline curvature and high mean shear. The reattachment point (x_r) further downstream is sensitive with respect to the turbulence intensity. High turbulence levels mix momentum in the cross-flow direction and causes the reattachment lenght to be smaller as compared to a flow with lower turbulence levels. The imposition of spanwise rotation enhances or suppresses the turbulence depending on the sense of frame rotation. In this case, $Ro > 0$ implies enhanced intensities along the stepped wall, upstream the step, whereas $Ro < 0$ implies reduced turbulence levels. It can thus be expected that the reattachment length is reduced in the former case as compared to a non-rotating backstep.

Figure 6 displays the variation of the reattachment length as a function of Ro. The predictions are in close agreement with the experimental data for $Ro > 0$. For $Ro < 0$, on the other hand, the predictions reveal substantial deviations from the experimentally observed behaviour; the impact of the imposed rotation is significantly overpredicted. In order to further investigate this, additional computations with the 'realisable $k - \varepsilon$ model', provided by Fluent were performed. Although this model exhibits a much weaker response to imposed rotation (see also figure 3) it still overpredicts the rotational effects for $Ro < 0$. It is therefore believed that the experimental data are characterised by three-dimensional effects and therefore not captured by the present predictions. It should finally be noted that the predictions with the standard $k - \varepsilon$ model seem totally insensitive to the imposed rotation.

CONCLUDING REMARKS

The objective of the present study was to investigate ewhether or not the modification (15), originally devised for a near-wall model, could be used in conjunction with the standard $k - \varepsilon$ model with wall functions. The results obtained with this modification showed encouraging improvements, as compared to the predictions obtained with the standard model. Further testing in complex industrial flows are therefore being conducted.

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FIGURES

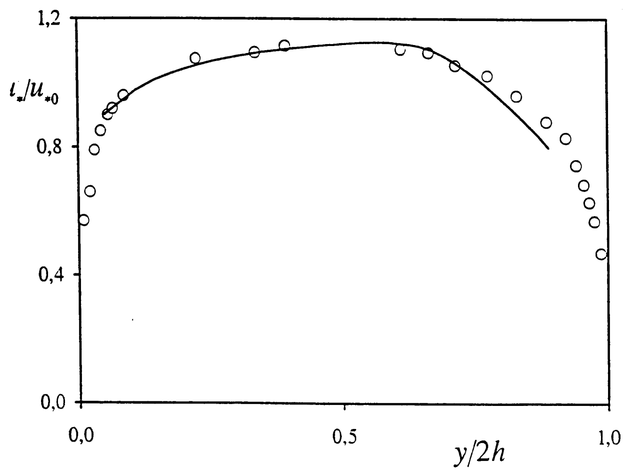

Figure 1: Mean velocity distribution in spanwise rotating channel flow. Re = 11500, Ro = 0.068. Symbols: experiments (Johnston *et al.* 1972); lines: present.

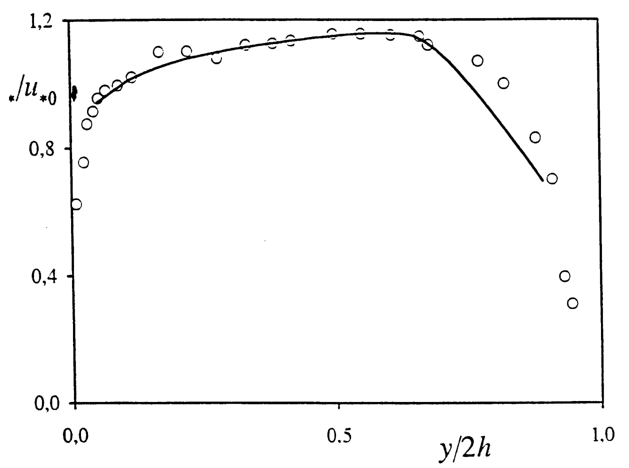

Figure 2: Mean velocity distribution in spanwise rotating channel flow. Re = 11500, Ro = 0.21. Symbols: experiments (Johnston *et al.* 1972); lines: present.

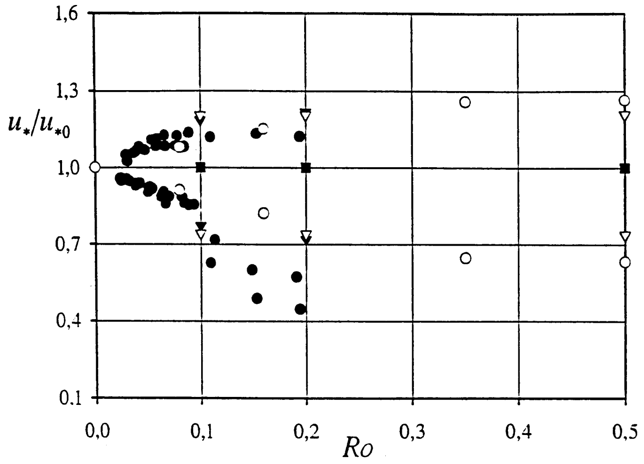

Figure 3: Spanwise rotating channel flow. Variation of friction velocity along the two walls as a function of Ro. Open circles: present; filled circles: experimental data (Johnston *et al.* 1972); open triangles: SMC (Pettersson and Andersson 1997); filled triangles: DNS (Kristoffersen and Andersson 1993); filled squares: the 'realisable $k-\varepsilon$' model provided by Fluent.

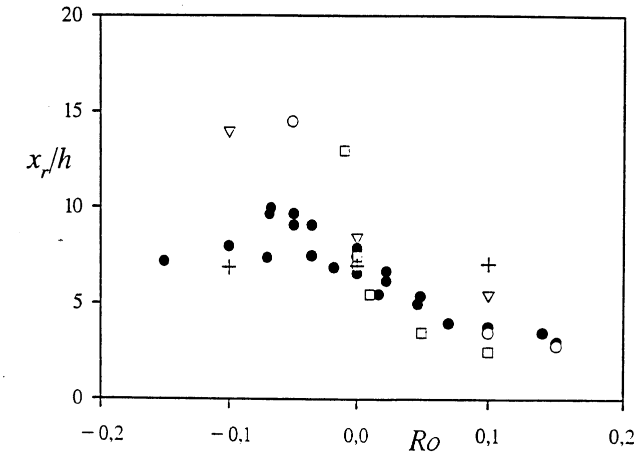

Figure 4: Rotating backstep. Variation of reattachment length as a function of Ro. Open circles: present ($Re \approx$ 10000); open squares: present ($Re \approx$ 18000); filled circles: experiments (Rothe 1975); plusses: standard $k-\varepsilon$ model; open triangles: the 'realisable $k-\varepsilon$' model provided by Fluent.