

# NEAR-WALL HOT-WIRE MEASUREMENTS: CONVECTIVE VELOCITY AND SPECTRA IN THE VISCOUS SUBLAYER

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## ABSTRACT

The convective velocity  $U_c$  of streamwise velocity fluctuations in the very near-wall region was obtained using a two-point correlation technique. It was found that in the viscous sublayer,  $U_c$  is approximately  $13u_\tau$  and  $15u_\tau$  respectively for the channel and boundary layer flows under investigation. Spectra data for the wall shear stress and streamwise velocity fluctuations in the viscous sublayer are also presented, and the normalized spectral plots for different flow conditions collapse at high frequencies or wavenumbers, thus indicating the possible presence of small scale universality at different Reynolds numbers even within the viscous sublayer.

## INTRODUCTION

For wall bounded turbulent shear flows, the wall serves as the source or sink of vorticity, and the wall can thus be construed as the anchoring point for the flow. In order to gain a fuller understanding of the physics governing near-wall turbulence, it is of utmost importance to obtain reliable flow measurements in the very near-wall region. Since skin friction drag constitutes the major component of drag for streamlined bodies, the various passive drag reduction schemes which have been proposed and investigated by numerous researchers all seek to alter the flow structures and velocity profile very near the wall. It is thus important to gain a more in-depth understanding of the flow characteristics very near the wall in order that such drag reducing schemes can be applied successfully. A further motivation for near-wall turbulence measurements arises from its application to turbulence modelling, where such measurements serve as a means for computational fluid dynamists to propose more accurate turbulence models in the near-wall region

and to check the validity of their computational results near the wall.

The hot wire's smallness in size and its ability to track a fluctuating velocity with a high degree of responsiveness establishes it as a primary means of turbulent velocity measurements. However, a hot wire operated in close proximity to a solid wall suffers from influence of wall effects. A hot wire that has been calibrated under free stream conditions thus fails to provide accurate time-resolved information of the velocity in the very near-wall region of the flow field under investigation. Chew et al. (1994) and Khoo et al. (1996) have previously proposed and verified various calibration techniques and procedures in order that a near-wall hot wire would be capable of yielding accurate time-resolved velocity measurements in the very near-wall viscous sublayer of a wall bounded turbulent shear flow.

## RESULTS AND DISCUSSION

### Convective Velocity

Velocity measurements were conducted in both a turbulent channel flow at different  $h^+$  ( $\equiv hu_\tau/\nu$ , where  $h$  is the channel half height) and flat plate boundary layer flow at different Reynolds numbers  $Re_\theta$  (based on momentum thickness and free stream velocity) using near-wall hot wires. The construction of the near-wall hot wire is available in Khoo et al. (1998), whereas details of the channel and the wind tunnel where measurements were taken can be found in Khoo et al. (1996).

Taylor's hypothesis, which can be stated mathematically as

$$\frac{\partial}{\partial t} = -U_c \frac{\partial}{\partial x}, \quad (1)$$

constitutes one of the most frequently used tools in turbulence research. It allows experimentalists to

convert the one-dimensional power density spectra from the frequency domain into the wavenumber domain. The spatial derivatives in the streamwise direction that can be obtained from derivatives with respect to time have also been applied to study coherent structures. In view of the important role played by Taylor's hypothesis in turbulence research, it is thus imperative to determine the convective velocity  $U_c$  (which differs markedly from the local mean velocity  $\bar{U}$  in the very near-wall region) accurately. Experiments were performed to determine the convective velocities  $U_c$  of the streamwise velocity fluctuations in the very near-wall region of the flows under investigation. This was achieved by employing 2 hot wires separated by a streamwise distance of 9 mm and 10 mm for the boundary layer and channel flows, respectively. The near-wall hot-wire probe, with the active element placed 50  $\mu\text{m}$  (corresponding to 2 wall units approximately) above the Perspex wall substrate, is positioned upstream. The downstream wire consists of a DANTEC 55P15 single-wire probe located 0.35 mm above the wall (corresponding to approximately 13 wall units). The output voltages from the 2 wires are sampled simultaneously, and the two-point correlation coefficient for the streamwise velocity is determined from

$$R_{u_1u_2}(T) = \frac{\overline{u_1(t)u_2(t+T)}}{u_1^2(t)}, \quad (2)$$

where  $u_1$  and  $u_2$  denote the instantaneous fluctuating streamwise velocities at the upstream and downstream locations, respectively. The time delay between the two velocities  $u_1$  and  $u_2$  is denoted by  $T$ . Results for the distribution of  $R_{u_1u_2}(T)$  are plotted against  $T^+$  ( $\equiv Tu_\tau^2/\nu$ ) in Fig. 1 for the channel and boundary layer flows. It can be observed that for the boundary layer flow, the correlation is stronger at lower  $Re_\theta$ . The time interval  $(T^+)_{\text{peak}} = T_1^+$  corresponding to the peak value of  $R_{u_1u_2}(T)$  signifies the time delay for the downstream instantaneous velocity to achieve a maximum correlation with the upstream instantaneous velocity. This value of  $T_1$  and the streamwise separation of the 2 wires are used to evaluate the average convective velocity  $U_c$  of the streamwise velocity fluctuations.

Figure 2 shows the results for  $U_c$  plotted against the elevation of the upstream near-wall hot wire,  $y^+$  ( $\equiv yu_\tau/\nu$ ), whereas the results of  $U_c$  for various probe separation distances  $s^+$  ( $\equiv su_\tau/\nu$ ) are plotted in Fig. 3. From both figures, it can be seen that the convective velocity  $U_c$  is approximately  $13u_\tau$  for the channel flow at  $h^+ = 390$  and  $15u_\tau$  for the boundary layer flow (within the range of  $Re_\theta$  investigated). From Fig. 2, it is observed that  $U_c$  remains fairly constant within the viscous sublayer and does not show any systematic dependence on  $y^+$  for the same flow configuration, thus suggesting that streamwise velocity perturbations propagate like waves in the very near-wall viscous sublayer region. This is

different from the normal concept of Taylor's hypothesis outside the viscous sublayer where the turbulence is assumed "frozen" and convected with mean velocity  $\bar{U}$ . If this is to be the case in the viscous sublayer,  $U_c$  would be equal to  $y^+u_\tau$  instead of  $13$  or  $15u_\tau$ . As reported by Kim and Hussain (1993) according to their DNS results at  $h^+ = 180$ , in the very near-wall region, streamwise vortices are temporally very persistent and do not lose their coherence for distances as long as  $1000\nu/u_\tau$ . The effective vertical mixing due to the presence of these vortices therefore suggests that the fluid particles very close to the wall are well correlated. It can be deduced from Fig. 3 that for identical flow conditions,  $U_c$  does not exhibit any systematic dependence on the probe separation for the range of separation distances investigated. The values of  $U_c$  derived for small probe separations are dominated by the small-scale motion and vice versa. The apparent invariance in  $U_c$  with probe separation thus suggests that very near the wall, events of different scales are convected at an almost constant velocity for the range of separation distances studied.

The experimental results of Krogstad et al. (1998) obtained at  $y^+ = 5$  for a turbulent boundary layer at  $Re_\theta = 1409$  suggest that large-scale structures move at higher convective velocities than small-scale events. Kim and Hussain (1993) performed bandpass filtering of DNS data for a turbulent channel flow at  $h^+ = 180$  to investigate the scale-dependence of  $U_c$ . In contrast to Krogstad et al., Kim and Hussain found that the large-scale events are convected at lower velocities than the small-scale events, although the variation turns out to be rather insignificant. However, this observation is inconsistent with DNS results of Jeon et al. (1999), who obtained the convective velocity of wall shear stress fluctuations for a turbulent channel flow at the same  $h^+$  of 180. Jeon et al. concluded that in general, large-scale fluctuations tended to have larger values of  $U_c$  as compared to small-scale fluctuations. However, when an overall convective velocity for the streamwise wall shear stress fluctuations was used to convert the one-dimensional frequency power spectrum into the streamwise wave-number power spectrum for the purpose of testing Taylor's hypothesis, there was excellent agreement between the streamwise wave-number spectrum using Taylor's hypothesis and the actual spectrum. It is thus evident that different researchers have reported conflicting trends for the scale dependence of  $U_c$  corresponding to the streamwise velocity fluctuations in the very near-wall region. However, the general consensus is that this scale dependence on  $U_c$  is probably very marginal in the very near-wall region. It is thus logical to assume a single representative or overall convective velocity for all scales in the very near-wall region. In adopting such an approach, substantial simplifications can be made

in the practical implementation of Taylor's hypothesis and turbulence modeling.

### **Spectra**

One of the major drawbacks of near-wall LDV measurements lies in its inability to yield spectral information due to the non-constant data rate arising as a consequence of the low particle count in the immediate neighbourhood of the wall. In contrast, the hot wire is capable of continuous and accurate velocity measurements even in the near-wall viscous sublayer region. Spectra of the wall shear stress spectra and the streamwise velocity fluctuations in the viscous sublayer at  $y^+ \approx 2$  obtained using the near-wall hot-wire probes are presented in Figs. 4 and 5 respectively, and compared to experimental and DNS results in the literature. From Fig. 4, it can be seen that the frequency power spectrum for the measured  $\tau$  at  $h^+ = 180$  compares favourably to the DNS results, especially for high frequencies corresponding to the small-scale fluctuations. It is further remarked that all the other measured spectral plots ( $h^+ = 390$ , and  $Re_\theta = 2900, 3400$  and  $4100$ ) tend to collapse at high frequencies. However, such a trend is not observed at low frequencies for the channel flow, where the normalized frequency power spectral density function tends to increase with increasing  $h^+$  for the same normalized frequency. For the boundary layer flow, the normalized spectra increases marginally with  $Re_\theta$  for the same normalized low frequency range. This observation of a mild Reynolds number dependence of the normalized spectra for low values of normalized wavenumber is similar to the experiments of Antonia et al. (1992), who obtained the spectra for the streamwise velocity fluctuations at  $y^+ \approx 32$  for turbulent channel flows at various Reynolds numbers using hot-wire anemometry. Antonia et al. verified that changes in the Reynolds number had significant effects on the low wavenumber (and hence frequency) portion of the spectra, whereas the high wavenumber portion of the spectra was negligibly affected by variations in the Reynolds number. The results of Antonia et al., however, are at odds with those of Wei and Willmarth (1989), who performed LDA measurements in a fully developed turbulent channel flow. Wei and Willmarth reported that the energy containing (low frequency) part of their spectra at  $y^+ \approx 15$  did not show appreciable variation with the Reynolds number, but the high wavenumber part of the spectra increased with Reynolds number, which they claimed was due to the formation of smaller eddies at higher Reynolds number, as a consequence of increased vortex stretching. However, they admitted that there was considerable scatter in their spectral data at lower frequencies. On the other hand, Antonia et al. substantiated their experimental observations using DNS data at  $y^+ \approx 40$ , which indicated that the increase in the spectral density

function with increasing Reynolds number was confined to small wavenumbers. This concurs with the concept of an 'inactive motion', first proposed by Townsend (1961) and Bradshaw (1967), which intensifies as the Reynolds number increases. This inactive motion which consists of the large-scale vorticity field and the pressure fluctuations of the large eddies in the outer layer is nominally irrotational. Such motions have very large wavelengths (of order  $\delta$ , the boundary layer thickness) and time-scales in comparison to the viscous (inner) layer scales. As the wall is approached, the normal ( $v$ ) component (to the wall) of the inactive motion has to be brought to rest due to the impermeability condition imposed by the wall, thus releasing their normal component of the energy into the other two orthogonal tangential components  $u$  and  $w$ . This 'splat effect' motion's influence on the shear stress is small, thus producing very little effect on the log law of the wall for the mean velocity, as observed in Khoo et al. (2000). As the Reynolds number increases, this inactive motion contributes appreciably to the low wavenumber components of the  $u$  and  $w$  spectra, thus causing the magnitudes of  $u^{+}$  (and hence  $u'/\bar{U}$ ) and  $w^{+}$  to increase with Reynolds number.

Furthermore, it can be observed from Fig. 4 that our experimental results compare favourably to the DNS results of Jeon et al. (1999), whereas most of the experimental results of other researchers do not. The early results of Sreenivasan and Antonia (1977) were obtained using a hot-film wall shear stress probe in air. It has been shown (Chew et al., 1998b) that the dynamic frequency response of such probes operating in air is excessively low, typically  $O(1)$  Hz). The  $l^+$  values of the transducer used by Keith and Bennett (1991) were 120 and 210 respectively for  $h^+$  values of 2669 and 3966. These values of  $l^+$  are very much greater than the customarily accepted value of 20 to 25 for near-wall turbulence measurements (Ligrani and Bradshaw, 1987a, Khoo et al., 1997), and are likely to suffer from spatial resolution problems. The length-to-diameter ratio of the probe employed by Wietrzak and Lueptow (1994) was 100, which is significantly smaller than the recommended value of at least 200 to ensure negligible heat loss to the prongs (Ligrani and Bradshaw, 1987b; Chew et al., 1998a).

It is of further interest to investigate the spectra of the streamwise velocity fluctuations in the viscous sublayer. However, frequency spectra of the streamwise velocity fluctuations in the viscous sublayer are not readily available in the literature. DNS results for the frequency spectra of the streamwise velocity fluctuations are also not available in the literature. However, wavenumber spectra of the streamwise velocity in the viscous sublayer are available from DNS results of channel flows at various values of  $h^+$ , and these prove to be invaluable in validating our spectral results. Taylor's

hypothesis was invoked to transform the frequency spectra obtained experimentally into the wavenumber spectra, using values of the convective velocity  $U_c$  determined in the previous section.  $U_c$  was assumed to be  $13u_\tau$  and  $15u_\tau$  respectively for the channel and boundary layer flows. The collapse between the experimental results for the channel flow at  $h^+ = 390$  and the DNS results of Antonia and Kim (1994) for  $h^+ = 395$  is excellent. This has one important implication: The excellent agreement attests to the validity of Taylor's hypothesis in transforming the frequency spectra into the wavenumber spectra by using the correct value for the convective velocity  $U_c$ . From Fig. 5, it is further evident that all the spectral plots tend to collapse together at high wavenumbers, which is identical to that observed for the wall shear stress spectra. This suggests the possible existence of small scale universality at different Reynolds numbers even within the viscous sublayer. For the boundary layer flow, the normalized spectra obtained at low wavenumbers tend to exhibit a marginal increase for the same normalized wavenumber corresponding to an increase in  $Re_\theta$ . Once again, this is similar to the results of Antonia et al. (1992) for their spectra of the streamwise velocity fluctuations obtained at  $y^+ \approx 32$  for turbulent channel flows at various Reynolds numbers as well as DNS results of a channel flow at  $y^+ \approx 40$  for different values of  $h^+$ . This may again be explained using the concept of the 'inactive motion', which contributes appreciably to the low wavenumber components of the streamwise velocity spectra, even within the viscous sublayer. Bradshaw and Langer (1995) has also commented that "it is of course well known that some parts of turbulence, notably the low wavenumber parts of the  $u$  and  $w$ -component spectra, do not scale on law-of-the-wall variables". This is supported by the present spectral results, which indicate that the large (energy containing) eddies which correspond to low wavenumbers do not scale on inner wall variables.

From Figs. 4 and 5, the spectral plots for the different flows tend to collapse at high frequencies and wavenumbers, thus suggesting the possible existence of small scale universality at different Reynolds numbers even within the viscous sublayer.

## CONCLUSION

Experiments performed to determine the convective velocity  $U_c$  of the streamwise velocity fluctuations in the viscous sublayer yielded values of  $13u_\tau$  and  $15u_\tau$  respectively for the channel flow at  $h^+ = 390$  and the boundary layer flows under investigation. It was also found that the value of  $U_c$  remains fairly constant within the viscous sublayer and does not show any systematic dependence on  $y^+$ , thus suggesting that streamwise velocity perturbations propagate like waves in the very near-wall viscous sublayer region.  $U_c$  was also found to be relatively independent of the

separation distance between the two hot-wire probes, thus implying that it is reasonable to assume a single representative or overall convective velocity for all scales in the very near-wall region, which leads to substantial simplifications in the application of Taylor's hypothesis and turbulence modelling.

Spectral data obtained for the wall shear stress and longitudinal velocity fluctuations in the viscous sublayer compared very favourably to existing DNS results in the literature. Moreover, all the respective normalized spectral plots for different flow conditions tend to collapse together at high frequencies or wavenumbers, thus suggesting the possible existence of small scale universality at different Reynolds numbers within the viscous sublayer. However, Reynolds number effects are evident for the normalized spectra obtained at low normalized frequencies or wavenumbers.

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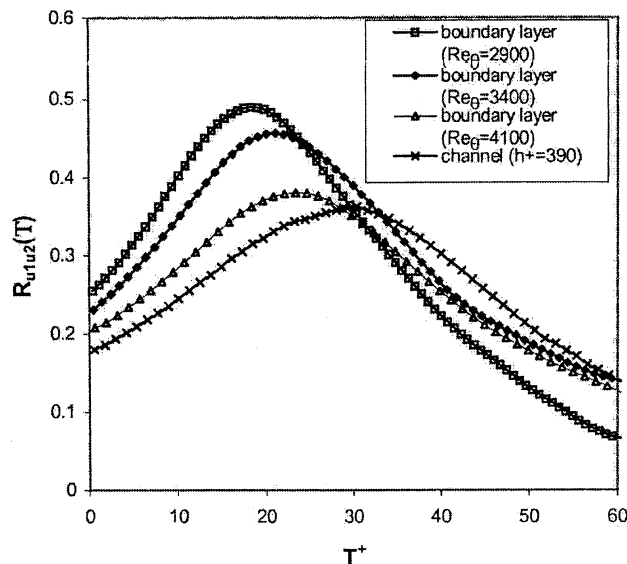


Figure 1 Distribution of two-point correlation coefficient with  $T^+$ .

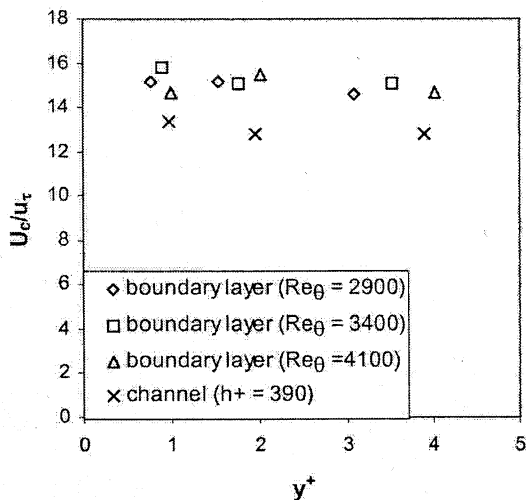


Figure 2 Convective velocity  $U_c$  for different values of  $y^+$  in the viscous sublayer.

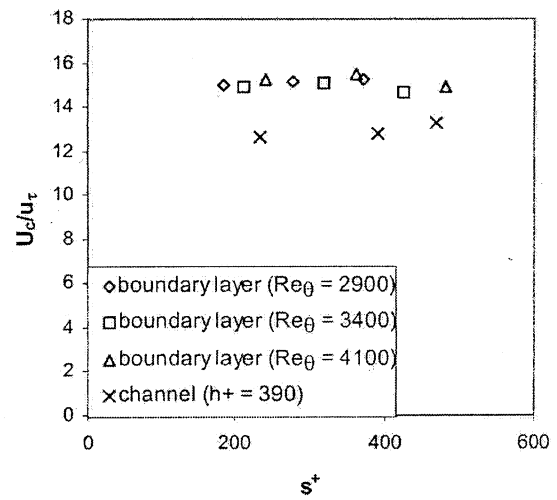


Figure 3 Convective velocity  $U_c$  for various probe separations  $s^+$  (in wall units).

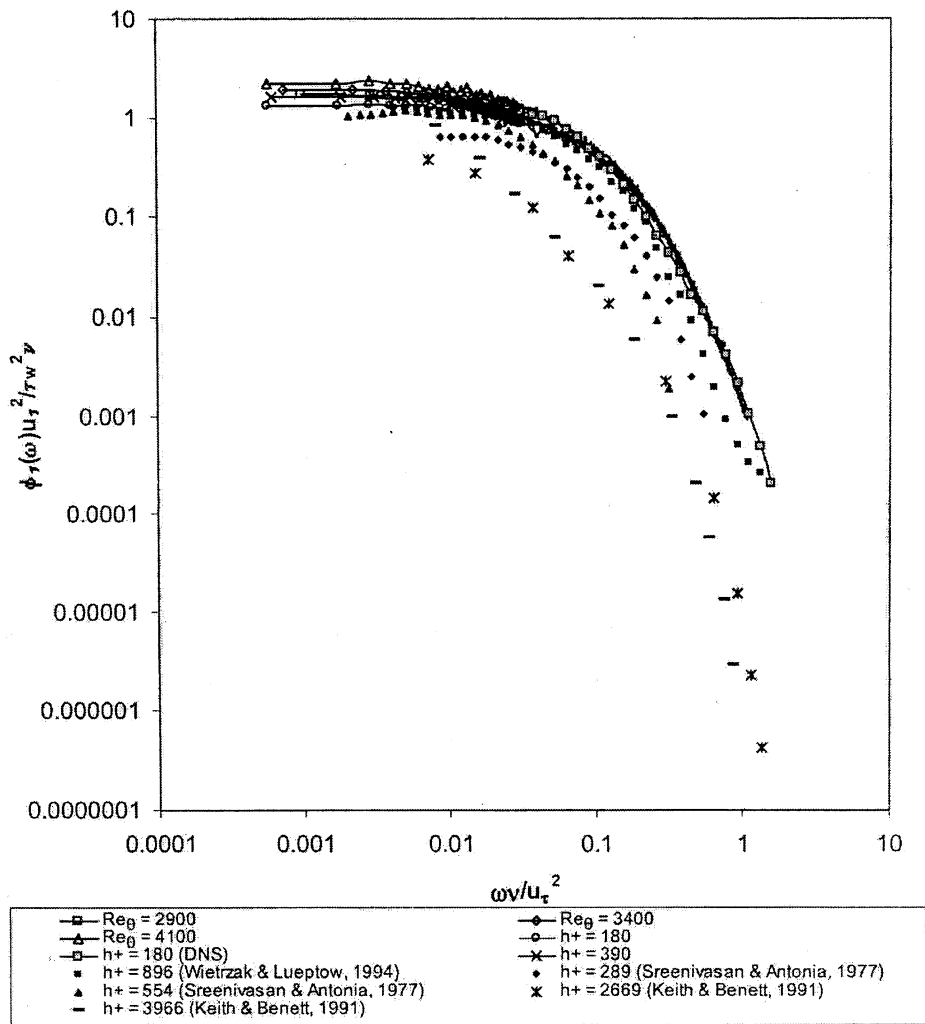


Figure 4 Normalized spectra of wall shear stress fluctuations.

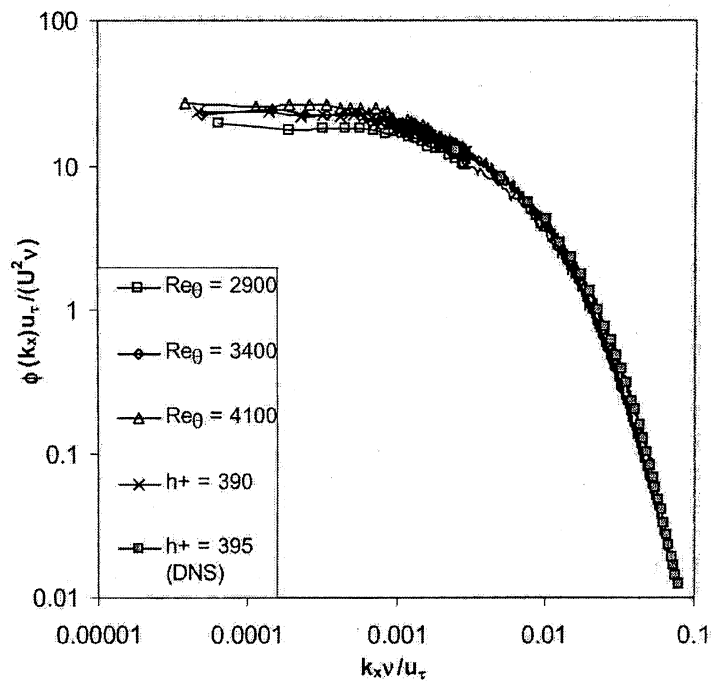


Figure 5 Normalized spectra of streamwise velocity fluctuations in the viscous sublayer at  $y^+ \approx 2$ .