SCALING LAWS FOR A TURBULENT BOUNDARY LAYER OVER A FLAT PLATE

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ABSTRACT

Experimental results concerning the behavior of the velocity structure functions in a turbulent boundary-layer are presented. Our analysis in terms of extended self-similarity confirms recent findings about a double-scaling regime in the logarithmic region. The crossover is controlled by the mean shear. At small separations, the exponents of the power-laws for the velocity structure functions are close to those usually experienced in homogeneous isotropic turbulence. At larger separations, where the mean shear strongly affects the velocity fluctuations, the exponents are substantially lower. The relationship between the present results and the existence of a new form of similarity, recently proposed for shear dominated turbulence, is thoroughly investigated.

INTRODUCTION

Scaling laws are well known in turbulence since the earliest works of Kolmogorov (Kolmogorov 1941, Kolmogorov 1962). In homogeneous and isotropic conditions, the structure functions S_p, i.e the statistical moments of the longitudinal velocity difference at separation r, are predicted to be scale invariant in the inertial range. When the dissipation $< \epsilon >$ and the scale r are the only relevant parameters, dimensional arguments yield a value $\zeta_p = p/3$ for the exponent

$$< \delta V^p > \propto < \epsilon >^{p/3} \; r^{p/3} \qquad (1)$$

which, for $n = 2$, implies the well-known law for the energy spectra $E(k) \sim k^{-5/3}$. Experiments as well as numerical simulations, however, have shown clear deviations of the higher-order exponents from this dimensional prediction. The anomalous correction is related to the statistical properties of the local dissipation field via the Refined Kolmogorov Similarity Hypothesis (RKSH)

$$< \delta V^p > \propto < \epsilon_r^{p/3} > \; r^{p/3} \,. \qquad (2)$$

Here ϵ_r denotes the local dissipation rate, spatially averaged over a volume of characteristic dimension r. Hence the scaling properties of the dissipation field ($< \epsilon_r^q > \propto r^{\tau_q}$) explain the observed values for the exponents of the structure functions.

Quite surprisingly, power laws have been observed also in very different conditions, such as wall bounded flows and homogeneous shear flows, using both DNS (Toschi et *al.* 1999, Gualtieri et *al.* 2000) and laboratory experiments (Antonia et *al.* 1998, Onorato et *al.* 2000).

Recently the physical origin of the scaling laws in shear dominated turbulence has been addressed in Benzi et *al.* (1999), where a new form of similarity law has been proposed and checked against numerical data. Here, we readdress the problem by considering experimental data for the logarithmic region of a boundary layer.

SCALING LAWS

To investigate the scaling properties of near-wall turbulence, we consider a flat plate boundary layer, obtained in a wind tunnel operated at 12 m/s. The Reynolds number, based on the momentum thickness, is $Re_\theta \simeq 2200$. Hot-wire measurements of the streamwise component of velocity have been carried out at several wall-normal distances. To characterize the statistical properties of turbulence, we consider the hierarchy of the longitudinal structure functions S_p

$$S_p := < \delta V^p > = < [v(x) - v(x+r)]^p > . \quad (3)$$

Brackets denote here time-averaging, and Taylor hypothesis is used to convert temporal into spatial increments.

At moderate Reynolds numbers the velocity fluctuations do not exhibit a well-defined inertial range, hence no clear power law can be extracted in terms of separation r. Nevertheless, a power-law behavior is exhibited in terms of Extended Self Similarity (Benzi et *al* 1996). This technique extends up to the dissipative range the scaling properties of the velocity increments and thus allows for an accurate evaluation of the scaling exponents (Frisch 1995). ESS employs as similarity variable the third order structure function instead of the separation r,

$$< \delta V^p > \propto < \delta V^3 >^{\zeta_p} . \quad (4)$$

Since S_3 in the classical inertial range is proportional to r, as follows from the Karman-Howarth equation, the relative exponents of S_p vs S_3 should equal those measured directly in terms of separation r. Whenever the turbulence is non-homogeneous and anisotropic, no general link between scaling in terms of separation and ESS exponents should be expected. Moreover, in wall-bounded flows, the statistical features of the flow, e.g. the structure functions in particular, will strongly depend on the distance from the wall.

In the present analysis, we restrict our attention to a single measurement point in the logarithmic region ($y^+ = 70$), which has been selected to illustrate the two basic mechanisms which characterize near-wall turbulence. As a general result, our experiments (Jacob et *al.* 2001) indicate that two distinct and coexisting scaling regimes characterize the wall region, confirming a previous analysis by Ciliberto et *al.* (2000). Depending on the distance from the wall or, more precisely, on the magnitude of the mean-shear, the relative extension of the two ranges varies.

Typical results obtained at $y^+ = 70$ are shown in figure 1, where the logarithm of S_6 is plotted versus the logarithm of S_3. Three distinct regions of nearly constant slope are observed. The "trivial" power-law behavior which characterizes the dissipative region ($\zeta_6 = 2$) is not shown in the figure. For larger separations a second interval emerges, and a least-square fit yields $\zeta_6 \simeq 1.78$, an estimation which falls in the range of values measured in homogeneous and isotropic conditions (Benzi et *al.* 1995). The values for the other exponents ($\zeta_5 \simeq 1.54$, $\zeta_4 \simeq 1.28$) confirm this finding.

At even larger scales, another range separates out, where a power-law with a quite different exponent ($\zeta_6 \simeq 1.54$) establishes. This value is actually very close to those measured in other conditions of very strong shear (Toschi et *al.* 2000, and references therein).

The abrupt transition which occurs between these two scalings can be better appreciated by looking at figure 2 which shows the local slope of the previous ESS plot. Such a representation evidences also the extension of the plateaux corresponding to the two different scalings. This transition is actually related to the behavior observed in near-wall velocity spectra (Hinze 1959, Townsend 1976): an intermediate region appears between the low wavenumber energy containing range and the inertial range (where the spectrum follows the $-5/3$ law). This intermediate region is characterized by a spectral decay as k^{-1}, and extends over a range of scales $(1/\delta) \leq k \leq (1/y)$, where δ is the boundary layer thickness and y is the distance from the plate. In our data, a too small separation of scales occurs to display this behavior directly in terms of wavenumber

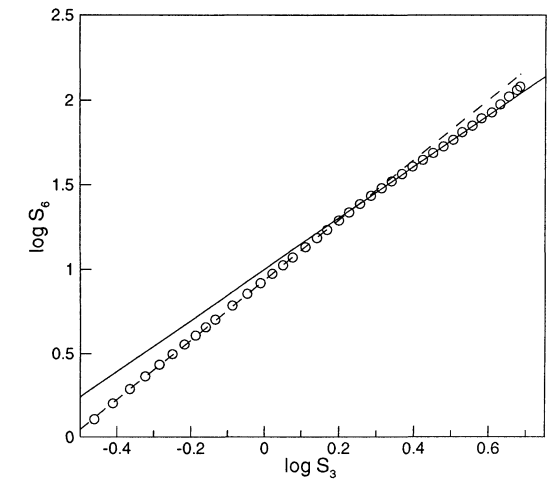

Figure 1: ESS: log$< \delta V^6 >$ versus log$< \delta V^3 >$ at $y^+ = 70$ (open circles). The slopes of the dashed and solid lines are 1.78 and 1.54 respectively.

in the energy spectrum. In terms of ESS the crossover between two rather different regions is instead very clear. Actually, this crossover is associated with a further length scale, introduced by the mean shear in addition to the two scales (dissipation and integral scale) which are typical of homogeneous isotropic turbulence.

The shear scale L_s may be estimated by equating the magnitude δU_S of the velocity fluctuations induced at scale r by the mean shear (proportional to Sr) and the fluctuations δU_ϵ associated to the classical energy cascade (proportional to $(\epsilon r)^{1/3}$). Hence, L_s follows as $L_s \propto (\epsilon/S^3)^{1/2}$. At scales $\eta \ll r \ll L_s$ (where η is the Kolmogorov scale) the influence of the mean shear is negligible. Vice-versa, at larger separations, the statistical properties of turbulence are strongly affected by the presence of the shear. Specifically, in the logarithmic region, the balance between energy dissipation and production leads to $L_s \propto ky$, hence the crossover between the two behaviors occurs at a wavenumber $k \propto y$. Clearly the exponents observed in the isotropic-like scaling region are expected to be explained in terms of the Refined Kolmogorov Similarity Hypothesis, in the form of equation (5) below. Under this respect, the values of the exponents we find are in complete agreement with those measured in isotropic conditions, suggesting that the anomalous correction to the dimensional exponent is still given in terms of the statistical properties of the dissipation field. The origin of the shear dominated-range is a priori less clear. We investigate this point in more detail in the next section.

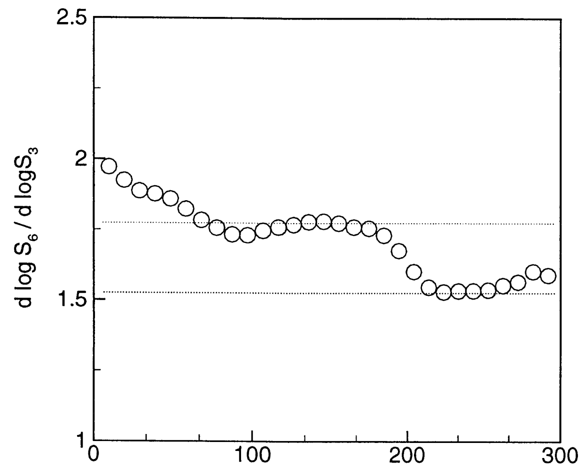

Figure 2: $d(log < \delta V^6 >)/d(log < \delta V^3 >)$ as a function of separation at $y^+ = 70$. Indicated are also the fits in the two scaling regions.

SIMILARITY LAWS

In order to gain insight in the physical nature of the observed ESS-scaling, let us consider the classical RKSH in its extended form

$$< S_p > \propto < \epsilon_r^{p/3} > < S_3 >^{p/3} . \qquad (5)$$

Alternatively, to try to avoid a direct use of the measured dissipation field, we can rewrite equation (5) in the form:

$$\frac{S_p(r)}{S_3(r)^{p/3}} \propto < \epsilon_r^{p/3} > . \qquad (6)$$

A plot of the left hand side of this equation is given in figure 3. The two distinct regions of scaling already observed in figure 1 are also appreciated here. Their estimated slopes are $s_1 \simeq -0.23$ and $s_2 \simeq -0.56$ for the range at small and large separations respectively. The validity of equation (6) in both ranges would imply a radical change in the statistical properties of the dissipation field at the crossover between the two regimes. This particular behavior is very difficult to be explained in physical terms. A more reasonable explanation is provided by the failure of the classical refined similarity law in the range where the shear is particularly relevant, i.e. for separations larger than the shear scale L_s.

By conjecturing a trend towards the recovery of an isotropic-like behavior at small scales we should expect equation (5) to correctly predict the dissipation in the range $\eta \ll r \ll L_s$. The value $s_1 = -0.23$ corresponds to the scaling behavior of the dissipation with respect to S_3. This figure is very close to that familiar to most people working in the field of scaling laws for homogeneous isotropic turbulence. Within experimental accuracy, it reproduces the scaling of $< \epsilon_r^2 >$ with respect to r, i.e. in the ESS context, with respect to S_3, which is given by $s_1 = -0.22$. Hence the present results are consistent, at small scales, with the classical form of refined similarity, and suggest that the behavior of the dissipation field is essentially the same as that of homogeneous isotropic turbulence.

Moreover, the considerations we have presented provide a strong feeling that at separations larger than L_S the classical form of RKSH should fail. In particular, if we assume a smooth behavior of the dissipation at the crossover between the two regions, the scaling exponent of $< \epsilon_r^q >$ with respect to S_3 should remain rather constant or, at least, should change smoothly. As we will see, this conjecture is consistent with the existence of

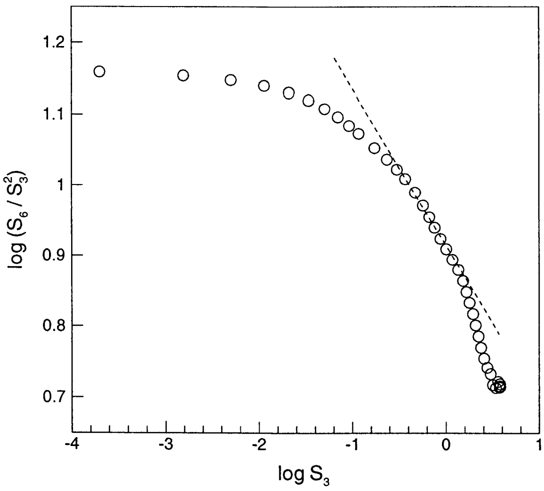

Figure 3: $\log{(S_6/S_3^2)}$ versus $\log S_3$. The dotted line gives the slope of $< \epsilon_r^2 >$ via the She-Lévêque model.

a new form of RKSH, originally proposed in Benzi et *al.* (1999). Actually, in the shear-dominated range, a dynamics quite different from the pure energy cascade takes place, since the energy balance is now dominated by the production term, dimensionally proportional to $S < \delta V^2 >$.

This circumstance has suggested a new form of similarity in terms of S_2 to replace the RSKH in the range of scales where the production term overwhelms the energy transfer term i.e. for $L_s \ll r \ll l_d$, where l_d is the integral scale:

$$< S_p > \propto < \epsilon_r^{p/2} > < S_2 >^{p/2} \ . \qquad (7)$$

We note explicitly that the k^{-1} behavior is entirely consistent with this new form of similarity, and that the estimate of the transition scale agrees with the classical results of wall bounded flows. Once again, the (measured) combination of structure functions $S_p/{S_2}^{p/2}$ can be isolated from the moments of the dissipation field. This quantity is plotted in figure 4 for $p = 6$. We obtain an estimated slope of $s_1 \simeq -0.35$ and $s_2 \simeq -0.61$ in the two scaling regions. In particular, the slope in the shear-dominated region is entirely consistent with the exponent of $< \epsilon_r^3 >$ with respect to S_3, as known from homogeneous isotropic turbulence. Actually, the dotted line in figure 4 has a slope $s \simeq -0.592$ given by the the value, for $q = 3$, of the expression

$$\tau_{sl}(q) \ = \ -\frac{2}{3}\, q \ + \ 2\ (1 - (2/3)^q\,) \qquad (8)$$

provided by the She-Lévêque model for homogeneous isotropic turbulence.

These results confirm that two quite distinct behaviors arise in the near-wall region: in the range of scales $\eta \ll r \ll L_s$ the inertial dynamics typical of homogeneous isotropic turbulence takes place, and the classical RKSH holds. In the range $L_s \ll r \ll L_d$ the dynamics is controlled by the energy production, and the features of turbulence can be described by the new form of similarity. The extension of these two ranges is controlled by the shear scale. When this latter falls halfway between the Kolmogorov scale and the integral scale, as in the present case, two distinct regions can be clearly observed. More precisely, the position of the shear scale can be estimated by two dimensionless parameters, namely S^* and S_c^*, defined respectively as :

$$S^* = \frac{S\, u_{rms}^2}{< \epsilon >} \propto (\frac{l_d}{L_s})^{2/3} \qquad (9)$$

and

$$S_c^* = S\,(\nu / < \epsilon >)^{1/2} \propto (\frac{\eta}{L_s})^{2/3} \qquad (10)$$

Moving towards the plate, the shear scale diminishes, and the extent of the classical inertial range reduces. Finally, in the buffer, the shear scale almost collapses on the dissipation scale, and the shear-dominated behavior extends over the entire range of scales.

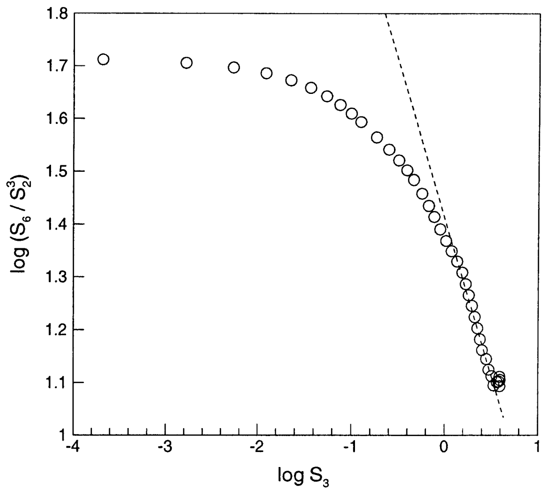

Figure 4: $\log{(S_6/S_2^3)}$ versus $\log S_3$. The dotted line gives the slope of $< \epsilon_r^3 >$ via the She-Lévêque model.

FINAL REMARKS

We have shown that two coexisting scaling behaviors characterize the log-region of a turbulent boundary layer. The associated power-laws extracted in terms of ESS are entirely consistent with the classical form of RKSH in the inertially dominated range. Above the shear scale, the exponents change radically. The new form of similarity law given in Benzi et *al.* (1999) has been shown able to fit the experimental data. The use of the two scaling laws has allowed for an estimation of the moments of the dissipation field in the two ranges in terms of the velocity structure functions. The values of the exponents we find strongly suggest that the statistical properties of the dissipation are not substantially altered with respect to homogeneous and isotropic turbulence. At present, this is only a strongly supported conjecture, which has been confirmed by DNS of wall turbulence and homogeneous shear flows (Gualtieri et *al.* 2000, Benzi et *al.* 1999). A direct experimental verification needs the availability of all the components of the instantaneous velocity gradient. By using a one-dimensional surrogate, we are able to attempt a preliminary evaluation of the dissipation, which is found in surprisingly good agreement with the above conclusions. This can be appreciated in figure 5, which gives the combination of structure functions

$$\frac{S_p(r)}{S_\alpha(r)^{p/\alpha}} \qquad \alpha = 3, 2, \qquad (11)$$

together with the moments of the dissipation $< \epsilon_r^2 >$ and $< \epsilon_r^3 >$ as estimated from the one-dimensional surrogate.

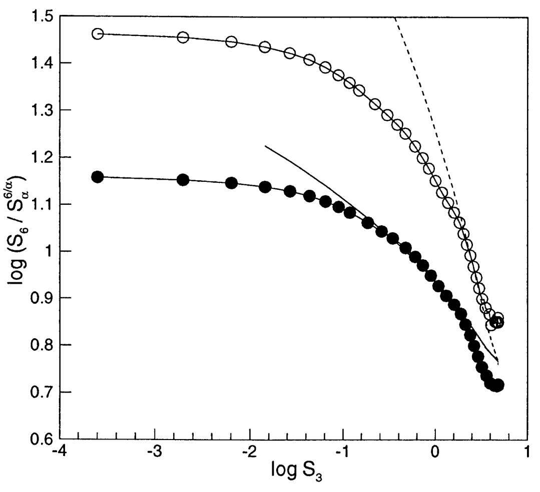

Figure 5: $\log(S_6/S_\alpha^{6/\alpha})$ versus $\log S_3$. Filled symbols: $\alpha = 3$, open symbols: $\alpha = 2$. The solid line represents the experimental evaluation of $< \epsilon_r^2 >$ via its one-dimensional surrogate, while the dotted lines represents the experimental evaluation of $< \epsilon_r^3 >$ via its one-dimensional surrogate.

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