LARGE-DOMAIN SIMULATIONS OF PLANE COUETTE AND POISEUILLE FLOW

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ABSTRACT

Direct numerical simulations of turbulent Poiseuille and Couette flow have been carried out in very large computational domains at different Reynolds numbers. Two-point correlations of velocities, and the pre-multiplied energy spectra are used to study the large scale structures. The spanwise scale of large structures in plane Couette flow is constant in the channel central region. For Poiseuille flow, it is variable, but is found to collapse under a particular power law outer scaling. The pre-multiplied energy spectra clearly distinguish between peaks that scale on inner (wall) variables, and peaks that scale with channel variables. Couette flow is found to contain two peaks that are more pronounced than in Poiseuille flow. A secondary peak in the streamwise spectra varies in inner variables, while a secondary peak in the spanwise spectra varies in outer variables.

INTRODUCTION

With the availablity of direct numerical simulation (DNS) databases, turbulence structures in wall bounded flow have attracted much attention. Small scale turbulent structures have been studied extensively, but the large scale structures, although very important, especially when sound radiation is concerned, are far from understood.

Jimenez (1998) studied the large scale structure in a plane Poiseuille flow by picking up peaks in a pre-multiplied energy spectrum using DNS data of Kim *et al* (1987) and later simulations. It was found that the spanwise peak positions of the pre-multiplied spectra grow approximately linearly with wall distance beyond $z^+ = 50$. The results, especially near the channel centre, showed considerable scatter from the linear profile, caused by the relatively small computational box with subsequent worsening of wavelength resolution at low wavenumbers.

The large scale structure in plane Couette flow is known to be different from Poiseuille flow (Komminaho *et al*, 1996 and others). Extremely long streamwise structures are found in the core region of plane Couette flow, which are not found in Poiseuille flow.

In the present study, direct numerical simulations of plane Poiseuille and Couette flow are performed in very large computational boxes. The scale of large structures is studied by analysis of two-point correlation functions and pre-multiplied energy spectra.

DNS METHOD

The governing equations of incompressible turbulent flow, the continuity and the momentum equation, are non-dimensionalized with the reference length L^*_{ref} chosen as the channel half width h^* and the reference velocity U^*_{ref} set as the friction velocity u^*_τ for Poiseuille flow, and the wall velocity u^*_w for Couette flow (both the upper and the lower wall move, with velocity u^*_w and $-u^*_w$ respectively).

$$\frac{\partial u_j}{\partial x_j} = 0 \qquad (1)$$

$$\frac{\partial u_i}{\partial t} = \epsilon_{ijk} u_j \omega_k + \delta_{1i} P_i - \frac{\partial q}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} \qquad (2)$$

where ω_i is the vorticity, $\omega_i = \epsilon_{ijk} \partial u_k / \partial x_j$; Reynolds number is $Re = U^*_{ref} h^* / \nu^*$, equal to $Re_\tau = u^*_\tau h^* / \nu^*$ for Poiseuille flow, and $Re_w = u^*_w h^* / \nu^*$ for Couette flow; ν^* is the kinematic viscosity of the fluid; $q = p + u_i u_i / 2$ is the non-dimensional modified pressure, and P_i is the driving body force, equal to 1 for Poiseuille flow and zero for Couette flow.

Fourier discretization is used for the two periodic directions, streamwise and spanwise, while Chebyshev discretization is applied to the wall-normal direction. Time advance

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is achieved with a third-order Runge-Kutta method for the convective term and the Crank-Nicolson method for the pressure and viscous terms. An implicit treatment is employed to avoid extremely small time steps in the near wall region owing to Chebyshev discretization. The '3/2 rule' de-aliasing has been applied whenever nonlinear quantities are calculated. The parallel implementation of Sandham and Howard (1998) is employed. All simulations start with an approximate mean turbulent flow with super-imposed artificial disturbances.

RESULTS

Simulations of plane Poiseuille and Couette flow were performed in very large computational domains at different Reynolds numbers, details of each simulation being shown in table 1. All the simulations were carried out on a Cray T3E super-computer. Statistical data are accumulated only after the influence of artificial conditions has disappeared and the flow has statistically settled down. The convergence is checked by comparing the statistical data in successive time segments, making sure that they are consistent. Simulations were then run over a long enough time to give good statistics. For example, case A was run over 100 non-dimensional time units with 64 processors, costing 16,000 PE hours.

Case	Type	Re	grid pts	L_x	L_z
A	Poiseuille	180	$256 \times 121 \times 256$	24	12
B	Poiseuille	360	$256 \times 161 \times 256$	12	6
C	Couette	1300	$1024 \times 81 \times 512$	192	48
D	Couette	3400	$512 \times 121 \times 256$	48	12

Table 1: Computational grids and box sizes.

Figures 1 and 2 show the two-point correlations at a wall-normal position near the maximum of kinetic energy for all cases. The box sizes of case A, B and C are large enough to get zero correlations at maximum separation, half the box length, in both streamwise and spanwise directions. This demonstrates that the present simulation domains are large enough to ensure zero two-point correlations, a condition that has not been achieved by previous Couette flow simulations. The box size of case D ensures zero spanwise two-point correlations, but leaves a streamwise two-point correlation residual of 0.0585 at the worst position (the channel centreline).

Statistics for Poiseuille flow case A have been compared with Kim *et al* (1987), with good agreement, and case B ($Re_\tau = 360$) with Moser *et al* (1999) for $Re_\tau = 395$, giving similarly good comparisons. Couette flow case C has been compared with Kristoffersen *et al* (1993), with good agreement for mean quantities. Differences between the present results and Kristoffersen *et al* (1993) for higher moments are caused by their smaller box size and coarser resolution. Further details are given in Hu *et al* (2001). Kinetic energy budgets have been computed for all simulations, and the energy budget error is typically about three orders of magnitude smaller than the maximum of production, as shown in figure 3 for Poiseuille flow case B and Couette flow case D.

Plane Couette flow is driven by the two walls moving in opposite directions, and has a typical S-shaped mean velocity profile, leading to a non-zero mean velocity gradient at the centreline. The non-dimensional value of this is 0.1924 for $Re_w = 1300$ and 0.1980 for $Re_w = 3400$ from the present simulations. Tillmark *et al* (1993) collected available experimental data and found that the non-dimensional mean velocity gradient at the centreline varies between 0.15 and 0.3 for $Re_w = 750 \sim 19000$. DNS of Komminaho *et al* (1996) for Reynolds number $Re_w = 750$ gave a value of 0.18. This mean velocity gradient gives Couette flow a finite shear stress at centreline, which leads to non-zero production as well as dissipation at the channel centreline (figure 3(b)). This is in contrast to Poiseuille flow where the production drops to zero at the centreline, and the shear stress varies linearly with distance.

In plane channel flows, the spanwise two-point correlation of streamwise velocity decreases with separation and then reaches a negative peak, the distance to which corresponds to half the dominant scale in the spanwise direction. The correlation of the spanwise velocity has the same peak position as the streamwise velocity, while the peaks for wall-normal velocity have the same trend but with half the separation distance.

The negative peak separations (half the characteristic spanwise wavelength) are plotted against wall-normal distance in figures 4 and 5 for Poiseuille and Couette flow, respectively. Peaks from another simulation of Poiseuille flow at $Re_\tau = 135$ are also plotted. In both flows, data from different Reynolds number simulations collapse towards $z^+ = 50$ for $y^+ < 15$ (figure 4(a) and 5(a)), consistent with the typical statement that the near wall structures have a spanwise length of 100 wall units. For Poiseuille flow, the peak separation then increases with wall-normal distance. For the low Reynolds number cases, a linear growth might be concluded (as in Jimenez, 1998), but this is obviously not true for higher Reynolds number. A collapse of the data is possible if a new non-dimensional peak separation, $z_{peak} = z^*_{peak}(1 - u^*/u^*_{max})^a/h^*$, is employed, as is plotted in figure 4(b). A best fit of the discrete points gives $a = 1.3$ and

$$z_{peak} = 0.1y^2 \quad (3)$$

The data collapse for $y^2 < 0.5$ ($y < 0.7$).

A sudden increase occurs near $y^+ = 15$ (more apparent for higher Reynolds number) for Couette flow, suggesting an overlap of the near-wall streaks and outer structures. The separation distance then increases with wall-normal distance before flattening out above $y = 0.5$. Outer scaling (figure 5(b)) shows that the peak positions appear with a constant spanwise separation of $1.6h^*$ for $Re_w = 1300$ and $1.73h^*$ for $Re_w = 3400$, implying a spanwise scale of $3.2h^*$ and $3.46h^*$. A corresponding negative peak is found at about $1.83h^*$ in Komminaho *et al* (1996) (their figure 10)for the centreline spanwise two-point correlation. This is confirmed by the similarity of contour plots of the fluctuating streamwise velocity in the channel central region, as shown in figure 6 for Couette flow case D.

Pre-multiplied energy spectra $\phi = k \times E(k)$ are shown in figure 8 for Poiseuille flow. Peaks are found at the same positions in wall units, for different Reynolds numbers, at $\lambda_z^+ = 108$ and $\lambda_x^+ = 1080$ for near-wall positions $y^+ < 15$. Further away from the wall, the peak wavelengths increase with wall distance. Another peak is found at $\lambda_z^+ = 400 \sim 500$ at the higher Reynolds number. Pre-multiplied energy spectra of Couette flow are shown in figure 9. The peaks at lowest λ appear at $\lambda_z^+ = 118$ and $\lambda_x^+ = 890$, which is consistent with the near wall structures of Poiseuille flow. A peak in the spanwise pre-multiplied spectra appears at $\lambda_z = 4$ for both Reynolds number cases, which is comparable with the results from the two-point correlation functions. In the streamwise direction, a secondary peak appears at $\lambda_x = 38.4$ for $Re_w = 1300$ and $\lambda_x = 16$ for $Re_w = 3400$, which correspond to $\lambda_x^+ = 3150$ and $\lambda_x^+ = 2990$. This suggests the large structure scales in wall units in the streamwise direction and in outer variables in the spanwise direction. The streamwise pre-multiplied energy spectra have a trend to become flatter at the higher Reynolds number, between $\lambda_x^+ = 1000 \sim 3000$, away from the wall. The energy spectra of the high Reynolds number Couette flow are shown in figure 7 for different wall-normal positions, with a -1 slope for comparison. No clear evidence for k^{-1} region is found from these simulations.

SUMMARY

Direct numerical simulations of turbulent plane Poiseuille and Couette flow have been carried out at different Reynolds numbers in very large computational domains. Statistics have been compared with available data, and energy budgets give satisfactory balances. The spanwise scale of large structures in Poiseuille flow collapses with a new outer scaling. The spanwise large structures of Couette flow scale with channel width and have a constant value in the central region. However in the streamwise direction, the large structures scale with wall units with λ_x^+ about 3000.

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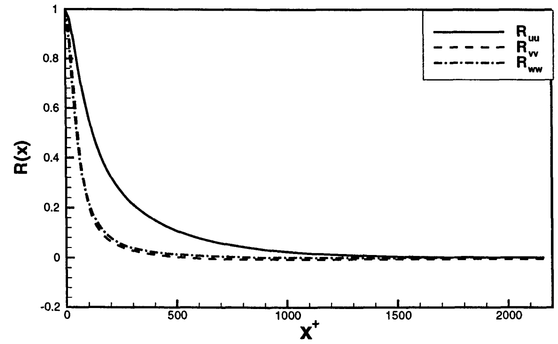
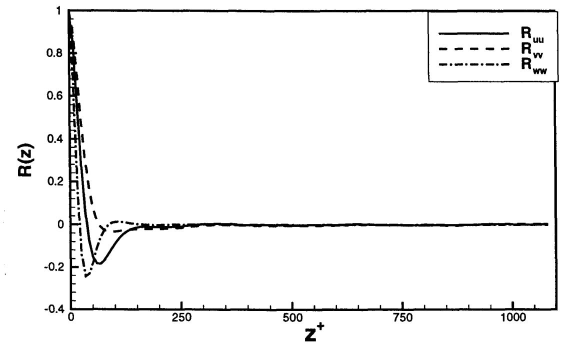


(a) Streamwise ($Re_\tau = 180$)



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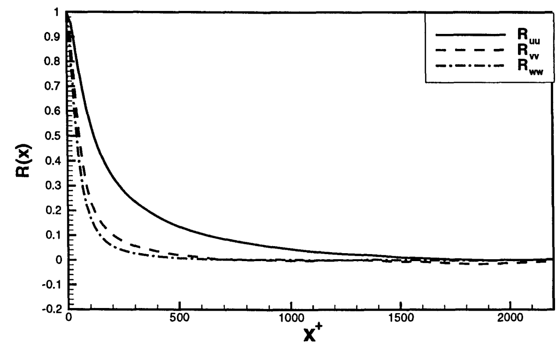

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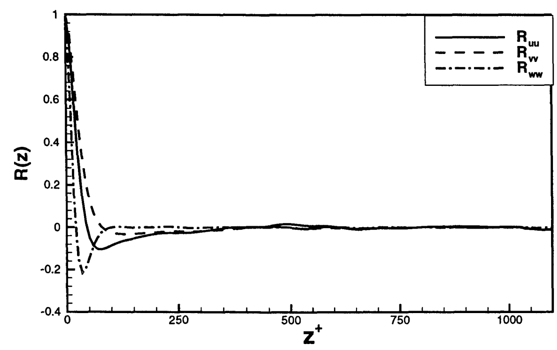
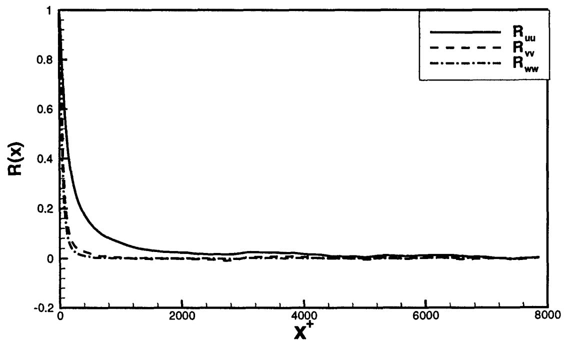


(d) Spanwise ($Re_\tau = 360$)

Figure 1: Two-point correlations of Poiseuille flow.



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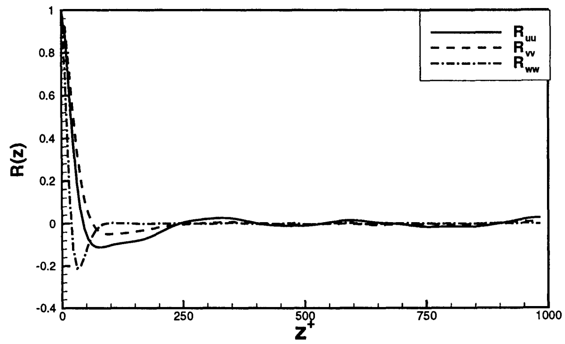

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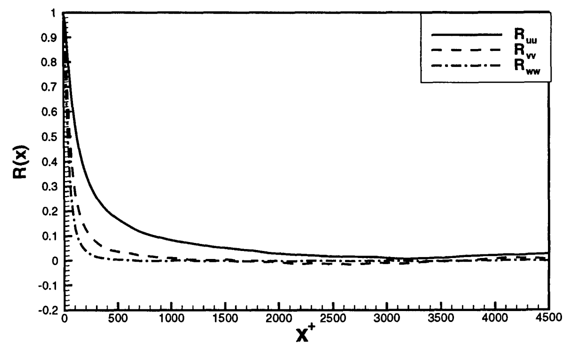
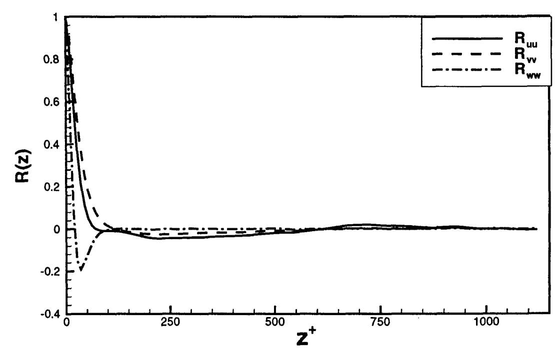
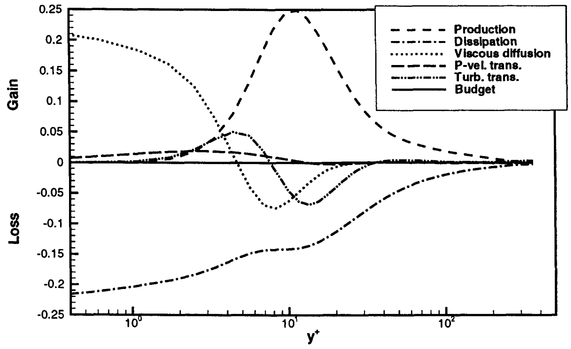


(c) Streamwise ($Re_w = 3400$)

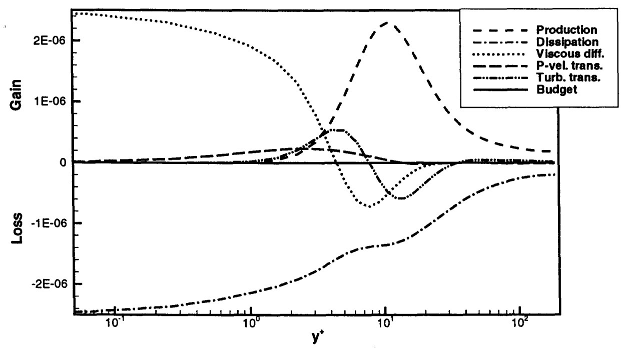


(d) Spanwise ($Re_w = 3400$)

Figure 2: Two-point correlations of Couette flow.

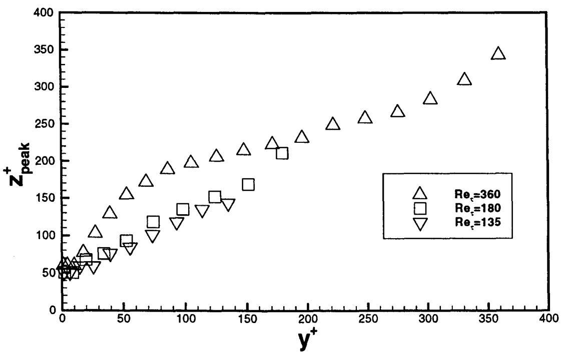


(c) Poiseuille flow, $Re_{\tau} = 360$

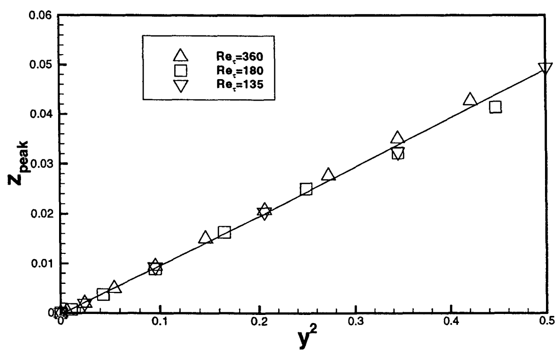


(b) Couette flow, $Re_w = 3400$

Figure 3: Energy budget of turbulent kinetic turbulent energy.

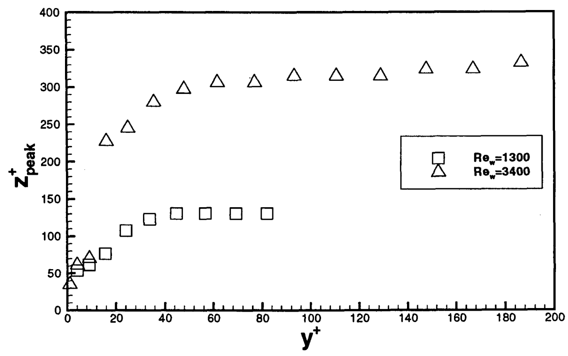


(a) Inner scale

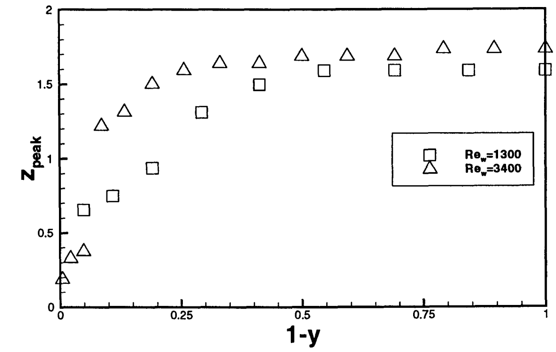


(b) Outer scale

Figure 4: Peaks of spanwise two-point correlations of Poiseuille flow.



(a) Inner Scale



(b) Outer scale

Figure 5: Peaks of spanwise two-point correlations of Couette flow.

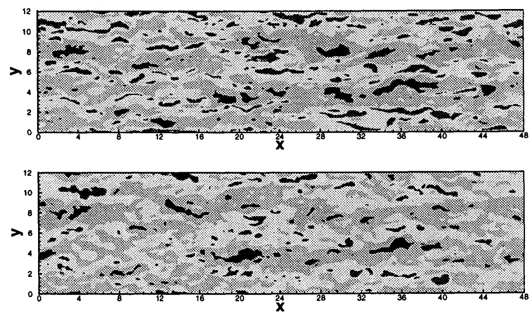


Figure 6: Couette flow ($Re_w = 3400$) fluctuation streamwise velocity contour at $y = 0.5$ (upper) and channel centreline ($y = 0$, lower), contour lever $u' = \pm 0.2$.

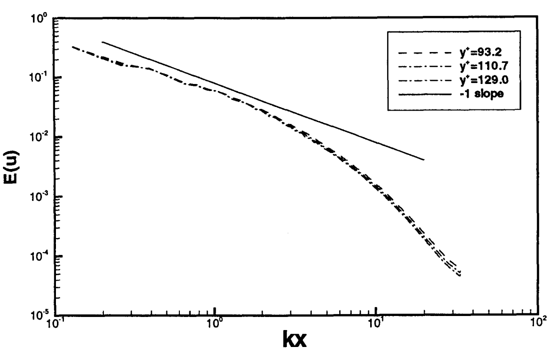
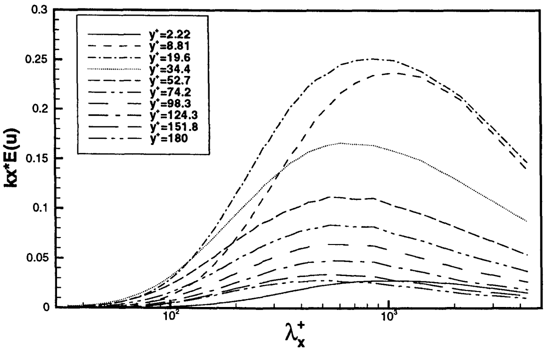
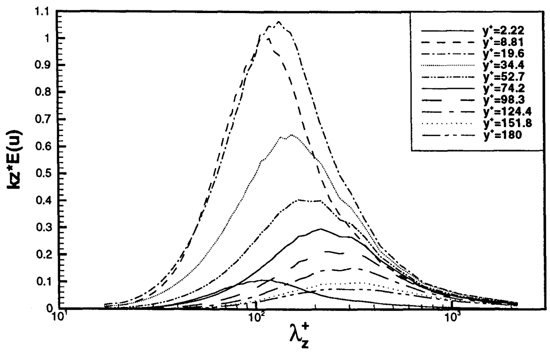


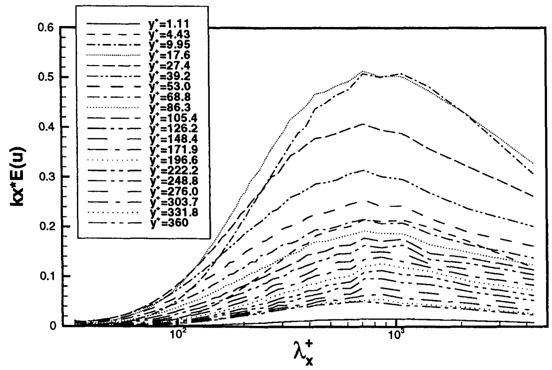
Figure 7: Energy spectra of Couette flow ($Re_w = 3400$)



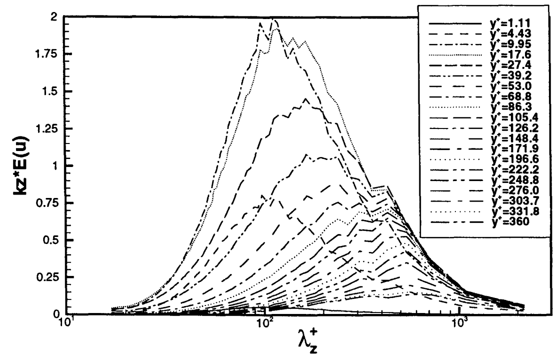
(a) Streamwise ($Re_\tau = 180$)



(b) Spanwise ($Re_\tau = 180$)

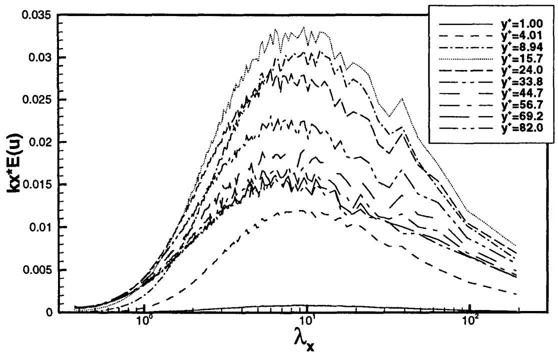


(c) Streamwise ($Re_\tau = 360$)



(d) Spanwise ($Re_\tau = 360$)

Figure 8: Pre-multiplied energy spectra of Poiseuille flow.



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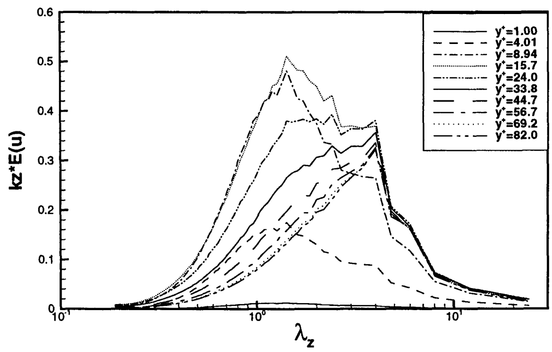

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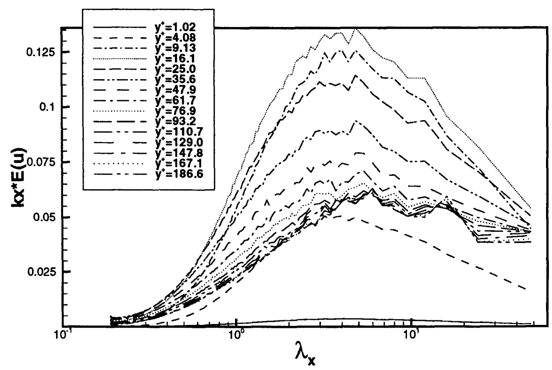
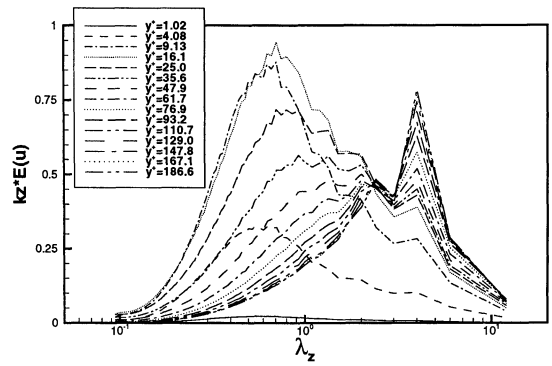


(c) Streamwise ($Re_w = 3400$)



(d) Spanwise ($Re_w = 3400$)

Figure 9: Pre-multiplied energy spectra of Couette flow