LES AND UNSTEADY RANS OF BOUNDARY-LAYER TRANSITION INDUCED BY PERIODICALLY PASSING WAKES

Frank Ham and Fue-Sang Lien
Department of Mechanical Engineering, University of Waterloo
Waterloo, Ontario, N2L 3G1, Canada
corresponding author: fslien@sunwise.uwaterloo.ca

ABSTRACT

Results from 3-dimensional Large-Eddy Simulation (LES) and 2-dimensional unsteady Reynolds-Averaged Navier Stokes (RANS) simulation of a spatially-evolving flat-plate boundary-layer undergoing transition induced by periodically passing wakes are presented and compared.

The LES simulations used a novel kinetic-energy conserving finite-volume discretization of the incompressible Navier-Stokes equations and the standard dynamic Smagorinsky subgrid-scale model. When compared to the direct numerical simulation (DNS) of Wu *et al.* (1999), the LES was able to correctly predict the onset of transition. Inspection of the instantaneous flow field in the transition region confirmed that intermittent turbulent spots were being distinctly resolved. A close inspection of the fluctuating velocities near the top of the boundary layer just prior to spot formation confirmed the presence of the "backward jet" inflectional velocity profile proposed by Wu *et al.* and Jacobs and Durbin (2000) as the precursor to turbulent spot formation, suggesting that the LES is actually capturing the bypass transition mechanism, at least in these initial stages. The transition length predicted by LES, however, was consistently shorter than the DNS result.

Unsteady RANS simulations were based on the STREAM code of Lien and Leschziner (1994), with the $v^2 - f$ turbulence model of Lien and Durbin (1996). RANS was able to correctly predict the onset of transition. The transition length predicted by RANS was also in agreement with the DNS, however the overshoot of average skin friction relative to the flat plate correlation (seen in both the DNS and present LES) was not observed.

INTRODUCTION

In orderly transition to turbulence, small disturbances in the laminar boundary layer lead to 2-dimensional Tollmien-Schlichting waves that are amplified through various stages leading eventually to a fully turbulent boundary layer. This relatively slow transition process has been extensively studied in relation to flow over aircraft wings, where the free-stream turbulence levels are generally low (Mayle 1991).

In the presence of disturbances external to the boundary layer, however, it is observed experimentally that transition can occur rapidly, "bypassing" the orderly route. Bypass transition is the dominant mode of transition in many turbomachinery applications, where free-stream turbulence intensities are usually well above the threshold level of about 0.5% (Yang *et al.* 1994). Furthermore, when the dominant free-stream disturbances are periodic in time, such as the periodically passing wakes generated by an upstream row of rotors or stators in a turbine cascade, the transition can also become periodic. In this case, the transition is referred to as "wake-induced" (Mayle 1991), but still fits under the broader classification of bypass transition (figure 1a).

In an effort to remove some of the geometric and physical complexity associated with the turbine cascade, Liu and Rodi (1991) experimentally investigated the wake-induced transition of a flat-plate boundary layer (figure 1b). In their experiments, periodic wakes were generated by a series of cylinders mounted on a rotating squirrel cage upstream of the flat plate. In the absence of the periodic wakes, the relatively low Reynolds number and low free-stream turbulence intensity of 0.3% resulted in a laminar boundary layer over the full length of the test plate. With the wakes, they found that transition first occurred in isolated stripes underneath the disturbed free-stream. The stripes traveled downstream and grew together to eventually form a fully turbulent boundary layer. They also found that the streamwise location of this merger moved upstream with increased wake-passing frequency.

The recent DNS of Wu *et al.* (1999) was designed following the experiment of Liu and Rodi (1991), and provided new insights into the mechanisms of bypass transition through a detailed analysis of the calculated flow fields (figure 1c). Wu *et al.* found that the transition to turbulence first occurred in isolated spots, which broaden and convect downstream where they eventually merged with the fully turbulent boundary layer. Analysis of the instantaneous flow field identified long backward jets contained in the fluctuating streamwise velocity field as precursors to turbulent spot formation. They proposed that the backward jets, located near the top of the boundary layer, were associated with a Kelvin-Helmholtz-like inflectional instability that interacts with the free-stream eddies, eventually leading to turbulent spot formation. More recent simulations of bypass transition under free-stream turbulence (Jacobs and Durbin 2000) also identified backward jets as consistent precursors to turbulent spot formation.

At 52 million and 71 million grid points respectively, the aforementioned transition simulations are some of the largest and most finely resolved ever reported. Interestingly, the bypass transition mechanism they uncovered - the backward jet - is actually a relatively large structure, spanning 100's of wall units in the streamwise direction, and about 60 wall units in the spanwise direction. This suggests that a coarser and significantly less expensive LES might be able to capture the bypass transition mechanism.

Yang *et al.* (1994) used LES to study bypass transition of a flat plate boundary layer subject to 5% freestream turbulence intensity. These simulations used the Smagorinsky subgrid scale model modified in an *ad hoc* manner to prevent excessive dissipation in the laminar portion of the boundary layer. They reported good agreement with available experimental data such as the average skin friction coefficient and shape factor, but did not report the resolution of turbulent spots or their precursors. In a more recent LES of natural transition, Huai *et al.* (1997) used the dynamic procedure (Germano *et al.* 1991) to avoid these *ad hoc* modifications. They reported that a localized version (Piomelli and Liu 1995) of the dynamic model gave accurate results both in a statistical sense and in terms of predicting the dynamics of the energy-carrying eddies.

In the present contribution, LES is used to study the wake-induced transition of a flat-plate boundary layer, with specific emphasis on the resolution of turbulent spots and their precursors. As a complementary effort, the same problem is simulated using 2-dimensional unsteady RANS. These two results, along with the DNS of Wu *et al.* (1999) represent a truly integrated analysis of this transitional flow, and provide a unique opportunity to assess the relative benefits and drawbacks of the various simulation technologies.

NUMERICAL METHOD

The numerical method used in the present work was based on a second-order accurate discretization of the spatially filtered incompressible Navier-Stokes equations that discretely conserves mass, momentum, and kinetic energy (in the inviscid limit) in both space and time. The details of the discretization are available elsewhere in these proceedings (Ham *et al.* 2001), and so are not repeated here. The sub-grid stresses were modeled using the standard dynamic Smagorinsky model (Germano *et al.* 1991, Lilly 1992) with averaging in the single homogeneous direction to avoid numerical instabilities.

PROBLEM DEFINITION

As in Wu *et al.* (1999), the present LES was designed following the experiment of Liu and Rodi (1991). Dimensions were scaled by the characteristic length scale, L, equal to the minimum distance from the upstream cylinders to the leading edge of the flat plate. Velocities were scaled by characteristic velocity scale U_{ref}, the freestream velocity in the absence of wakes. The problem Reynolds number was $Re = U_{ref}L/\nu = 1.5 \times 10^5$. The downward velocity of cylinders was $U_{cyl}/U_{ref} = 0.7$, and the passing wake period was $T = 1.67L/U_{ref}$, corresponding to case number 4 in the experiments of Liu and Rodi (1991).

Boundary Conditions

The application of boundary conditions followed the procedure described in Wu *et al.* (1999) with one exception: the precomputation of the self-similar plane wake used as the inlet condition was appropriately filtered for the coarser grid spacing of the present simulation.

Computational Domain

Figure 1c schematically illustrates the computational domain used in the present LES. In an effort to minimize the problem size, the domain selected was only a fraction of the DNS domain of Wu *et al.* (1999). In the streamwise direction, the domain was shortened to just include the transition, $0.1 < x/L < 1.75$. In the spanwise direction, the domain width was $0 < z/L < 0.1$, and in the wall-normal direction, $0 < y/L < 0.3$.

Grid Spacing and Time Step

The grid spacing requirements for accurate DNS of bypass transition have been well established through grid independence studies performed as a part of recent simulations. For the bypass transition simulations of Jacobs and Durbin (2000) the grid spacing (based on maximum friction velocity) was $\Delta x^+ = 11.7$, $\Delta z^+ = 6.0$. This is in agreement with the earlier recommendations of Rai and Moin (1993). The DNS of Wu *et al.* (1999) used a slightly coarser spacing of $\Delta x^+ = 24$, $\Delta z^+ = 11$ (based on friction velocity at $x = 3$).

The grid spacing requirements for accurate LES of bypass transition, however, are less well established. In the bypass transition LES of Yang *et al.* (1994), the grid spacing was $\Delta x^+ = 80$ and $\Delta z^+ = 14$. Based on the experience gained through the present research, we believe this streamwise spacing to be too coarse to resolve discrete turbulent spots. In the present work, the finest grid size used was $256 \times 64 \times 48$, which corresponded to a grid spacing based on maximum friction velocity of $\Delta x^+ = 45$ and $\Delta z^+ = 17$. In the wall-normal direction, spacing at the wall was $\Delta y^+ = 2$.

The computational time step was set constant at $\Delta t = 0.003L/U_{ref}$, which corresponded to a time step in wall units (based on maximum friction velocity) of $\Delta t^+ = \Delta t u_\tau^2/\nu = 1.3$.

Computational Details

The combination of reduced domain size, increased grid spacing, and increased computational time step yielded a reduction in problem size by a factor of about 160 relative to the DNS. Computations were carried out on a parallel PC cluster at the University of Waterloo. Simulations were typically run for 10 wake passing periods (about 5500 time steps), and required about 3 days using 32 nodes of the cluster.

RESULTS

Instantaneous Fields

The relatively small size of the present LES (about 700,000 grid points) afforded some experimentation with the grid spacing. The first simulations were performed on relatively coarse grids, with $\Delta x^+ = 95$ and $\Delta z^+ = 35$. Although transition was observed to occur at approximately the correct location (when compared to the DNS), inspection of the instantaneous velocity fields did not reveal isolated turbulent spots. Further, the spanwise resolution in these coarse simulations was certainly not capable of properly resolving the backward jet structures, which appear to have a width of about 60 wall units.

In the finest LES, however, distinct turbulent spots were clearly discernable. Figure 2 uses the fluctuating velocities in the wall normal direction at 5 equally spaced times to visualize the transition. The interaction between the passing wake and the laminar boundary layer appears as elongated puffs at $t/T = 0$, and breakdown to a turbulent spot does not occur until some time closer to $t/T = 0.4$. At $t/T = 0.4$, the isolated spot is clearly discernable with its characteristic arrowhead pointing upstream. As the turbulent spot is convected downstream, it grows and eventually merges with the fully turbulent portion of the boundary layer.

While qualitatively similar to the DNS result, the following important differences are noted:

(1) The front separating the laminar and fully turbulent portion of the boundary layer is much more irregular in the LES, with long fingers of turbulence reaching significantly upstream. These fingers thin in the spanwise direction, but do not dissipate or convect downstream with the rest of the boundary layer when not being fed by spots.

(2) In the LES, the streamwise location at which the turbulent spots merge with the fully turbulent portion of the boundary layer is in the range of $x = 1$ to 1.2. In the DNS, this location was further downstream at $x = 1.5$ to 1.7.

(3) Close inspection of the turbulent spot reveals the presence of $2 - \Delta$ fluctuations in the velocity field near the upstream edge,

indicating under-resolution in the streamwise direction particularly.

Average Quantities

To make our investigation of this transitional flow more comprehensive, unsteady RANS simulations were also performed. These simulations used the STREAM code of Lien and Leschziner (1994), and the $v^2 - f$ turbulence model of Lien and Durbin (1996). Computing time per simulation was approximately 2 hours on a desktop PC. This represents a reduction in computational effort of about 2 orders of magnitude compared to the LES.

Figure 3 compares the average skin friction calculated by the LES and RANS to the DNS of Wu *et al.* (1999). Both the LES and RANS correctly predict the onset of transition. In the case of the LES, however, the transition length is under-predicted. This is consistent with the observation that the turbulent spots merge with the fully turbulent boundary layer further upstream than in the DNS. There are several possible explanations for this, although the most compelling is that the dissipation rate given by the standard dynamic model is simply too low in the transition region, where the spanwise averaging used in calculating the model constant includes significant regions of laminar flow. This explanation suggests that another incarnation of the dynamic model might be more appropriate for modeling this type of transitional flow, even when a homogeneous direction is present, such as the dynamic localization model of Ghosal (1995), or the Lagrangian dynamic model of Meneveau (1996).

The transition length predicted by RANS agrees with the DNS result, although the overshoot of average skin friction relative to the flat plate correlation (seen in both the DNS and present LES) is not observed. No tuning of model constants for this particular flow was done.

CONCLUSIONS

LES and unsteady RANS simulations have been made of a spatially-evolving flat-plate boundary-layer undergoing transition induced by periodically passing wakes. The LES used a novel kinetic-energy conserving finite-volume discretization of the incompressible Navier-Stokes equations and the standard dynamic Smagorinsky subgrid-scale model. RANS simulations were based on the STREAM code of Lien and Leschziner (1994), with the $v^2 - f$ turbulence model of Lien and Durbin (1996).

Overall, RANS and LES both have benefits when applied to this flow. RANS predicts a slightly superior average skin friction coefficient, at enormously reduced computational cost, although the performance under more complex conditions (for example, flow in a turbine cascade involving complex geometry, higher freestream turbulence intensity, and pressure gradients) remains to be tested.

The LES was able to resolve both turbulent spots and their backward jet precursors, consistent with the recent DNS results of Wu *et al.* (1999) and Jacobs and Durbin (2000). The location of the onset of transition agreed with the DNS result; however the transition length was under-predicted. This discrepancy may be related to the spanwise averaging used in the calculation of the subgrid scale model coefficient, and other implementations of the dynamic model might give superior results. The potential improvement from these other models remains to be tested, and is the subject of ongoing research.

Acknowledgement

Most of this work was completed while the authors were in attendance at the Stanford Summer Program 2000. The authors a grateful to X. Wu, M. Wang, and P. Durbin of Stanford University for their assistance during that period.

References

Germano, M., Piomelli, U., Moin, P., & Cabot, W. H., 1991, "A dynamic subgrid-scale eddy viscosity model", *Phys. Fluids A*, **3**, no. 7, pp. 1760-1765.

Ghosal, S., Lund, T. S., Moin, P. & Askelvoll, K., 1995, "A dynamic localization model for large-eddy simulation of turbulent flows", *J. Fluid Mech.*, **286**, pp. 229-255.

Ham, F. E., Lien, F. S. & Strong, A. B., 2001, "A new method for direct numerical simulation that discretely conserves mass, momentum, and kinetic energy", *Proceedings of the Second International Symposium on Turbulence and Shear Flow Phenomena - 2001 TSFP-2, KTH, Stockholm, Sweden.*

Huai, X., Joslin, R. D. & Piomelli, U., 1997, "Large-eddy simulation of transition to turbulence in boundary layers", *Theoret. Comput. Fluid Dynamics*, **9**, pp. 149-163.

Jacobs, R. G., & Durbin, P. A., 2000, "Simulations of bypass transition", *J. Fluid Mech.*

to appear.

Lien, F. S. & Durbin, P. A., 1996, "Non-linear $k - \epsilon - v^2$ modelling with application to high lift", *Proceedings 1996 Summer Program, Center for Turbulence Research, Stanford University*, pp. 5-22.

Lien, F. S. & Leschziner, M. A., 1994, "A general non-orthogonal collocated FV algorithm for turbulent flow at all speeds incorporating second-moment closure", *Comp. Meth. Appl. Mech. Eng.*, **114**, pp. 123-167.

Lilly, D. K., 1992, "A proposed modification of the Germano subgrid scale closure method", *Phys. Fluids A*, **4**, no. 3, pp. 633-635.

Liu, X. & Rodi, W., 1991, "Experiments on transitional boundary layers with wake-induced unsteadiness", *J. Fluid Mech.*, **231**, pp. 229-256.

Mayle, R. E., 1991, "The role of laminar-turbulent transition in gas turbine engines", *ASME J. of Turbomachinery*, **113**, pp. 509-537.

Meneveau, C., Lund, T. S. & Cabot, W. H., 1996, "A Lagrangian dynamic sungrid-scale model of turbulence", *J. Fluid Mech.*, **319**, pp. 353-385.

Piomelli, U. & Liu, J., 1995, "Large eddy simulation of rotating channel flows using a localized dynamic model", *Phys. Fluids A*, **7**, p. 839.

Rai, M. M. & Moin, P., 1993, "Direct numerical simulation of transition and turbulence in a spatially evolving boundary layer", *J. Comput. Phys.*, **109**, pp. 169-192.

Wu, X., Jacobs, R. G., Hunt, J. C. R. & Durbin, P. A., 1999, "Simulation of boundary layer transition induced by periodically passing wakes", *J. Fluid Mech.*, **398**, pp. 109-154.

Yang, Z., Voke, P. R. & Savill, M. A., 1994, "Mechanisms and models of boundary layer receptivity deduced from large-eddy simulation of by-pass transition", Direct and Large-Eddy Simulation I, P. R. Voke *et al.* (eds.), Kluwer Academic, Dordrecht, pp. 225-236.

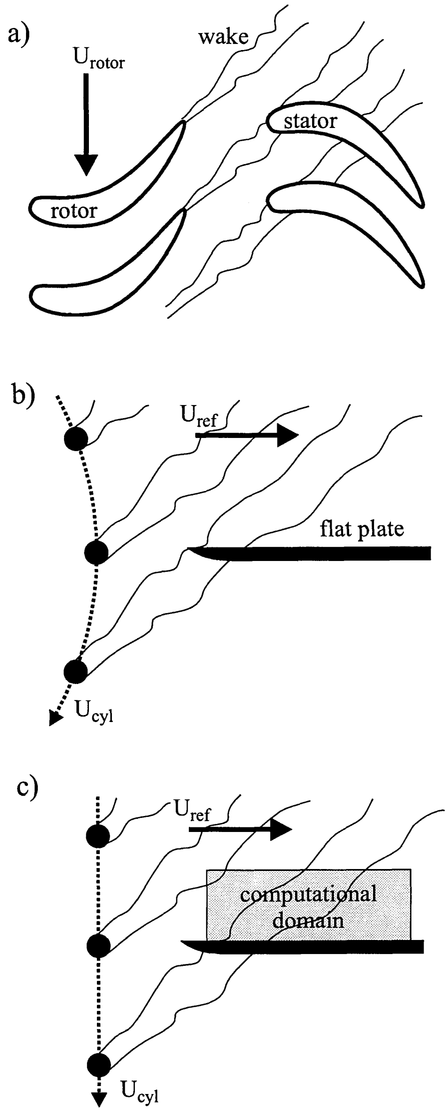

Figure 1: a) Skematic of rotor-stator wake interaction; b) layout in the experiments of Liu and Rodi (1991); c) layout in the DNS of Wu *et al.* (1999), and the present LES and RANS.

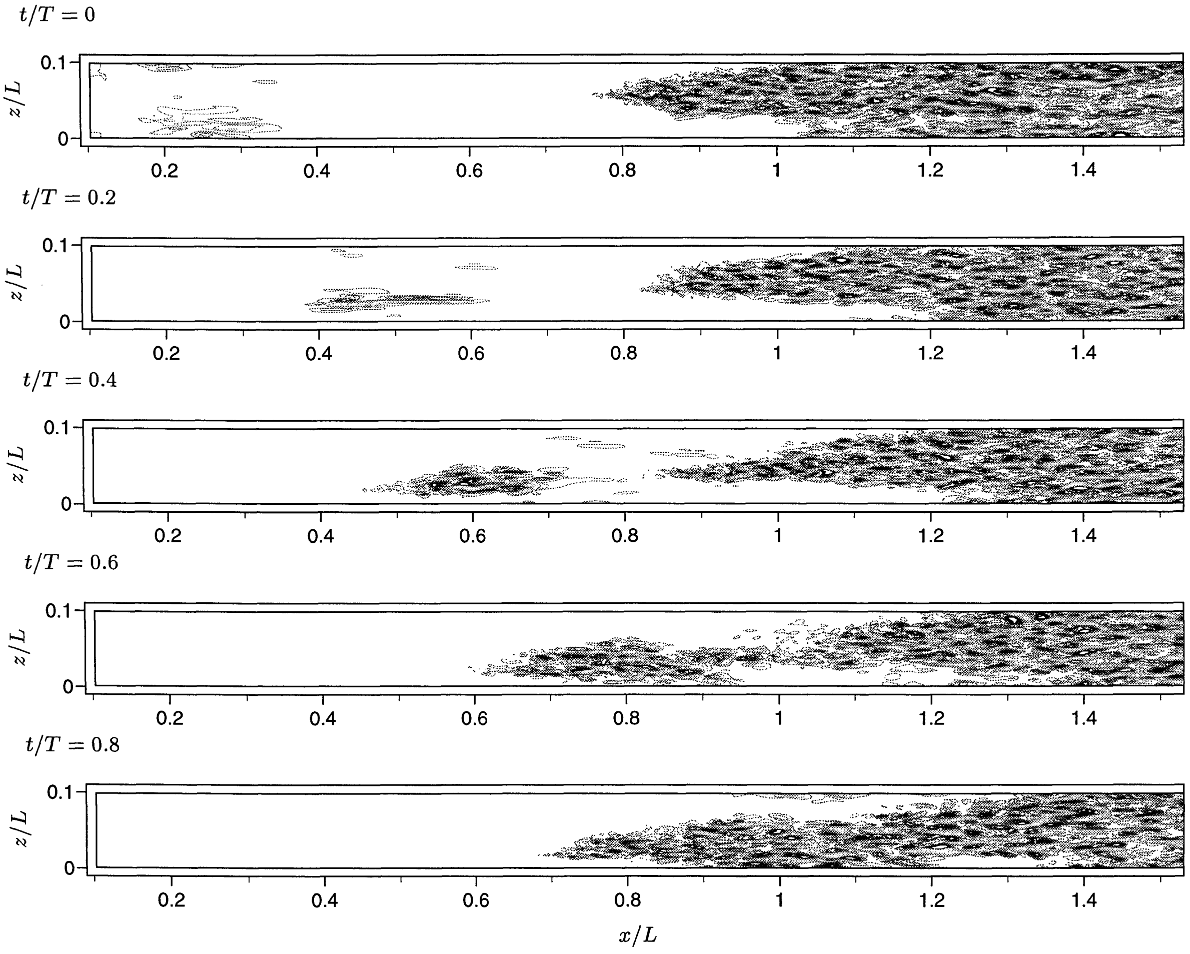

Figure 2: Visualization of turbulent spot formation and growth using v-component of fluctuating velocity in the x-z plane near the wall ($y/\delta_{99} = 0.4$ at $x/L = 0.8$). Contours represent $-0.1 < v/U_{ref} < 0.1$ in 0.01 increments.

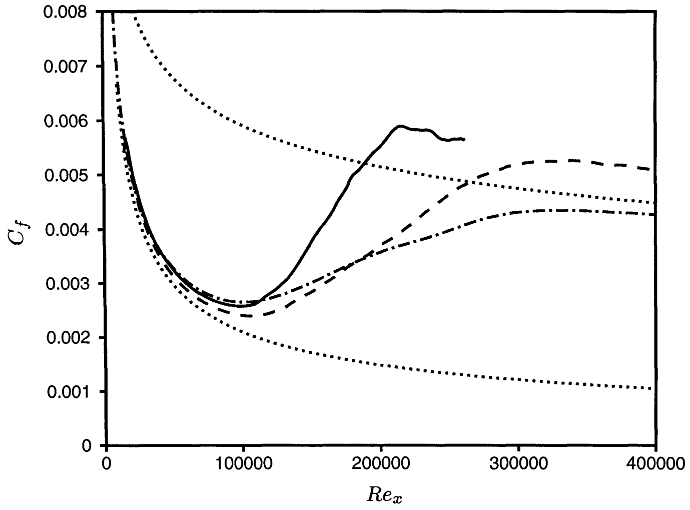

Figure 3: Comparison of time averaged skin friction coefficient along flat plate, ——— LES; —·— RANS; - - - - DNS of Wu *et al.* (1999); ······· laminar and turbulent correlations.