EXPLICIT ALGEBRAIC STRESS MODELLING OF HOMOGENEOUS AND INHOMOGENEOUS FLOWS

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ABSTRACT

Capability of the explicit algebraic stress models to predict homogeneous and inhomogeneous shear flows are examined. The importance of the explicit solution of the production to dissipation ratio is first highlighted by examining the algebraic stress models performance at purely irrotational strain conditions. Turbulent recirculating flows within sudden expanding pipes are further simulated with explicit algebraic stress model and anisotropic eddy viscosity model. Both models show a better stress-strain interaction, showing a reasonable shear layer development. The anisotropic stress field are also accurately predicted by the models, though the anisotropic eddy viscosity model of Craft et al. returns marginally better results.

INTRODUCTION

Separation and reattachment are commonly found phenomena in engineering flows, such as aerofoil with separation bubbles, diffuser and combustors. The separation with a slow pressure recovery can cause significant loss of lift of the aerofoil, however on the other hand, the presence of the recirculation region may help to stabilize the combustion zone within the combustors. It is, therefore, essential to understand and to predict correctly these phenomena. The sudden-expanding pipe provides the simplest geometry to study the phenomena and this has been the focus of research both experimentally and numerically.

The dominant features of the flow are the separation at the expansion, the shear layer with a slight streamline curvature, reattachment of the shear layer and the recovery of the flow. The complex physical features, despite its geometric simplicity, serves the purposes of testing the performance of the turbulence models. It is widely accepted that the linear eddy-viscosity type of turbulence models can not, without modifications, account for the streamline curvature effect. Therefore, the natural route is to apply Second-Moment Closures in predicting the recirculating flows. However, the extra computational cost incurred due to the solution of the transport equations of the Reynolds stresses prevents the model from being widely used, especially in three-dimensional environments.

One alternative is to adopt a non-linear stress and strain relationship of the Reynolds stresses.This can be achieved by assuming that the Reynolds stresses are taken to be non-linear function of the mean velocity gradients. However, there are many approaches in deriving the coefficients and determining the order of the tensorially independent groups. Most of the models are formulated at the quadratic level, and few adopt cubic stress and strain relationships. These models are termed by Gatski and Speziale (1993) as the anisotropic eddy viscosity models, because these formulations have no direct relation with the Reynolds stress models.

Based on the algebraic stress model (ASM) of Rodi (1972) and with the aid of the Caley-Hamilton theorem, Pope (1975) proposed that the most general form of anisotropy tensor can be expressed in terms of the mean strain and vorticity tensors of ten tensorially independent groups (up to fifth order) and coefficients. This was motivated by the fact that the implementations of the algebraic stress models are not straightforward, because the stress-strain relation is not explicit. The explicit algebraic stress models are attractive, because the Reynolds stresses are related to the mean velocity gradients implicitly through the Reynolds stress closures.

Gatski and Speziale (1993) further extended Pope's formulation to three-dimensional turbulent flows in non-inertial frame. One drawback of the above model is the adoption of the equilibrium value of the ratio of production to dissipation (P/ϵ=1.89) in the model coefficients. This, as pointed out by Girimaji (1996), is internally inconsistent. In order to account for the turbulent flows with localized strain rates that are large, the Gatski and Speziale's explicit algebraic stress model has been regularised. Girimaji has indicated that the production to dissipation ratio can in fact be determined analytically, and this has potential benefits in computing complex flows with strong shear layers.

Therefore, the focus of the study is to examine the importance of the explicit solution of the P/ϵ ratio at large strain rates for explicit algebraic stress models. Attention will be focusing on the model performance in homogenous flow under rotational and irrotational strains. The capability of the explicit algebraic stress model and anisotropic eddy viscosity model to predict inhomogeneous flow within the sudden expanding pipe geometry is also investigated.

THE COMPUTATIONAL MODEL

The Governing Equations

The behaviour of the flow is in general governed by the fundamental principles of classical mechanics expressing the conservation of mass and momentum. The time-averaged equations for high-Reynolds-number flow, may be described by the equations (in cartesian tensor):

$$\frac{\partial(\rho U_i)}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial(\rho U_i U_j)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}[\mu_l(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}) - \rho\overline{u_i u_j}] \qquad (2)$$

where $\overline{u_i u_j}$ is turbulent flux arising from the time-averaging process. The tensorial form of the momentum equation represents the U and V momentum solved.

Turbulence models

The adopted anisotropic eddy viscosity model of Craft et al.(1993) can be expressed as,

$$-[\frac{\overline{u_i u_j}}{k} - \frac{2}{3}\delta_{ij}] = 2C_\mu S_{ij}$$
$$+C_1 4C_\mu[S_{ik}S_{kj} - \frac{1}{3}\delta_{ij}S_{kl}S_{kl}]$$
$$+C_2 4C_\mu[W_{ik}S_{kj} + W_{jk}S_{ki}]$$
$$+C_3 4C_\mu[W_{ik}W_{kj} - \frac{1}{3}\delta_{ij}W_{kl}W_{kl}]$$
$$+C_4 8C_\mu^2(S_{ki}W_{lj} + S_{kj}W_{li} - \frac{2}{3}S_{km}W_{lm}\delta_{ij})S_{kl}$$
$$+C_6 8C_\mu^2 S_{ij}S_{kl}S_{kl} + C_7 8C_\mu^2 S_{ij}W_{kl}W_{kl} \qquad (3)$$

where $C_1 = -0.1$, $C_2 = 0.1$, $C_3 = 0.26$, $C_4 = -1$, $C_6 = -0.1$, $C_7 = 0.1$.

$$C_\mu = \frac{0.3}{1. + 0.35[max(S,\Omega)]^{1.5}}[1 - e^{[\frac{-0.36}{e^{-0.75max(S,\Omega)}}]}]$$

$$S_{ij} = \frac{1}{2}\frac{k}{\epsilon}(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}), \quad W_{ij} = \frac{1}{2}\frac{k}{\epsilon}(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i})$$

$$S = \sqrt{2S_{ij}S_{ij}}, \quad \Omega = \sqrt{2W_{ij}W_{ij}}$$

Following Gastki and Speziale(1993), the two-dimensional algebraic stress model can be expressed as,

$$\frac{\overline{u_i u_j}}{2k} - \frac{1}{3}\delta_{ij} = -\frac{3\alpha}{3 - 2\eta_1 - 6\eta_2}$$
$$[S_{ij}^* + (S_{ik}^* W_{kj}^* + S_{jk}^* W_{ki}^*) - 2(S_{ik}^* S_{kj}^* - \frac{1}{3}S_{kl}^* S_{kl}^* \delta_{ij})] \qquad (4)$$

where $\alpha = \frac{C_2 - 4/3}{C_3 - 2}$, $\eta_1 = S_{ij}^* S_{ij}^*$, $\eta_2 = W_{ij}^* W_{ij}^*$, $g = (C_1/2 + P/(\rho\epsilon) - 1)$ and

$$S_{ij}^* = \frac{2 - C_3}{2g}S_{ij}$$

$$W_{ij}^* = \frac{2 - C_4}{2g}[W_{ij} + \frac{C_4 - 4}{C_4 - 2}e_{mji}\Omega_m]$$

In the present study, the linear pressure-strain model adopted is the Speziale, Sarkar and Gatski (1991) model (SSG). The model coefficients are $C_1 = 6.8$, $C_2 = 0.36$, $C_3 = 1.25$ and $C_4 = 0.4$.

It was indicated by Gatski and Speziale (1993) that for sufficiently large strain rates η_1, singularities can occur. Therefore, the regularised model is proposed by Gatski and Speziale, i.e.

$$\frac{3}{3 - 2\eta_1 - 6\eta_2} \sim \frac{3(1 + \eta_1)}{3 + \eta_1 - 6\eta_2(1 + \eta_1)} \qquad (5)$$

This new formulation ensures the coefficient to be positive. However, there remains unknown ratio of P/ϵ. To propose an explicit ASM, the equilibrium value of $P/\epsilon = 1.89$ is adopted by Gatski and Speziale (1993).

An analytic solution of P/ϵ can be obtained by multiplying equation 4 with $S_{i,j}^*$, i.e.

$$-\overline{u_i u_j}S_{ij}^* = k\alpha\frac{6\eta_1}{3 - 2\eta_1 - 6\eta_2} \qquad (6)$$

and the above equation produces a cubic order polynomial for g, which is function of P/ϵ. The general solution of this equation was obtained by Girimaji (1996) and is adopted here. It should be pointed out that by adopting this approach, no regularization procedure is needed, because the approach produces non-singular behaviour of equation 4 (Girimaji, 1996, Wallin and Johansson, 2000).

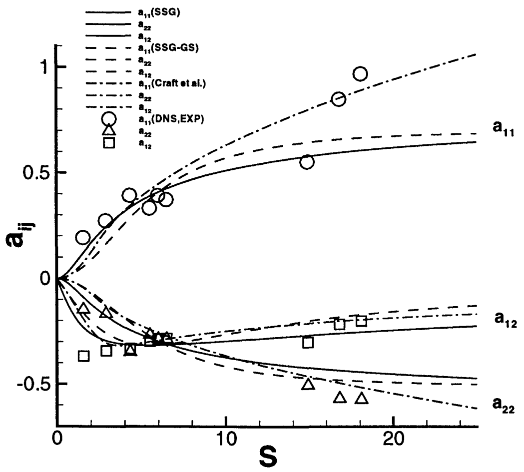

Figure 1: Variation of anisotropy at different strain rate-homogeneous shear.

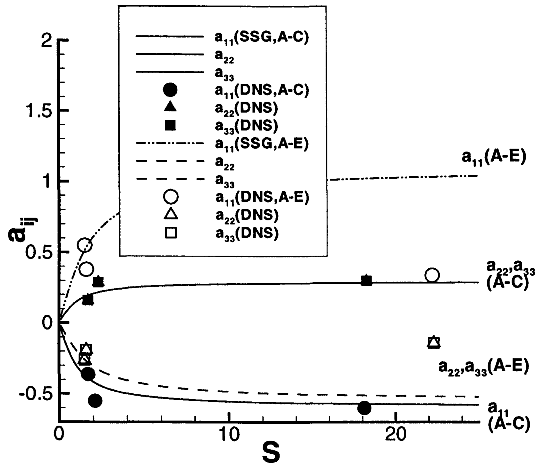

Figure 2: Anisotropy at different strain rate-axi-symmetric contraction and expansion(SSG model).

NUMERICAL ALGORITHM

This scheme solves discretised versions of all equations on a staggered finite-volume arrangement. The principle of mass-flux continuity is imposed indirectly via the solution of pressure-correction equations according to the SIMPLE algorithm (Patankar, 1980). The flow-property values at the volume faces contained in the convective fluxes which arise from the finite-volume integration process are approximated by the QUICK scheme (Leonard, 1979).

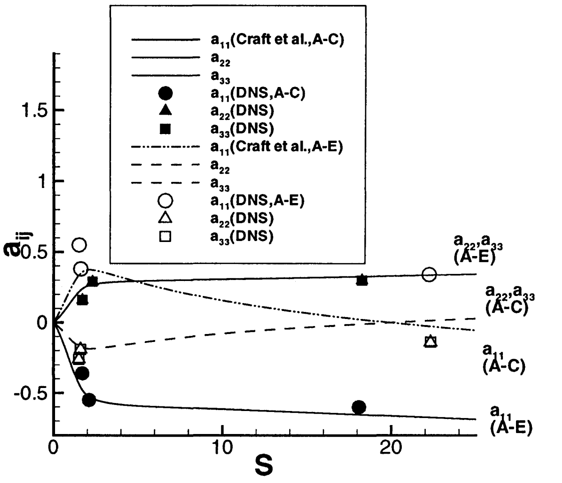

Figure 3: Anisotropy at different strain rate-axi-symmetric contraction and expansion(Craft et al. model).

The numerical meshes, 120x100 and 90x60, are non-uniform both in the x and y directions. Initial tests on the influences of the grid density revealed that the differences between the two meshes were small. Therefore, in subsequent computations, the mesh 90x60 will be adopted.

RESULTS

The performance of the model is first examined by applying to the homogeneous shear flow in equilibrium state at different strain rates

$$2S_{ij} = \begin{pmatrix} 0 & S & 0 \\ S & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The anisotropy a_{ij} is defined as $\overline{u_i u_j}/k - 2/3\delta_{ij}$. The predicted results are contrasted with DNS data (Roger and Moin, 1987) and measurements (Champagne et al., 1970, Harris et al., 1977, Tavoularis and Corrsin, S., 1981). Three models are contrasted here, namely SSG, SSG-GS and Craft et al.. The SSG-GS is the regularised explicit ASM model by Gatski and Speziale (shown in equation 5), while the SSG is the fully explicit ASM, i.e. the P/ϵ is obtained by equation 6. As shown in Figure 1, it can be clearly seen that all the models perform reasonably well, though at higher and lower strain rates the models behave differently.

Further, attention is directed to the homogeneous turbulence field induced by the irrotational strains under axi-symmetric contraction(A-C)

$$2S_{ij} = \begin{pmatrix} S & 0 & 0 \\ 0 & -S/2 & 0 \\ 0 & 0 & -S/2 \end{pmatrix}$$

and axi-symmetric expansion(A-E).

$$2S_{ij} = \begin{pmatrix} -S & 0 & 0 \\ 0 & S/2 & 0 \\ 0 & 0 & S/2 \end{pmatrix}$$

Here, the results by the the fully explicit ASM and Craft et al.'s anisotropic eddy viscosity model are shown in Figures 2 and 3. The SSG-GS is not included because it produces unrealistic stress field. This is apparent by examining the regularised model, as shown in equation 5. At high irrotational strain rates ($\eta_2 = 0$), the coefficient becomes constant, and the stress field is then proportional to S^*_{ij}. For constant P/ϵ ratio, this implies that the S^*_{ij} is again proportional to the strain rate, and hence so is the case for the stress field. For the fully explicit ASM, the P/ϵ ratio increases in tandem with strain rates, and hence produces a bounded stress field. Another advantage of the explicit solution of the P/ϵ ratio is that the P/ϵ ratio is always positive and the coefficient for the S_{ij} in the stress and strain relation is always negative, which also ensures stable solution numerically.

For the axi-symmetric contraction case, the SSG model agrees well with the DNS data. However, for the axi-symmetric expansion case, the model produces a too high level of anisotropy at higher strain rates. The Craft et al.'s model agrees perfectly with DNS data for the axi-symmetric contraction case, but produces the wrong trend under the axi-symmetric expansion condition. It should be pointed out that a revised model proposed by Craft et al. (1997) by including the transport equation for the second invariant, can deliver much better results.

Next the computations are applied to a simple dump combustor with the expansion ratio of 1.5, as shown in Figure 4. The inlet centreline velocity was maintained at 19.2 m/s, corresponding to the Reynolds number of $1.25x10^5$. The inlet of the computational domain was located at X/H=0.38, which is the first downstream position at which measurements are available. H is the difference of the radius of the expanding and inlet pipe. The predicted results are contrasted with the measurements of Ahmed and Nejad (1992).

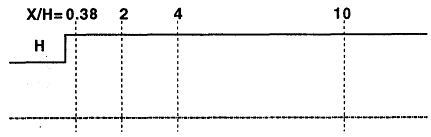

Figure 4: Geometry.

Figure 5 shows the predicted axial velocity distributions at four selected locations. It can be clearly seen that the linear $k-\epsilon$ model shows a more diffusive profile. The performance of the SSG and the anisotropic models shows the best results, where the shear layer is correctly predicted. The performance of the models can be further ascertained by reference to the shear stress distributions, shown in Figure 6. The correct development of the shear layer is the reflection of the accurate shear stress level predicted. The elevated level of diffusive cross stream transport of the $k - \epsilon$ prediction can be seen from the $\overline{uv}$ at X/H=2. The performance of the anisotropic model is marginally better than the explicit algebraic stress model. The level of the turbulent kinetic energy is related to its generation term, where for simple shear flow is $P_k \sim -\overline{uv}\partial U/\partial y$, and is shown in Figure 7.

Figures 8 to 10 show the predicted turbulence intensity profiles. As expected, the linear model indicates an isotropic stress field. Both SSG ASM and the Craft et al.'s model show a better anisotropic stress field.

CONCLUSIONS

Capability of the explicit algebraic stress models to predict homogeneous and inhomogeneous shear flows are examined in the present study. The importance of the explicit solution of the production to dissipation ratio is highlighted by examining the model performance at purely irrotational strain conditions. The regularised ASM was shown to produce unrealistic stress field under large irrotational strain, whereas both the explicit ASM and anisotropic eddy viscosity model remains bounded. Turbulent recirculating flows within sudden expanding pipes are further simulated with explicit algebraic stress model and anisotropic eddy viscosity model. Both models show a better stress-strain interaction, showing a reasonable shear layer development. The correct development of the shear layer is the reflection of the accurate shear stress level predicted. The anisotropic stress field are also accurately predicted by the models, though the Craft et al.'s model returns marginally better results.

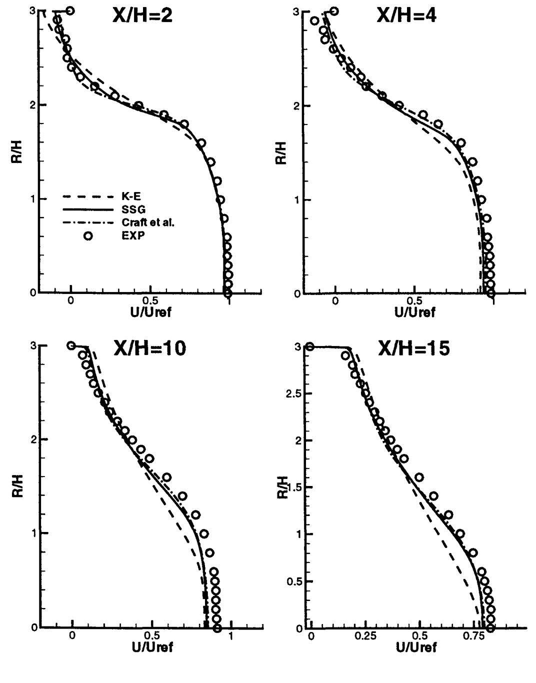

Figure 5: Axial velocity.

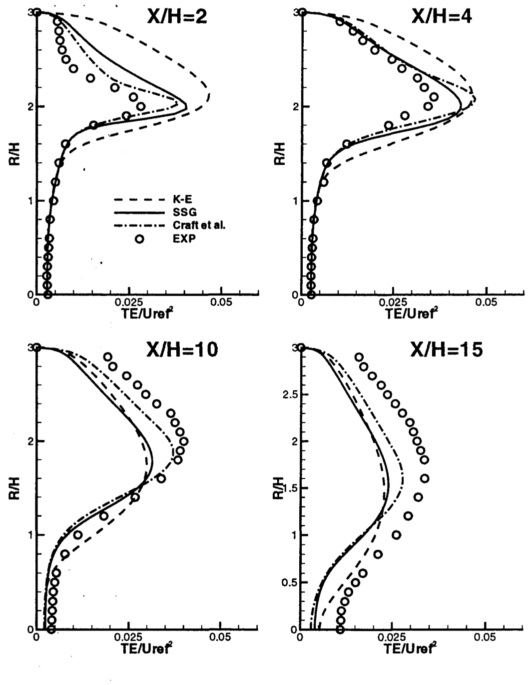

Figure 7: Turbulent kinetic energy.

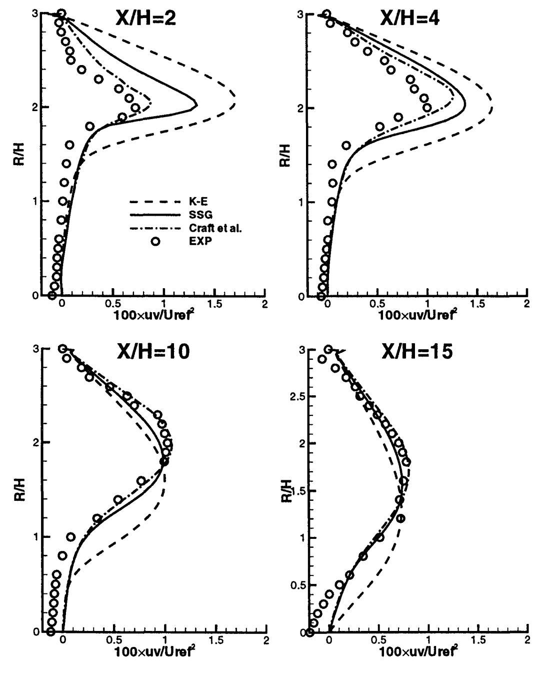

Figure 6: Shear stress.

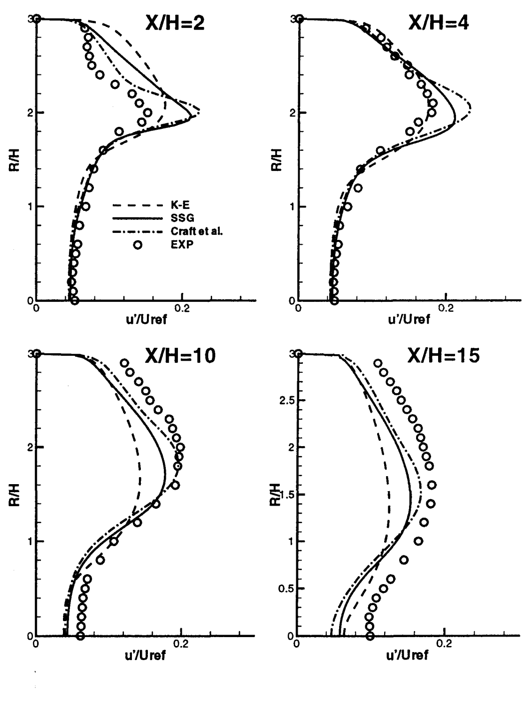

Figure 8: $\sqrt{u^2}$ distribution.

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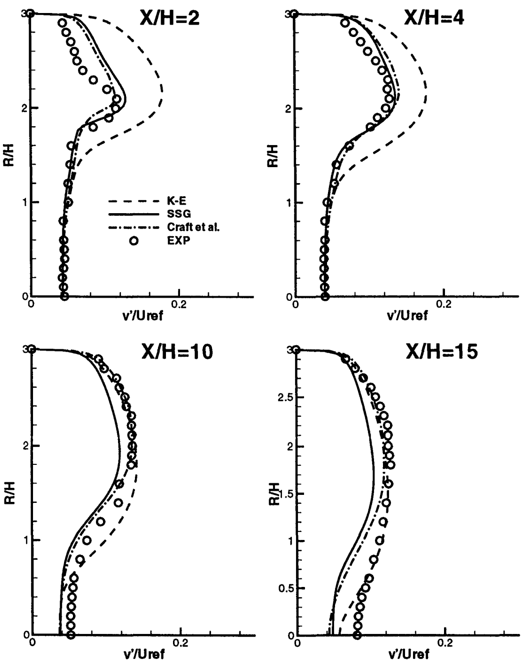

Figure 9: $\sqrt{\overline{v^2}}$ distribution.

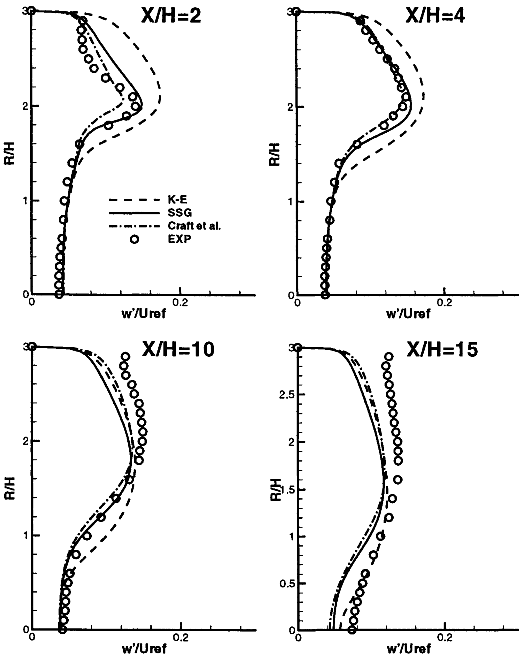

Figure 10: $\sqrt{\overline{w^2}}$ distribution.

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