AN IMPROVED TURBULENCE MODEL FOR ROTATING SHEAR FLOWS

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ABSTRACT

The objective of the present study is to construct a turbulence model based on a low-Reynolds-number non-linear k-ε model for turbulent flows in a rotating channel. Two-equation models, in particular the non-linear k–ε model, are very effective for solving various flow problems encountered in engineering applications. In channel flows with rotation, however, the explicit effects of rotation only appear in the Reynolds stress components. The exact equations for k and ε do not have any explicit terms concerned with the rotating effects. Moreover, a Coriolis force vanishes in the momentum equation for a fully developed channel flow with spanwise rotation. Consequently, in order to predict rotating channel flows, after proper revision the Reynolds stress equation model (RSM) or the non-linear eddy viscosity model (NLEVM) should be used. In this study, we improve the non-linear k–ε model so as to predict rotating channel flows. In the modelling, the wall-limiting behaviour of turbulence is also considered. First, we evaluated the non-linear k–ε model using the direct numerical simulation (DNS) database for a fully developed rotating turbulent channel flow. Next, we assessed the non-linear k–ε model at various rotation numbers. Finally, based on these assessments, we reconstruct the non-linear k–ε model to calculate rotating shear flows.

GOVERNING EQUATIONS FOR NLEVM

The incompressible Reynolds averaged Navier-Stokes equations in a reference frame rotating at a constant angular velocity Ω_k are described as follows:

$$\bar{U}_{i,i} = 0 \tag{1}$$

$$\frac{D\bar{U}_i}{D\tau} = -\bar{P}_{\text{eff},i}/\rho + \left(\nu\bar{U}_{i,j} - \overline{u_i u_j}\right)_{,j} - 2\epsilon_{ikl}\Omega_k\bar{U}_l \tag{2}$$

where the effective pressure $\bar{P}_{\text{eff}}$ includes the centrifugal force ($= -\rho\Omega^2 r^2/2$). The Reynolds stress expression in the quadratic NLEVM is given by the following form (Abe, Kondoh and Nagano, 1997):

$$\overline{u_i u_j} = 2k\delta_{ij}/3 - (1/f_R)\left[2\nu_t S_{ij} + 4C_D k\tau_R^2(S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}) - 4C_D k\tau_R^2(S_{ik}S_{kj} - S_{mn}S_{mn}\delta_{ij}/3)\right] \tag{3}$$

$$\nu_t = C_\mu f_\mu (k^2/\varepsilon) \tag{4}$$

$$\frac{Dk}{D\tau} = \nu k_{,jj} + T_k + \Pi_k + P_k - \varepsilon \tag{5}$$

$$\frac{D\varepsilon}{D\tau} = \nu\varepsilon_{,jj} + T_\varepsilon + \Pi_\varepsilon + (\varepsilon/k)(C_{\varepsilon 1}P_k - C_{\varepsilon 2}f_\varepsilon\varepsilon) + E \tag{6}$$

where τ_R is the characteristic time scale, $S_{ij}(= (\bar{U}_{i,j} + \bar{U}_{j,i})/2)$ is the strain-rate tensor, $\Omega_{ij}(= (\bar{U}_{i,j} - \bar{U}_{j,i})/2)$ is the vorticity tensor, T_k and T_ε are the turbulent diffusion terms, Π_k and Π_ε are the pressure diffusion terms, and E is an extra term ($= 0$ in the NLAKN model). Note that, in rotating flows, the vorticity tensor should be replaced with the absolute vorticity tensor, i.e., $W_{ij} = \Omega_{ij} + \epsilon_{mji}\Omega_m$ (Speziale *et al.,* 1991). The above model is according to Abe, Kondoh and Nagano (1997), and thus we refer to this as the NLAKN model in the following.

The turbulent and pressure diffusions are modeled using the GGDH in the NLAKN as follows:

$$T_k + \Pi_k = [C_s f_{t1}(\nu_t/k)\overline{u_j u_\ell} k_{,\ell}]_{,j} \quad (7)$$
$$T_\varepsilon + \Pi_\varepsilon = [C_\varepsilon f_{t2}(\nu_t/k)\overline{u_j u_\ell}\varepsilon_{,\ell}]_{,j} \quad (8)$$

The model functions and constants used in the NLAKN model are shown below.

$$f_R = 1 + (C_D \tau_R)^2 \times \left[(22/3)W^2 + (2/3)(W^2 - S^2) f_B\right] \quad (9)$$
$$f_B = 1 + C_\eta (C_D \tau_R)^2 (W^2 - S^2) \quad (10)$$
$$f_\mu = \left\{1 + \left(35/R_t^{3/4}\right) \exp\left[-(R_t/30)^{3/4}\right]\right\} \times [1 - f_w(26)] \quad (11)$$
$$f_\varepsilon = \{1 - 0.3\exp[-(R_t/6.5)]\} \times [1 - f_w(3.7)] \quad (12)$$
$$f_{t1} = 1 + 5.0 f_w(5) \quad (13)$$
$$f_{t2} = 1 + 4.0 f_w(5) \quad (14)$$
$$C_D = 0.8, C_\mu = 0.12, C_\eta = 5.0, C_{\varepsilon 1} = 1.45$$
$$C_{\varepsilon 2} = 1.9, C_s = 1.4, C_\varepsilon = 1.4 \quad (15)$$

where $S^2 = S_{ij}S_{ij}$, $W^2 = W_{ij}W_{ij}$, and τ_R is defined as $\tau_R = \nu_t/k$. The wall-refection function is defined using the dimensionless distance $n^* = (\nu\varepsilon)^{1/4} n/\nu$ as follows:

$$f_w(\xi) = \exp\left[-(n^*/\xi)^2\right] \quad (16)$$

where $f_w(26)$ in (11) means $\exp\left[-(n^*/26)^2\right]$. Note that the wall distance n is defined as "the distance between that point and the nearest point on the whole surface in a flow field" (Abe *et al.*, 1997).

EVALUATION OF MODELED EQUATIONS IN ROTATING CHANNEL FLOWS

First, we evaluate the modeled ε–equations of importance for determining the time-scale, and the modeled expressions for Reynolds shear stress by using the trustworthy DNS data of Nishimura and Kasagi (1996). Figure 1 shows the results of assessment for ε-equations in the rotating channel flow. The AKN model (Abe, Kondoh and Nagano, 1994), which is a linear eddy viscosity model, is included for comparison. It can be seen that the NLAKN slightly overpredicts the ε near the wall on the pressure side. On the suction side, however, none of the ε models give quantitative agreement with the DNS. The expressions for Reynolds shear stress are evaluated as shown

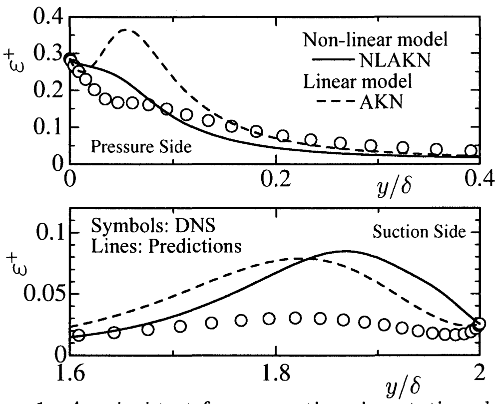

Figure 1: *A priori* test for ε–equations in rotating channel flow

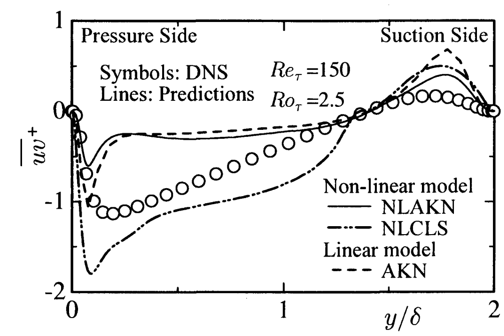

Figure 2: *A priori* test for Reynolds shear stress expressions in rotating channel flow

in Fig. 2, which includes the NLCLS model (Craft, Launder and Suga, 1997), a cubic non-linear k–ε model. It can be seen from this figure that the Reynolds shear stresses by both the linear and non-linear expressions are underpredicted on the pressure side and overpredicted on the suction side. The NLCLS model, however, overpredicts these expressions only on the pressure side. *A priori* test for expressions for normal stress components of non-linear model is shown in Fig. 3. Figure 4 indicates the wall-limiting behaviour of normal stress components which is predicted by the NLAKN and the NLCLS models on the pressure side. Disagreements are observed in all normal stress components, and the NLCLS model gives negative values for $\overline{v^2}$ and $\overline{w^2}$. Obviously, the turbulence model can not be realized. Moreover, the wall-limiting behaviour of normal-stress components is not satisfied in both models. From these assessments, it is found that the transport equation for the dissipation rate of turbulence energy ε and the expressions for the Reynolds stresses give results in disagreement with the DNS data.

Next, we explore the rotation number dependence of the model prediction with the aid of the DNS data from Kristoffersen and Andersson (1993). The evaluation has been performed at different rotation numbers (Ro_τ =

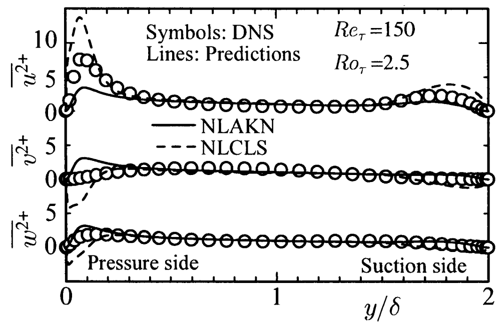

Figure 3: *A priori test* for normal stress expressions

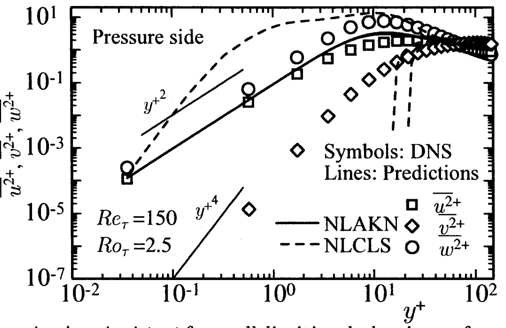

Figure 4: *A priori test* for wall-limiting behaviour of normal stress expressions in rotating channel flow

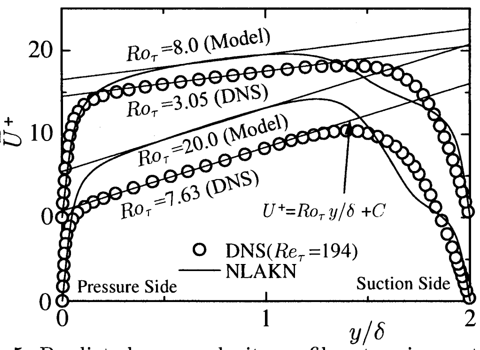

Figure 5: Predicted mean velocity profiles at various rotation numbers

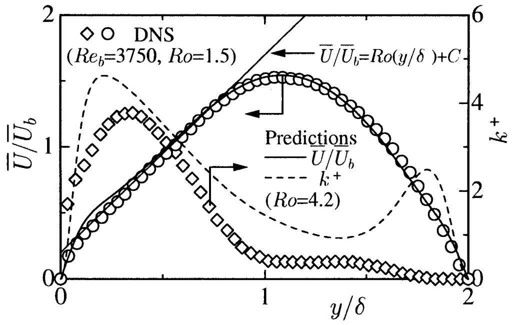

Figure 6: Profiles of turbulent quantities at high rotating number ($Ro = 1.5$)

$2\Omega\delta/u_\tau$) from 3.05 to 7.63. Figure 5 shows the predictions of the NLAKN model at various rotation numbers in comparison with the DNS. It is well known that a region exists in a rotating channel where the vorticity ratio, $S = -2\Omega/(d\bar{U}/dy)$, becomes $S = -1$, which represents neutral stability (Kristoffersen and Andersson, 1993). This relation yields the following equation:

$$\bar{U}^+ = Ro_\tau(y/\delta) + C \tag{17}$$

The mean velocity profile with the gradient Ro_τ does exist in the region of neutral stability in the DNS. However, in order to satisfy the DNS-based relation with the NLAKN model, a rotation number about three times as large as the DNS has to be provided in the calculation, thus indicating the weak rotation number dependence of the NLAKN model.

We have also assessed the model performance at a much higher rotation number (Lamballais et al., 1996). The rotation number based on the bulk velocity, the channel half width and the angular velocity ($= 2\Omega\delta/U_b$) is 1.5, which is larger than the maximum value ($Ro = 0.5$) of the above DNS (Kristoffersen and Andersson, 1993), and the corresponding Reynolds number ($= 2\bar{U}_b\delta/\nu$) is 3750. However, in the calculations using the NLAKN model, the rotation number 4.2 is used for the above-mentioned reason. From Fig. 6, it can be seen that the model can not represent laminarization phenomena on the suction side, i.e., no observable vanishing of either the turbulence energy or the Reynolds shear stress. From these results, it can be concluded that predictions from the existing non-linear k–ε model indicate weak dependence on the rotation number.

As demonstrated in the foregoing, there are crucial weak points in the NLAKN model for the prediction of rotating flows, which should be amended.

RECONSTRUCTION OF TURBULENCE MODEL

In this section, we reconstruct the NLAKN model based on the above-mentioned evaluation. In order to satisfy the wall-limiting behaviour of normal stress components in the NLEVM, we introduce a new time scale into the Reynolds stress expression (3) as follows:

$$\tau_R^2 = \tau_{Ro}^2 + \tau_{Rw}^2 \tag{18}$$

where $\tau_{Ro}(= \nu_t/k)$ is the original part of the time scale defined by NLAKN (1996), and τ_{Rw} is introduced to satisfy the wall-limiting behaviour. The time scale τ_{Rw} is modelled in consideration of the wall-limiting behaviour of the normal stress components as follows:

$$\tau_{Rw} = \sqrt{\frac{1}{6}\frac{f_R/C_D}{f_{SW}}}\left(1 - \frac{3C_{v1}f_{v2}}{8}\right) f_{v1}^2 \tag{19}$$

where $f_{v1} = \exp\left[-\left(R^*_{tm}/45\right)^2\right]$, $f_{v2} = 1 - \exp\left(-\sqrt{R_t}/C_{v2}\right)$, $C_{v1} = 0.4$, $C_{v2} = 2 \times 10^3$, and f_{SW} is given by:

$$f_{SW} = W^2/2 + S^2/3 - f^{\Omega}_{SW} \qquad (20)$$

f^{Ω}_{SW} is related with the rotation number defined as follows:

$$f^{\Omega}_{SW} = \left[\left(\sqrt{S^2/2} - \sqrt{W^2/2}\right) f_w(1)\right]^2 \qquad (21)$$

This represents Ω_k^2 implicitly in the rotating channel flows. By introduction of the time scale τ_{Rw}, the wall-limiting behaviour of normal stress components is satisfied, i.e., $\overline{u^2} \propto y^2$, $\overline{v^2} \propto y^4$ and $\overline{w^2} \propto y^2$.

The modified turbulence Reynolds number R_{tm} is proposed in (19) for capturing the laminarization on the suction side as follows:

$$R_{tm} = C_{tm} n^* R_t^{1/4} / (C_{tm} R_t^{1/4} + n^*) \qquad (22)$$

where C_{tm} is the model constant set at 1.3×10^2.

Next, in order to modify the rotation number dependence of the model, we adopt the rotation-influenced additional term of Shimomura (1989) in the ε–equation, which can represent the asymmetry in turbulence quantities of rotating channel flows:

$$R = C_\Omega f_\Omega k \epsilon_{ij\ell} W_{ij} \Omega_\ell \qquad (23)$$

where $C_\Omega = -0.045$, and the following model function f_Ω is newly introduced to reflect the low-Reynolds-number and rotation number effects:

$$f_\Omega = C_{f_\Omega} \exp\left[-\left(R_\Omega/10\right)^{0.2}\right] \qquad (24)$$

where $C_{f_\Omega} = 6.0$ is the model constant, and R_Ω is a parameter defined as follows:

$$R_\Omega = \eta_t \sqrt{f^{\Omega}_{SW}} \qquad (25)$$

where $\eta_t (= \sqrt{\nu/\varepsilon})$ is the Kolmogorov time scale. η_t is sensitive to the low-Reynolds-number effect, and thus we introduce this parameter into f_Ω.

Considering information obtained previously (Abe, Kondoh and Nagano, 1994; Nagano and Shimada, 1996), the following revised model functions are employed in the proposed model:

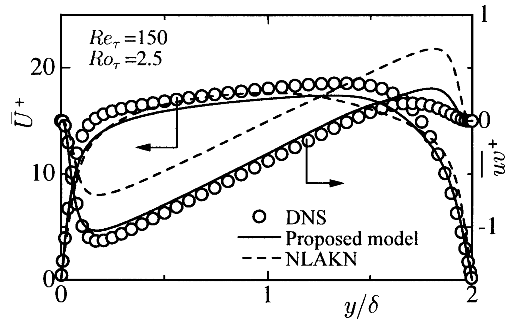

Figure 7: Distributions of mean velocity and Reynolds shear stress in rotating channel flow ($Ro_\tau = 2.5$)

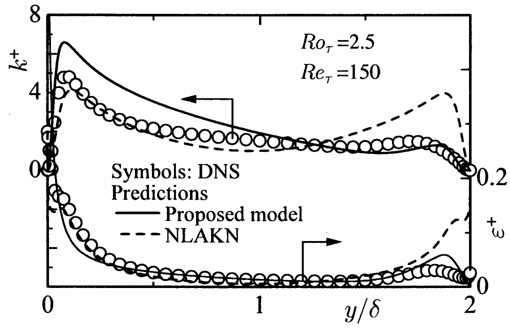

Figure 8: Distributions of turbulence energy and its dissipation rate in rotating channel flow ($Ro_\tau = 2.5$)

$$f_\mu = \left\{1 + \left(40/R_t^{3/4}\right) \exp\left[-\left(R_t/35\right)^{3/4}\right]\right\} \times \left[1 - f_w(32)\right] \qquad (26)$$

$$f_\varepsilon = \left\{1 - 0.3 \exp\left[-\left(R_t/6.5\right)\right]\right\} \times \left[1 - f_w(3.7)\right] \qquad (27)$$

$$f_w(\xi) = \exp\left[-\left(R_{tm}/\xi\right)^2\right] \qquad (28)$$

The turbulent diffusion terms in k– and ε–equations are modeled with the GGDH similar to the NLAKN model:

$$T_k = \left[C_s f_{t1} (\nu_t/k) \overline{u_j u_\ell} k_{,\ell}\right]_{,j} \qquad (29)$$

$$T_\varepsilon = \left[C_\varepsilon f_{t2} (\nu_t/k) \overline{u_j u_\ell} \varepsilon_{,\ell}\right]_{,j} \qquad (30)$$

where $C_s = C_\varepsilon = 1.4$. The model functions in (30) are modified for introduction of τ_{Rw} as follows:

$$\left.\begin{aligned} f_{t1} &= \left[1 + 9.0 f_w(8)\right] / \left[1 - f_w(32)\right]^{1/2} \\ f_{t2} &= \left[1 + 5.0 f_w(8)\right] / \left[1 - f_w(32)\right]^{1/2} \end{aligned}\right\} \qquad (31)$$

The pressure diffusion terms in k– and ε–equations, which are often ignored in conventional modeling, are introduced to satisfy exactly the wall-limiting behaviour, and the same extra production term is also added to the ε–equation as in the NS model (Nagano and

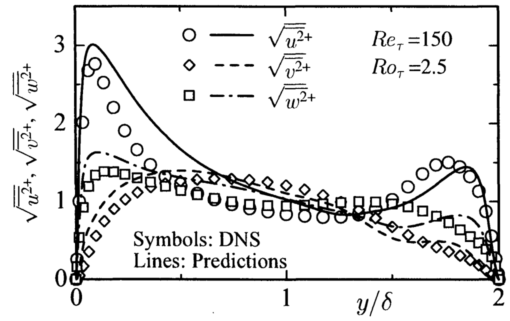

Figure 9: Distributions of normal stress components in rotating channel flow ($Ro_\tau = 2.5$)

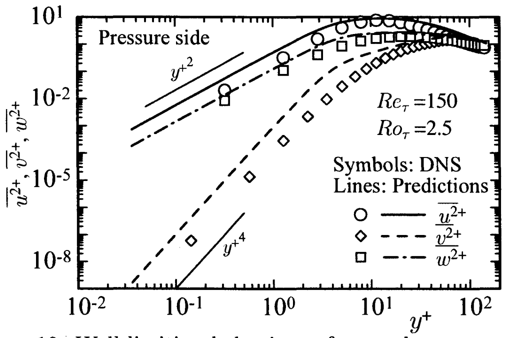

Figure 10: Wall-limiting behaviour of normal stress components in rotating channel flow ($Ro_\tau = 2.5$)

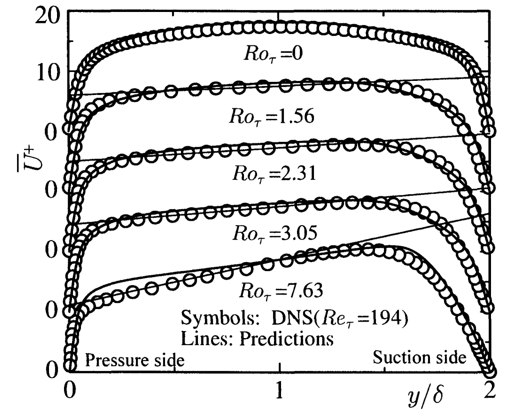

Figure 11: Mean velocity profiles in various rotation number flows

Shimada, 1995) as follows:

$$\Pi_k = \max\left\{-0.5\nu\left[(k/\varepsilon)\,\varepsilon_{,j} f_w(1)\right]_{,j}, 0\right\} \quad (32)$$

$$\Pi_\varepsilon = C_{\varepsilon 4}\left\{[1 - f_w(5)]\,(\varepsilon/k)k_{,j} f_w(5)\right\}_{,j} \quad (33)$$

$$E = C_{\varepsilon 3}\nu(k/\varepsilon)\overline{u_j u_\ell}\bar{U}_{i,\ell k}\bar{U}_{i,jk} + C_{\varepsilon 5}\nu(k/\varepsilon)(\overline{u_j u_k})_{,j}\bar{U}_{i,k}\bar{U}_{i,jk} \quad (34)$$

where $C_{\varepsilon 3} = 0.02$, $C_{\varepsilon 4} = 0.5$ and $C_{\varepsilon 5} = 0.015$. The other model constants are the same as those of the original NLAKN model.

RESULTS AND DISCUSSION

In order to demonstrate the performance of the improved NLAKN model, various rotation and Reynolds number flows have been calculated using the proposed model. Figure 7

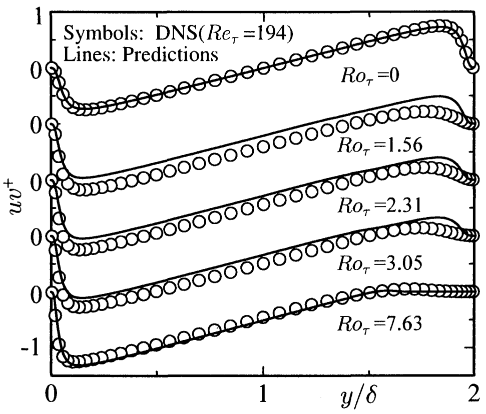

Figure 12: Distributions of Reynolds shear stress in various rotation number flows

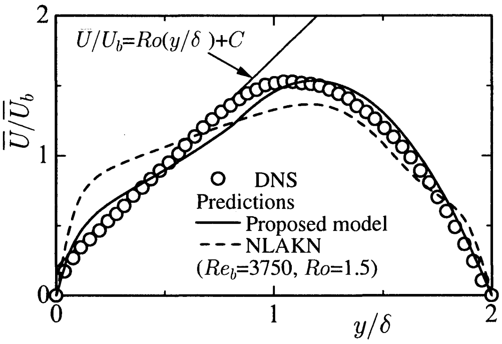

Figure 13: Mean velocity profiles in high rotation number flow ($Ro = 1.5$)

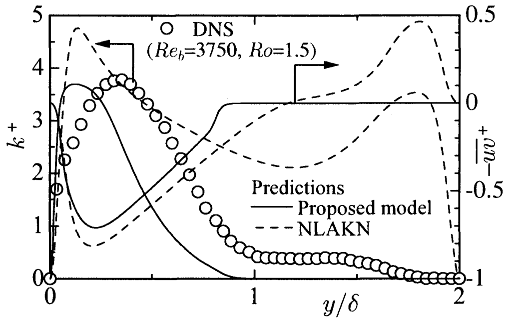

Figure 14: Distributions of turbulence energy and Reynolds shear stress in high rotation number flow ($Ro = 1.5$)

shows the distributions of mean velocity and Reynolds shear stress in comparison with the original model predictions and the DNS data (Nishimura and Kasagi, 1996; $Re_\tau = 150$, $Ro_\tau = 2.5$). The turbulence energy and its dissipation rate are indicated in Fig. 8. It can be seen that predictions with the improved model agree with the DNS data. Obviously, the proposed model adequately captures turbulent quantities on the suction side, and the introduced model functions are found to work effectively. The predicted normal stress components are shown in Fig. 9. Since the introduced time scale τ_{Rw} functions effectively near the wall like a wall-reflection term of a pressure strain term in the RSM, the proposed model reproduces exactly the normal stress components in the rotating channel flow. Figure 10 shows the wall-limiting behaviour of normal stress components, while indicating that the proposed model can predict the wall-limiting behaviour of normal stress components. Especially, the wall-normal component, $\overline{v^2}$, which is important quantity for the turbulent diffusion term, is reproduced appropriately in the present model.

Next, we appraised the rotation number dependence of the improved model. Figures 11 and 12 show the mean velocity and Reynolds shear stress profiles in various rotation number flows together with the related DNS data (Kristoffersen and Andersson, 1993; $Re_\tau = 194$, $Ro_\tau = 0 \sim 7.63$). In Fig. 11, it can be seen that the calculated mean velocities are considerably refined in comparison with Fig. 5. Also, the Reynolds shear stresses predicted by the present model are in good agreement with the DNS data in various rotation number flows.

Finally, we have calculated a high rotation-number flow ($Ro = 1.5$). In the foregoing calculations corresponding to the DNS of Kristoffersen and Andersson (1993), the maximum rotation number is $Ro = 0.5$. Thus, the DNS data for $Ro = 1.5$ (Lamballais et al., 1996) used here significantly increase the validation range for the proposed model. In Fig. 13, predictions for mean velocity are presented; in Fig 14, corresponding predictions for turbulence energy and Reynolds shear stress are shown in comparison with the original model predictions and DNS data. In the present case, since there is a laminar region (or no turbulence) in the channel, this is one of the most difficult problems for evaluation of a turbulence model. The proposed model predicts the laminar region on the suction side, but the original model produces turbulence there. Hence, the mean velocity profile predicted by the improved model comes to show good agreement with the DNS data.

CONCLUSIONS

We have developed a non-linear two-equation turbulence model to predict rotating channel flows, in which the wall-limiting behaviour and redistribution of normal stress components are also considered. The predictions with the proposed quadratic model give good agreement with the DNS data for various rotation number flows. The proposed model also satisfies the wall-limiting behaviour of the normal stress components exactly, and can adequately predict redistribution of the normal stress components.

ACKNOWLEDGEMENTS

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