A TERM-BY-TERM MODEL OF TURBULENCE ENERGY AND STRESS DISSIPATION CONSISTENT WITH NEAR WALL LIMITS

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ABSTRACT

A DNS-based new model of the transport equation for the turbulence energy dissipation rate ε and an algebraic expression for its tensor ε_{ij}, all satisfying the wall-limits without using topological parameters, are derived in terms of their 'homogeneous' parts ε^h and ε^h_{ij}. The approach is based on the two-point velocity covariance analysis of Jovanović, Ye and Durst (1995), with re-interpretation of the viscous term. *A priori* evaluation of ε_{ij} using DNS data for quantities other than ε_{ij}, and the computation with the full $\overline{u_iu_j}$-ε^hmodel of flows in a pipe, plane channel, boundary layer, behind a backward-facing step and in an axially-rotating pipe, show good near-wall behavior of all terms in accord with the DNS data.

INTRODUCTION

The transport equation for turbulence energy dissipation rate ε has been widely used to close single-point $k-\varepsilon$ eddy-viscosity and second-moment (Reynolds stress) models. Yet, the term-by-term scrutiny of even simple equilibrium wall flows shows that none of the modelled term reflects the corresponding terms in the exact equation for ε. In this paper we revisit the dissipation equation for low-Re-number near-wall flows following a different approach with two novelties. First, the derivation of the equation for ε has been based on the two-point analysis (Jovanovic *et al.* 1995) which leads to the equation for the 'homogeneous' dissipation rate

$$\varepsilon^h = \varepsilon - \frac{1}{2}\nu\frac{\partial^2 k}{\partial x_l \partial x_l} = \varepsilon - \frac{1}{2}\mathcal{D}^\nu_k \tag{1}$$

where $\mathcal{D}^\nu_k$ is the viscous diffusion of the kinetic energy of turbulence. Second, a *term-by-term* modelling for the individual terms in the ε^h equation is presented, combined with the algebraic relationship for ε_{ij} in term of ε^h, stress and dissipation tensor invariants and turbulence Reynolds number Re_t. This interpretation offers a better reproduction of the DNS results as well as other advantages.

Two-point interpretation of ε_{ij}

The viscous term in the exact transport equation for the turbulent stress tensor $\overline{u_iu_j}$ is usually split into the viscous diffusion and dissipation rate

$$V_{ij} = \nu\frac{\partial^2 \overline{u_iu_j}}{\partial x_k \partial x_k} - 2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} = \mathcal{D}^\nu_{ij} - \varepsilon_{ij} \tag{2}$$

The same term in the transport equation for the two-point correlation $\overline{(u_i)_A(u_j)_B}$ expressed in the local coordinate system with the origin at midpoint between A and B, with $\xi_k = x^B_k - x^A_k$ and $x^{AB}_k = 1/2(x^A_k + x^B_k)$, reads

$$V^{AB}_{i,j} = \frac{1}{2}\nu\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_{AB}\overline{(u_i)_A(u_j)_B} + 2\nu\frac{\partial^2}{\partial \xi_l \partial \xi_l}\overline{(u_i)_A(u_j)_B} \tag{3}$$

In homogeneous turbulence all derivatives with respect to $(x_k)_{AB}$ vanish and, for $A \to B$, $V^{AB}_{i,j} \to V_{ij}$ represents the dissipation in a homogeneous flow, ε^h:

$$V^{AB}_{i,j} \to V_{ij} = \left[2\nu\frac{\partial^2}{\partial \xi_l \partial \xi_l}\overline{(u_i)_A(u_j)_B}\right]_{\xi=0} = -\varepsilon^h_{ij} \tag{4}$$

In inhomogeneous turbulence, for $A \to B$:

$$V^{AB}_{i,j} \to V_{ij} = \frac{1}{2}\nu\frac{\partial^2 \overline{u_iu_j}}{\partial x_k \partial x_k} - \varepsilon^h_{ij} = \frac{1}{2}\mathcal{D}^\nu_{ij} - \varepsilon^h_{ij} \tag{5}$$

Comparison with the single-point derivation, equation (2), yields:

$$\varepsilon_{ij} = \varepsilon^h_{ij} + \frac{1}{2}\mathcal{D}^\nu_{ij} \tag{6}$$

The dissipation tensor has contributions due to flow inhomogeneity that is treated as diffusive transport. Clearly, no algebraic interpolation for the ε_{ij} can account for the dissipation due to gradients of the Reynolds stresses.

Modelling Implications

We can now apply the common form of the algebraic expression for ε_{ij}, but in terms of the *homogeneous* part of the dissipation rate tensor,

$$\varepsilon_{ij}^h = (1-f_s)\frac{2}{3}\delta_{ij}\varepsilon^h + f_s\frac{\overline{u_iu_j}}{k}\varepsilon^h \tag{7}$$

The components of ε_{ij} can now be obtained from equation (6) where $\mathcal{D}_{ij}^{\nu}$ is the viscous diffusion of the corresponding stress component, computed from the solution of the stress transport equations. The advantage of using this approach is apparent: ε_{ij} satisfies exactly the wall-limits of each normalized component $(\varepsilon_{ij}/\varepsilon)(k/\overline{u_iu_j})$, (no sum on repeated indices) which is 1 for $i{=}j{=}1$, $i{=}j=3$, and $i=1$, $j=3$ and equals 2 if $i=1$, $j=2$ and $i=2$, $j=3$. A slight discrepancy appears for the wall-normal component, $i=j=2$, for which the model gives $\varepsilon_{22}/\varepsilon\cdot k/\overline{u_2^2}=3.5$, instead of the exact 4.

Obtaining ε^h

Equations (6) and (7) enable now to determine ε_{ij}. To close these expressions it is necessary to provide ε^h. The computationally most convenient way to obtain ε^h is to solve its own transport equation: in fact the stress transport equations can be written with ε_{ij}^h as the sink term with corresponding reduction of the viscous diffusion term by factor of two:

$$\frac{\mathrm{D}\overline{u_iu_j}}{\mathrm{D}t} = \ldots + \underbrace{\frac{1}{2}\frac{\partial}{\partial x_k}\left(\nu\frac{\partial\overline{u_iu_j}}{\partial x_k}\right)}_{\mathcal{D}_{ij}^{\nu}} - \varepsilon_{ij}^h + \cdots \tag{8}$$

Derivation of the equation for ε^h from the equation for the two-point correlation yields approximately (some higher order terms omitted, Jovanović *et al.*, 1995):

$$\frac{\mathrm{D}\varepsilon^h}{\mathrm{D}t} = \frac{\mathrm{D}\varepsilon}{\mathrm{D}t} - \underbrace{\frac{1}{2}\frac{\partial}{\partial x_k}\left(\nu\frac{\partial\varepsilon^h}{\partial x_k}\right)}_{\mathcal{D}_{\varepsilon}^{\nu}} \tag{9}$$

where $\mathrm{D}\varepsilon/\mathrm{D}t$ can be replaced by the RHS of the conventional or any other low-Re-number ε-equation. It should be noted that the definition of $\tilde{\varepsilon}$ (the 'isotropic' part of dissipation rate, $\tilde{\varepsilon}|_{wall}=0$), used in some models to handle the near wall behaviour and the wall boundary conditions should be modified, i.e:

$$\tilde{\varepsilon}^h = \varepsilon^h - \nu\left(\frac{\partial k^{1/2}}{\partial x_n}\right)^2; \varepsilon^h\mid_{x_n=0} = \nu\left(\frac{\partial k^{1/2}}{\partial x_n}\right)\Bigg|_{x_n=0}^2 \tag{10}$$

In addition to better physical foundation of the ε^h equation, major advantage is achieved in evaluating the individual dissipation rate components ε_{ij} or ε_{ij}^h, which become important for second-moment closures.

Now a model for the ε equation is required. A more accurate term by term modelling for this equation is given in the next section. We show that decisive advantages are achieved if the equation for the homogeneous dissipation rate ε^h is solved instead of total ε. In fact, in the final model we abandon completely the ε as a variable and use instead ε^h as the scale providing variable, as will be shown below. However, the discussion that follows is applicable equally to ε and ε^h equations.

TERM-BY-TERM MODELLING OF THE ε EQUATION .

A reformulation of some of the terms in the exact equation for the dissipation rate ε (see e.g. Mansour *et al.*, 1988, for details) is now described.

The mixed production term

First consider the 'mixed' production term

$$P_\varepsilon^1 + P_\varepsilon^2 = -2\nu\left(\overline{\frac{\partial u_i}{\partial x_l}\frac{\partial u_j}{\partial x_l}} + \overline{\frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_j}}\right)\frac{\partial U_i}{\partial x_j} \tag{11}$$

This term is regarded as negligible in high-Re-number flows. The major production of ε, $P_{\varepsilon 4}$ (Eq. 19) , associated with the self-stretching of the fluctuating vortices is usually modelled in terms of the energy production $P_k = -\overline{u_iu_j}\partial U_i/\partial x_j$, scaled with the characteristic turbulence time scale k/ε.

The first term in the brackets is in fact ε_{ij}. The second term is closely related to ε_{ij} - they contain common terms. Now, if ε_{ij} can be modelled satisfactory by equation (6), this term can be retained in its exact form. Of course, away from the wall and at high Re numbers, ε_{ij} becomes isotropic irrespective of the stress anisotropy and this term is not sufficient to account for total production of ε so that the standard production term should be retained, although with a smaller coefficient. The new model consists of the sum of the new and standard term, with $C_{\varepsilon 1}=1$,

$$P_\varepsilon^1 + P_\varepsilon^2 = -\varepsilon_{ij}\frac{\partial U_i}{\partial x_j} - 1.0\,\overline{u_iu_j}\frac{\partial U_i}{\partial x_j}\frac{\varepsilon}{k} \tag{12}$$

The new model of $P_\varepsilon^1 + P_\varepsilon^2$, equation (12) reproduces very well the DNS data for a plane channel, both using the total dissipation ε, Fig. 1a, or homogeneous dissipation ε^h, Fig. 1b.

The new model is useful in non-equilibrium flows subjected to strong linear straining. This becomes obvious if P_ε^1 is expanded into components. For 2-D flows:

$$P_\varepsilon^1 = -\varepsilon_{12}\frac{\partial U_1}{\partial x_2} - (\varepsilon_{11}-\varepsilon_{22})\frac{\partial U_1}{\partial x_1} \tag{13}$$

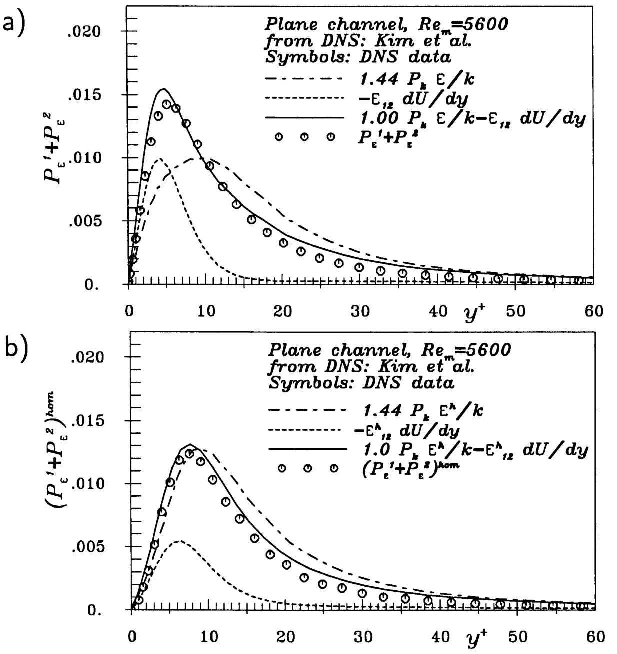

Fig. 1 The new model of $(P_\varepsilon^1+P_\varepsilon^2)$ for conventional (a) and new (b) ε-equation

Hanjalić and Launder (1980) proposed to sensitize the production P_ε to irrotational strain by increasing the coefficient $C_{\varepsilon 1}$ from 1.44 to $C'_{\varepsilon 1} \approx 4.44$, and introduced a new term, $(C'_{\varepsilon 1} - C_{\varepsilon 1})C_\mu k\Omega_{ij}\Omega_{ij}$, ($\Omega_{ij}$ is mean flow vorticity). The net effect is visible in two-dimensional flows, where in addition to the conventional production, another source term, $(C'_{\varepsilon 1}-C_{\varepsilon 1})(\overline{u_1^2}-\overline{u_2^2})\partial U_1/\partial x_1$ appears in the ε equation.

Such an enhancement of the production of ε is now accounted by the new formulation of P_ε^1, and no additional term is necessary. While $\overline{u_1u_2}/(\overline{u_1^2} - \overline{u_2^2})$ is of the order of magnitude of 1, making the production by both the rotational and irrotational strain of equal importance, $\varepsilon_{12}/(\varepsilon_{11}-\varepsilon_{22})$ is much smaller than 1, except very close to the wall (Hanjalić and Jakirlić, 1993). Hence, the term will itself distinguish the effect of the irrotational from the rotational strain in the production of ε.

A direct validation of Eq. (12) for more complex flows is missing because of the lack of DNS or other data. However, some insight can be gained by comparing the terms in Eq. (12) with the conventional model of the production of ε, i.e. $P_\varepsilon^1 + P_\varepsilon^2 = 1.44 P_k\varepsilon/k$. Fig. 2 shows such a comparison within the separation bubble in a back-step flow.

The gradient production term

The gradient production of ε, denoted as P_ε^3, reads exactly

$$P_\varepsilon^3 = -2\nu \overline{u_k \frac{\partial u_i}{\partial x_l}} \frac{\partial^2 U_i}{\partial x_k \partial x_l} \tag{14}$$

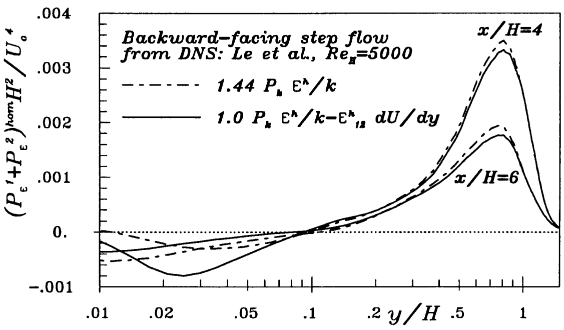

Fig. 2 The new and traditional models of ε $(P_\varepsilon^1 + P_\varepsilon^2)$ for new ε-equation at two locations in the back-step flow

Current practice assumes a simple gradient model $\overline{u_k\frac{\partial u_i}{\partial x_l}} \propto \tau\overline{u_j u_k}\frac{\partial^2 U_i}{\partial x_j \partial x_l}$, where $\tau = k/\varepsilon$, yielding the term with the squared second velocity derivative. Such a model does not allow for a proper sign of the curvature of the mean velocity profile. Bernard's vorticity transport theory (Bernard, 1990) provides a more rational method. The turbulent velocity gradient flux is expanded into

$$\overline{u_k \frac{\partial u_i}{\partial x_l}} = \left(\frac{\partial \overline{u_k u_i}}{\partial x_l} - \overline{u_i s_{kl}} - \overline{u_i \omega_{kl}} \right) \tag{15}$$

where s_{kl} and ω_{kl} are the fluctuating strain rate and vorticity respectively. The first term is now exact. The second term needs modelling. The third term is omitted since it is antisymmetric in its indices while the velocity curvature term is symmetric. For the 2D near-equilibrium wall layer the term $\overline{u_1 s_{22}}$ can be expanded, using the continuity, to produce

$$\overline{u_1 s_{22}} = -\frac{1}{2}\frac{\partial(\overline{u_1^2} - \overline{u_3^2})}{\partial x_1} - \frac{\partial \overline{u_1 u_3}}{\partial x_3} - \overline{u_3\omega_2} \tag{16}$$

where $\omega_i = \epsilon_{ijk}\frac{\partial u_j}{\partial x_k}$. The first two terms can be neglected because of spanwise and streamwise homogeneity. The results of Bernard (1990) is used to close $\overline{u_3\omega_2}$:

$$\overline{u_3\omega_2} = \frac{1}{2}\frac{Q_4}{1+Q_3Q_4(\partial U_1/\partial x_2)^2}\frac{\partial \overline{u_3^2}}{\partial x_2}\frac{\partial U_1}{\partial x_2} \tag{17}$$

where Q_3 and Q_4 are the Lagrangian integral scales. For a fully developed channel flow, Bernard (1990) recommended $Q_3 = 0.65$ and $Q_4 = 10.8$.

The profile of $\overline{u_3\omega_2}$ obtained from expression (17) for the channel flow agrees qualitatively well with the DNS data (not shown here). However, the insertion of $\overline{u_3\omega_2}$ in equation (15) and subsequently in equation (14) for the complete production term, yields P_ε^3 which differs both in magnitude and in sign from the DNS results in the near-wall region. A substantial improvement is achieved if $\partial\overline{u_3^2}/\partial x_2$ is replaced by $\partial\overline{u_2^2}/\partial x_2$, and the Bernard time scale function $Q_4/(1+Q_3Q_4(\partial U_1/\partial x_2)^2)$ by k/ε, Fig. 3a, yielding the expression for P_ε^3

$$P_\varepsilon^3 = -2\nu\frac{\partial\overline{u_1u_2}}{\partial x_2}\frac{\partial^2 U_1}{\partial x_2 \partial x_2} - 0.4\nu\frac{k}{\varepsilon}\frac{\partial\overline{u_2^2}}{\partial x_2}\frac{\partial U_1}{\partial x_2}\frac{\partial^2 U_1}{\partial x_2\partial x_2} \tag{18}$$

The plot of each term in expression (18) and of their sum, i.e. the complete model of $P_{\varepsilon 3}$ computed from DNS data for a plane channel, is presented in Fig. 3b, showing good agreement with the DNS.

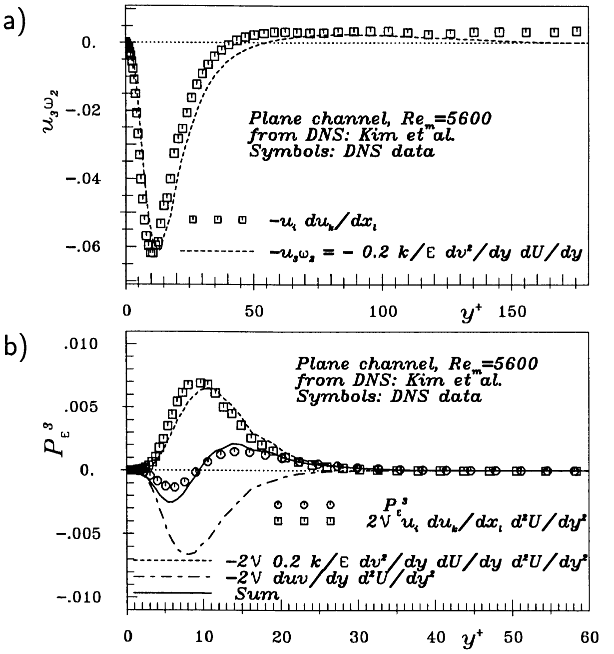

Fig. 3 Modified new model of (a) vorticity flux $\overline{u_3\omega_2}$ and (b) "gradient" production of ε - P_ε^3

Production - destruction term

For the two remaining terms, representing the difference between the turbulent production and viscous destruction of ε, which represents the major source of dissipation at high Re numbers, Hanjalić and Launder (1976) proposed a joint model

$$P_\varepsilon^4 - Y = -2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_l}\frac{\partial u_k}{\partial x_l}} - 2\overline{\left(\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}\right)^2} = -C_{\varepsilon 2} f_\varepsilon \frac{\varepsilon\tilde{\varepsilon}}{k} \quad (19)$$

where $\tilde{\varepsilon} = \varepsilon - 2\nu(\partial k^{1/2}/\partial x_l)^2$. The plot of Eq. (19) with the original function f_ε and the modified one proposed by Coleman and Mansour (1993), shows poor agreement close to the wall for both models, Fig 4a. For illustration, the proposal of Durbin (1991) to replace in the model (19) the time scale $\tau = k/\varepsilon$ by Kolmogorov scale $\tau_K = \sqrt{\nu/\varepsilon}$, when τ_K becomes larger than $\tau/6$, is presented, showing also poor agreement. In contrast, the application of the same model using the homogeneous dissipation rate, i.e. $-C_{\varepsilon 2} f_\varepsilon \varepsilon^h \tilde{\varepsilon}^h/k$ (see eq. (10) for $\tilde{\varepsilon}^h$), yields much better agreement with the DNS data, as shown in Fig. 4b.

The complete ε^h equation

The final form of the new model dissipation equation, expressed solely in terms of ε^h can now be written as

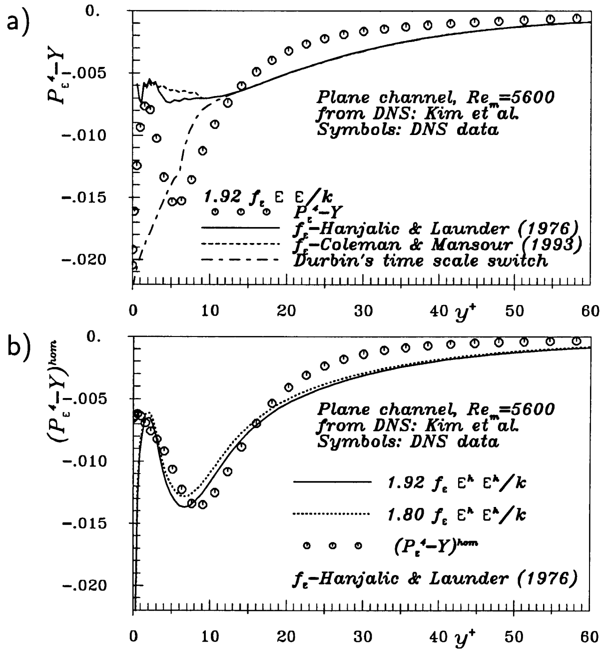

Fig. 4 Model of $P_\varepsilon^4 - Y$ for conventional (a) and new (b) ε-equation

$$\frac{\mathrm{D}\varepsilon^h}{\mathrm{D}t} = \underbrace{-\varepsilon_{ij}^h\frac{\partial U_i}{\partial x_j} - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j}\frac{\varepsilon^h}{k}}_{P_{\varepsilon^h}^1 + P_{\varepsilon^h}^2}$$

$$\underbrace{-2\nu\left(\frac{\partial\overline{u_i u_k}}{\partial x_l}\frac{\partial^2 U_i}{\partial x_k \partial x_l} + C_{\varepsilon 3}\frac{k}{\varepsilon^h}\frac{\partial\overline{u_k u_l}}{\partial x_j}\frac{\partial U_i}{\partial x_k}\frac{\partial^2 U_i}{\partial x_j \partial x_l}\right)}_{P_{\varepsilon^h}^3}$$

$$\underbrace{-C_{\varepsilon 2} f_\varepsilon\frac{\varepsilon^h\tilde{\varepsilon}^h}{k}}_{P_{\varepsilon^h}^4 - Y} + \underbrace{\frac{\partial}{\partial x_k}\left(\frac{1}{2}\nu\delta_{kl} + C_\varepsilon\frac{\varepsilon^h}{k}\overline{u_k u_l}\frac{\partial\varepsilon^h}{\partial x_l}\right)}_{\frac{1}{2}\mathcal{D}_{\varepsilon^h}^\nu + \mathcal{D}_{\varepsilon^h}^t} \quad (20)$$

This equation can now be solved with the model equation for turbulent stress $\overline{u_i u_j}$ that contains conventional modifications for near wall and viscosity effects, but with ε_{ij}^h as the sink term and with the factor 1/2 in front of the viscous diffusion term, as shown in equation (8). It is noted that the full dissipation rate ε does not appear at all in the model and need not be considered.

MODEL PERFORMANCE

Fig. 5 shows an *a priori* test of the equation for ε^h alone for a plane channel: only equation (20) is solved using DNS data for all input variables. Both ε^h and ε show very good agreement with DNS.

Next, in Figs 6 to 8 we present some results computed with the new ε_{ij}^h equation (20) in conjunction with the low-Re-number model for $\overline{u_i u_j}$ equations of Hanjalić and Jakirlić (1995, 1998), for fully developed channel and pipe flows, flow in an axially rotating pipe, and for flow over backward-facing step.

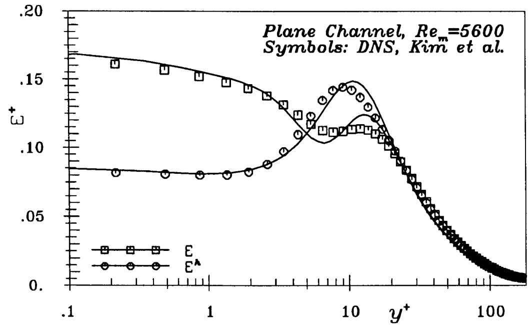

Fig. 5 Computed ε^h and ε in channel flow by new model of ε^h-eq. (only ε^h-eq. solved)

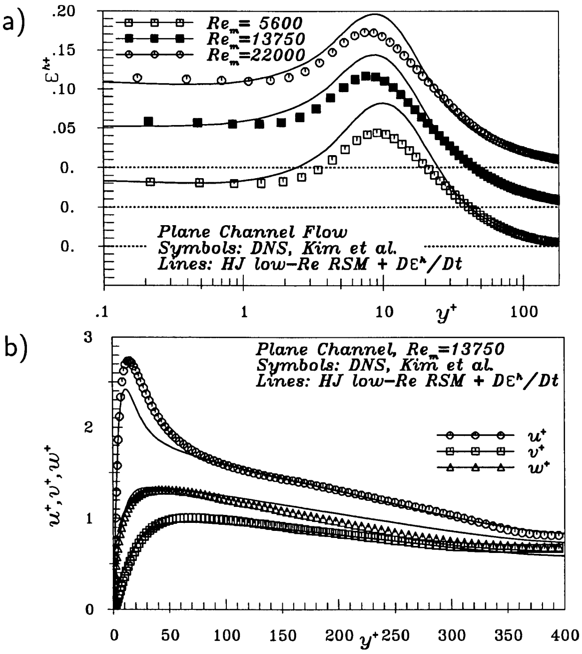

Fig. 6 Computed (a) ε^h and (b) normal stresses in channel flow (Re_m = 5600, 13750 and 22000) with ε^h-eq. in the HJ low-Re RSM

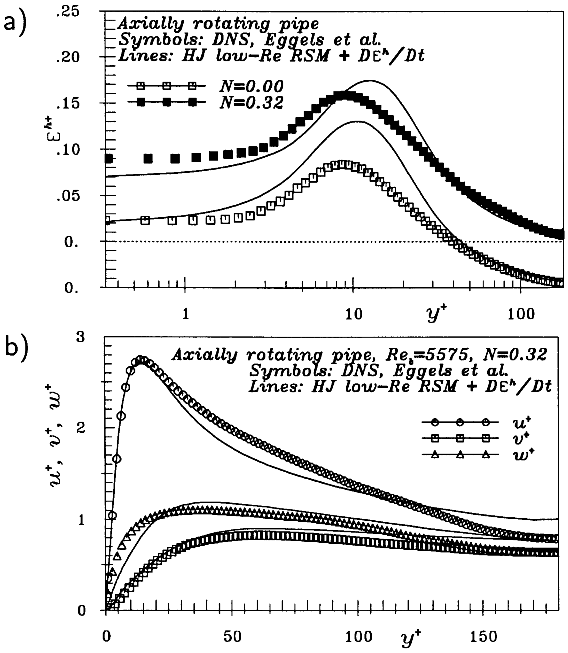

Fig. 7 Computed (a) ε^h and (b) normal stresses in axially rotating pipe flow (N=0.0 and 0.32) with ε^h-eq. in the of HJ low-Re RSM

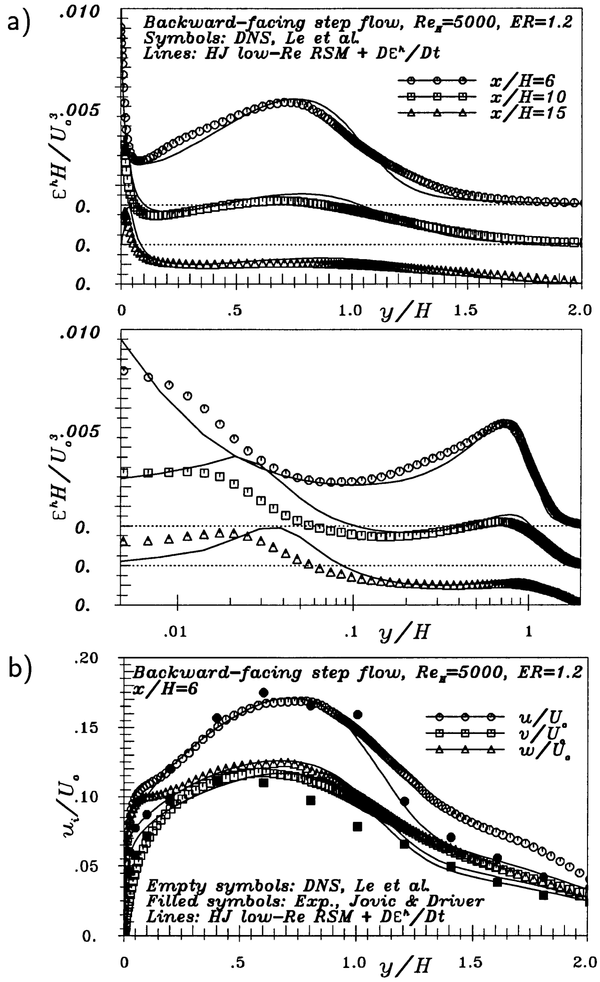

Fig. 8 Computed (a) ε^h and (b) normal stresses at some locations in the back-step flow with ε^h-eq. in the HJ low-Re RSM

We regard these results as satisfactory, particularly in view of the fact that a single model with a single set of empirical coefficients and functions was used to compute several flows with distinct features, including separation, reattachment and flow rotation.

Admittedly, there is still room for improvement, e.g. for rotating pipe, and for reproducing better the DNS value of dissipation at the wall. Two sources of discrepancies were discovered: inadequate representation of off-diagonal components of ε^h_{ij}, and a slight imbalance in the model equation for the wall-normal stress $\overline{u_2u_2}$ at the wall. The latter affects only the asymptotic wall behaviour of $\overline{u_2u_2}$ and has only a marginal effect on other variables. This can be cured by introducing e.g. the 'pressure diffusion' correction of Launder and Tselepidakis (1993) The problem of off-diagonal components of ε^h_{ij} is discussed briefly in the next section.

REVISION OF THE MODEL FOR ε_{IJ}

The model (7) for ε_{ij} reproduces the DNS data for diagonal components in most flows considered, but less satisfactory for the off-diagonal ones (here

ε_{12}). In the here proposed approach ε_{12} plays an important role in reproducing the production term P_ε^1 close to a solid wall. We propose an improvement in the model of ε_{ij} with a particular focus on off-diagonal components, by addition of an extra term to equation, i.e.

$$\varepsilon_{ij}^h = \left[(1-f_s)\frac{2}{3}\delta_{ij} + f_s\,\frac{\overline{u_i u_j}}{k} + 2f\,S_{ij}\tau_K\right]\varepsilon^h \quad (21)$$

Adopting the newly introduced function as

$$f = min\left\{AA_2\,,\ \left[1 - exp\left(-\frac{Re_t}{150}\right)\right]^3\right\} \quad (22)$$

yields a significant improvement in the reproduction of ε_{12} in, e.g., a backward-facing step flow, as shown in Fig. 9.

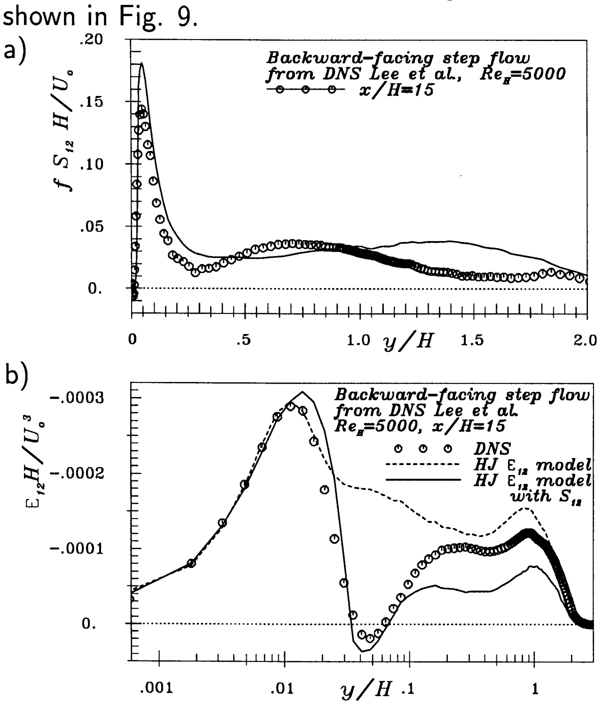

Fig. 9 Profiles of the product fS_{12} and of ε_{12} obtained by new model (equation 21) in the recovery region of the back-step flow

Expression (21) follows from the proposal of Hanjalić and Launder (1980) to enhance the effect of irrotational strain in the ε equation by adding the term $Ck\Omega_{ij}\Omega_{ij}$. Combined with $P_\varepsilon^1 = \varepsilon_{ij}(\partial U_i/\partial x_j)$ (equation 11), these two terms reduce for thin shear flows (for $\Omega_{ij} \approx S_{ij} \approx \frac{1}{2}\partial U_i/\partial x_j$) to

$$(\varepsilon_{ij} + Cf\tau S_{ij}\varepsilon)\,\frac{\partial U_i}{\partial x_j} \quad (23)$$

Replacing ε_{ij} by expression (7) and τ by τ_K, applied to ε^h instead to ε, leads to expression (21).

The above proposal serves more as an illustration of the necessity to account for the effect of mean rate of strain on ε_{ij} rather than to propose a new model. Further work, with more extensive testing in a wider variety of flows, is needed before a more definite model is proposed.

CONCLUSIONS

Based on two-point covariance analysis and reformulation of the models of each term using some arguments from the vorticity transport theory, a transport equation is derived homogeneous dissipation $\varepsilon^h = \varepsilon - 1/2\nu(\partial^2 k/\partial x_l \partial x_l)$, which should replace the classic ε equation in both the $k-\varepsilon$ and Reynolds stress models (RSM) for near-wall flows. An algebraic model for ε_{ij} is also proposed for the RSM. A consistent use of ε^h and ε_{ij}^h in the complete model provides several benefits: it ensures to satisfy wall-limits without using any topology parameter, reduces the necessity for empirical inputs and enables better term-by-term reproduction of DNS data. Both, *a priori* and full model computations of turbulent flows in a plane channel, constant-pressure boundary layer, behind a backward-facing step and in axially rotating pipe, produced results for second-moment turbulence correlations and stress budget in good agreement with the available DNS results.

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REFERENCES

Bernard, P. S. (1990): *Theoretical and Computational Fluid Dynamics* **2**, 165-183.

Coleman, G.N. and Mansour, N.N. (1993): *Turbulent Shear Flows*, **8**, Editors: Durst, F. *et al.*, Springer, 269-282,

Durbin, P.A. (1991): *Theoret. Comput. Fluid Dynamics*, **3**, 1-13

Eggels, J.G.M., Boersma, B.J., and Nieuwstadt, F.T.M. (1994): Lab. for Aero- und Hydrodynamics, Delft University of Technology, October

Hanjalić, K. and Jakirlić, S. (1993): *Appl. Scientific Research* **51**, 513-518, 1993

Hanjalić, K. and Jakirlić, S. (1998): Computers and Fluids, **27**, No. 2, 137-156.

Hanjalić, K. and Launder, B.E. (1976): *J. Fluid Mech.* **74**, 593-610.

Hanjalić, K. and Launder, B.E. (1980): *J. of Fluids Engineering.* **102**, 34-40.

Jakirlić, S. and Hanjalić, K. (1995): Proc. 10th Symp. on Turbulent Shear Flows, The Pennsylvania State University, Vol. 3, 3.25-23.30

Jovanović, J., Ye, Q.-Y. and Durst, F. (1995): *J. Fluid Mech.* **293**, 321-347.

Launder, B.E. and Tselepidakis, D.P. (1993): In *Turbulent Shear Flows* **8**, eds. F. Durst *et al.* Springer, Berlin, 81-96

Le, H., Moin, P. and Kim, J. (1997): *J. Fluid Mech.*, Vol. 330, pp 349-374

Mansour, N. N., Kim, J. and Moin, P. (1988): *J. Fluid Mech*, **194**, 15-44.