SUBGRID-SCALE DYNAMIC MODELLING IN LES OF TURBULENT BUBBLY FLOWS

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Abstract The contribution deals with the large-eddy simulation of a turbulent, vertical mixing layer laden with bubbles at low void-fraction. The subgrid-scale modelling is based on the Smagorinsky kernel, in both its original form and the dynamic procedure of Germano. A new model is proposed for possible bubble-induced turbulence modulation, in which the mixing-length of the dispersed phase at the subgrid-scale is inferred dynamically from the resolved flow field. A two-dimensional idealization of the flow permitted to examine the role of the ratio of the cut-off filter to the bubble diameter, the effect of varying the lift coefficient, and the performance of the subgrid-scale models.

INTRODUCTION

Three-dimensional mixing of multiphase flows may occur in a multitude of applications. Industrial applications include gas stirring of liquid metal ladles in several metallurgical processes, and venting of vapour mixtures to liquid pools in chemical and nuclear reactors. Bubbly flows may also play an important role in environmental processes such as the aeration of lakes, mixing of stagnant water and, generally, destratification of water reservoirs. For all these applications, the basic need is to determine the currents induced by the gaseous phase evolving in the surrounding liquid and thereby the consequent mixing and partition of energy, or species concentration, in the core flow.

The computational methodology that has to be followed in this context is the interpenetrating, two-fluid approach; this requires the correct representation of interfacial forces, interphase transfer mechanisms, and the turbulence induced by the shear and the bubbles. In general, apart from the uncertainties with regard to the values of the coefficients associated with the lift, drag and added-mass forces, the remaining open question is how to faithfully reproduce the turbulence contribution, in particular that induced by the bubbles (*Pseudo-Turbulence*). Various models have been proposed to simulate the effect of bubble-induced turbulence, most developed from single-phase two-equation turbulence models (Lopez de Bertodano *et al.*, 1994; Sato *et al.*, 1981; Smith & Milelli, 1998). Within the RANS approach, the ideas were often materialized in terms of a linear superposition of the shear-induced and bubble-induced stresses in the equations for the liquid phase.

The problem with most of these approximations is the presence of *ad-hoc* tunable coefficients. A global strategy dispensing entirely with what has been proposed hitherto appears therefore to be needed. In this paper, we explore the use of the Large Eddy Simulation (LES) approach for bubbly flows. In LES, the large scales of turbulence are directly solved, whereas the smallest ones are modelled. The interest in adopting LES in the present context is to permit the bubbles to directly interact with eddies which have at least the same size, but not with the smaller ones. The dissipation of turbulent energy at the subgrid-scale level requires a subgrid-scale model (SGS).

The work aims precisely at testing two different models for approximating the subgrid-scale global dissipative effect. This is completed in the context of the SGS modelling strategy based on the Smagorinsky (1963) kernel, in both its original form and the dynamic procedure (DSM) of Germano et al. (1991). The test case selected here is the bubbly mixing layer studied experimentally by Roig *et al.* (1997). The experimental facility, shown schematically in Fig. 1, consists of a vertical square-channel air-water loop. The convergent channel is divided at the bottom into two parts

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by a splitter plate, each side being supplied independently by a mixture of bubbles and water at specified rates. The advantage of opting for this case study is that the flow can be considered as statistically two-dimensional, which helps conducting sensitivity runs focusing on the influence of the width of the cut-off filter, the effect of varying the lift coefficient, and the predictive performance of the SGS models.

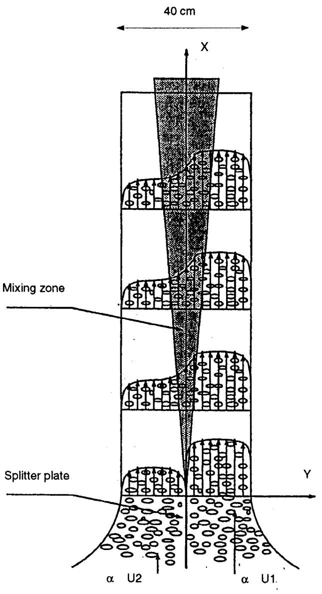

Figure 1: Experimental set-up (Roig *et al.*, 1997)

GOVERNING EQUATIONS

Using the interpenetrating media formalism, separate conservation equations are required for each phase, together with the inter-phase exchange terms. The phases are treated as two interpenetrating continua: each point in the mixture is occupied simultaneously (in variable proportions) by both phases. See Lahey & Drew (1988) for more details. For isothermal gas-liquid mixtures without phase change, the filtered transport equations can be formulated as follows:

$$\partial_t(\widehat{\rho}^k) + (\widehat{\rho}^k \widetilde{U}_j^k)_{,j} = 0, \quad (1)$$

$$D_t\left(\widehat{\rho}^k \widetilde{U}_i^k\right) = -(\alpha^k \widetilde{p})_{,i} + \widetilde{\sigma}_{ij,j}^k - \tau_{ij,j}^k + F_j^k \quad (2)$$

where the superscript k refers either to the liquid or the gas phase, α^k the filtered void-fraction, $\widehat{\rho}^k = \alpha^k \rho^k$ the apparent density, and $\widetilde{\sigma}_{ij}^k$ the filtered viscous stress. The subgrid stress defined as $\tau_{ij}^k \equiv \widehat{\rho}^k \widetilde{u_i u_j}^k - \widehat{\rho}^k \widetilde{u_i}^k \widetilde{u_j}^k$ results from filtering the instantaneous equations; the subgrid-scale field $u^k(\mathbf{x}')$ includes possible bubble-induced fluctuations. Note that since the pre-filtered equations are already volume averaged (or phase filtered), the cut-off filter $\widetilde{\Delta}$ must in all cases be larger than the length-scale characteristic of the dispersed phase, e.g. the bubble diameter. The source term F^k encompasses the buoyancy, drag, lift and added-mass forces: $F^g = -F^l$ denotes the averaged force exerted by the liquid on the bubble (c.f. Smith, 1998). The drag and virtual mass coefficients can be set to $C_{vm} = 0.5$ and $C_D = 0.44$, respectively (Davidson, 1990). However, in contrast to Drew & Lahey's (1987) recommendations according to which the lift coefficient C_L should be set to 0.5, changing this value is suspected to significantly alter the results. Outputs of various runs with variable C_L values will be discussed herein.

SUBGRID-SCALE MODELLING

The subgrid-scale stress tensor was approximated by invoking the conventional Boussinesq hypothesis in which the deviatoric part of τ_{ij} is linearly tied to the resolved strain rate tensor $\widetilde{S}_{ij}$. According to Smagorinsky (1963), the turbulent viscosity can be written as

$$\mu_{sgs}^k \equiv \ell^2/\tau = (C_s\widetilde{\Delta})^2 \rho^k |\widetilde{S}| \quad (3)$$

where the cut-off filter is associated with the cell volume ($\ell \equiv C_s\widetilde{\Delta}$), and the inverse of the time scale τ is represented by $|\widetilde{S}|$, the second invariant of $\widetilde{S}_{ij}$. The model was employed in its original form, with $C_s = 0.12$ (c.f. Tran, 1997), and using the dynamic procedure of Germano *et al.* (1991), in which the C_s is determined locally from the filtered velocity field[1].

The SGS modelling for the bubble-induced dissipation in the liquid phase follows the paths prevailing in the RANS framework: this consists of promoting the deviatoric part of Reynolds stresses (or indirectly the eddy viscosity). The first approach tested here is that of Tran (1997):

$$\mu_t^l = \mu_{sgs}^l (1 + C_f \alpha^g 6\pi \frac{D\mu^l}{\widetilde{\Delta}\mu_{sgs}^l})^{\frac{1}{3}}, \quad (4)$$

in which μ^l is the molecular viscosity, D the bubble diameter, and $C_f = 0.17$ a model constant. The model is in effect grossly dissipative since the coefficients C_f is fixed. What is proposed here is a *hybrid* approach that takes directions from the scaling adopted by Sato *et al.* (1981), but the mixing-length of the

[1]Here the ratio of the test filter to the grid filter was taken equal to 2

dispersed phase is now inferred from the resolved flow field. The argument is that since the bubbles are known to have the tendency to break the largest subgrid-scale eddies into their size, it is likely that their mixing length (say ℓ_b) may be comparable to the cut-off length-scale ℓ, but not their velocity scale (only if the bubbles follow the liquid motion). This idea also finds support in Bardina et al.'s (1980) Scale-Similarity Principle according to which the smallest resolved scales are similar to the largest modelled ones; the latter are in this case the bubbles themselves. Now, if we argue that the excess in momentum diffusivity of the liquid caused by the relative motion between the phases can be scaled as $\mu_b \equiv \ell_b v_b$, then the effective subgrid-scale viscosity should take the following form

$$\mu_t^l = \mu_{sgs}^l + (C_s\Delta)\rho_l\alpha_g^{1/n}|u_g - u_l| \qquad (5)$$

Proceeding so means that the mean eddy fluctuation v_b can only be triggered by the slip velocity. The only unknown in Eq. (5) is the exponent n^{-1} which is introduced as an indicator for the influence of bubble concentration on μ_b. According to Lance & Bataille (1991), for $\alpha^g < 2\%$ the bubble-induced turbulence varies quasi-linearly with void fraction, thus, n can be taken equal to unity. Eq. (5) reflects a precise physical mechanism: that is, the dissipation of turbulent kinetic energy that may occur at the subgrid-scale level due to the superposition of the liquid fluctuations and those induced by the bubbles are dictated by the energy-containing eddies. Furthermore, the incorporation of locally determined length scales $\ell_b(\mathbf{x}, t) \equiv (C_s\widetilde{\Delta})$ for the dispersed phase may also be interpreted as a measure for accounting for possible increase in turbulence (whenever $C_s > 0$) or modification of the length scales towards isotropy via a reduction of the shear stress (whenever $C_s < 0$). This physical mechanism was in particular evoked by Lance & Bataille (1991).

SIMULATION SET-UP

In the sensitivity part of the investigation the flow has been calculated in two dimensions, taking the risk of violating the LES method which is conceptually three-dimensional. The computational domain was deliberately truncated compared to the experiment ($30cm$ width and $60cm$ height), and slip boundary conditions were thereby imposed on the lateral planes. For this particular case (in 2D), this measure was found to be equivalent to extending the domain and imposing non-slip wall conditions[2]. On top of the domain a constant pressure boundary condition was imposed. At the bottom, the liquid and the gas were injected with the profiles measured at the end of the splitter plate (at $x = -1cm$), a level that corresponds to the inlet plane in the present simulations. The grid was uniformly distributed in order to keep a constant filter width ($\widetilde{\Delta} \equiv dx$); the finest grid consists of a 100×200 nodes ($dx = 3mm$) and the coarse one of 30×40 nodes ($dx = 10mm$). The inlet void fraction, liquid and gas averaged velocities, and liquid U_{rms} velocity profiles were taken from the experiment. The other rms components were assumed to be random deviates of a Gaussian probability distribution with zero mean and standard deviation deduced from the profile of U_{rms}. The situation considered here consists of a constant void fraction distribution of 1.9% with bubbles of $3mm$ diameter. The injected liquid velocities were $0.22m/s$ in the slow channel (left), and $0.54m/s$ in the fast channel (right). The slip velocity at injection was $\approx 30cm/s$. The calculations were performed using an unstructured, multi-block, multi-grid finite-volume code employing a fully co-located storage arrangement. The calculations were carried out for about 5000 time steps (with $\Delta t = 0.001s$) to reach a statistically steady-state solution. A second-order central differencing scheme was used for spatial discretization, and a second-order fully implicit backward differencing scheme for time marching. The solution was iterated to convergence using a pressure-correction approach.

CALCULATIONS

The implications of both the filter width and the lift coefficient on the results were examined within the context of the standard Smagorinsky SGS model. Results of U_{rms} distributions obtained with varying C_L for the coarse and fine grids are compared in Fig. 2 with the experiments. They clearly indicate that independently from the value assigned to C_L, rigorous resolution requires the cut-off filter to be comparable to the bubble size. With the coarse grid the rms velocities were overall grossly exaggerated and the filtered void fraction and velocity profiles rather flat (results not shown here). More precise indications on the effect of varying the lift coefficient can be seen in the context of Fig. 3 comparing the void fraction

[2] In this case, resort was made to the wall-function approach of Werner & Wengle (1989).

distributions at two elevations using the fine grid. But, looking at Fig. 2 already suggests that without the lift force the simulation misrepresents the lateral spreading of the plume at the expense of a strong oscillation in the *rms* magnitude. Figure 3 confirms this result throughout the ragged profiles of void fraction for $C_L = 0$, but it also reports overpredicted peaks for $C_L = 0.5$. Judging from this Figure in particular, one is tempted to conclude that $C_L = 0.25$ is the best compromise for this class of flow.

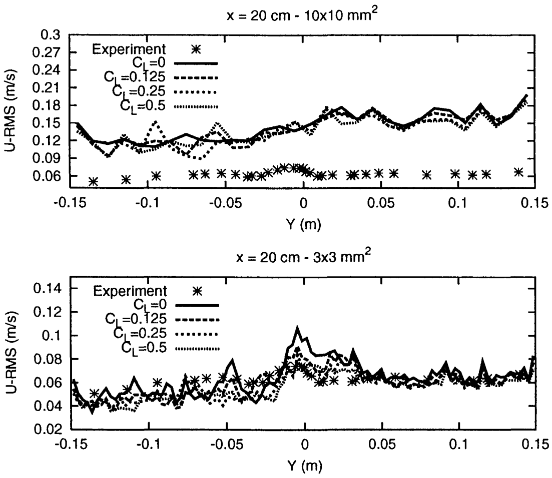

Figure 2: U_{rms} distributions for the two meshes. Calculations with variable C_L and SGS model with $C_s = 0.12$

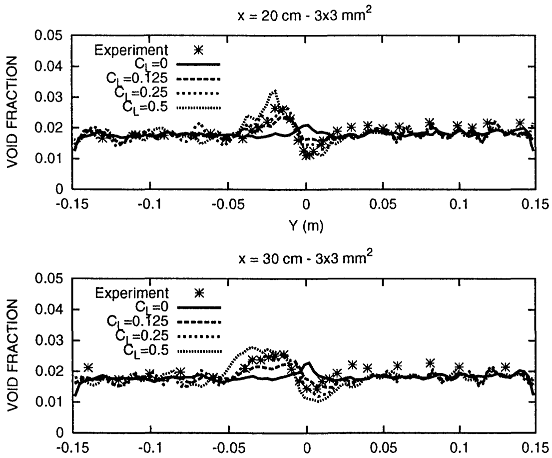

Figure 3: Void fraction distributions for the fine mesh. Calculations with variable C_L and SGS model with $C_s = 0.12$

In summary, it appears that with a cut-off length scale significantly larger than the bubble size ($\widetilde{\Delta}/D = 3.33$), the simulation cannot capture all the important scales involved in the flow. The best results were obtained with $\widetilde{\Delta}/D = 1.5$. On the other hand, it is likely that in the absence of bubbles a rigorous LES of this flow would require an equal grid resolution, owing to the high rate of dilution of the dispersed phase (1.9% only). But apart from that, a large ratio of ℓ/ℓ_b will leave an unresolved gap of eddies of intermediate length-scales interacting with the bubbles that need to be modelled.

A further optimization exercise, in which $\widetilde{\Delta}/D = 1.6$ and $C_L = 0.25$, led to the results displayed in Fig. 4. There it is shown that both the velocity distributions and void fraction distributions compare well with the experiment, but not the *rms* values away from the mixing zone. This points to two plausible reasons: either the three-dimensionality of the flow has an appreciable *global* effect on the fluctuating field, in which case a 2D idealization is restrictive, or the bubble-induced fluctuations at the large-scale level in weak-shear regions are not well captured.

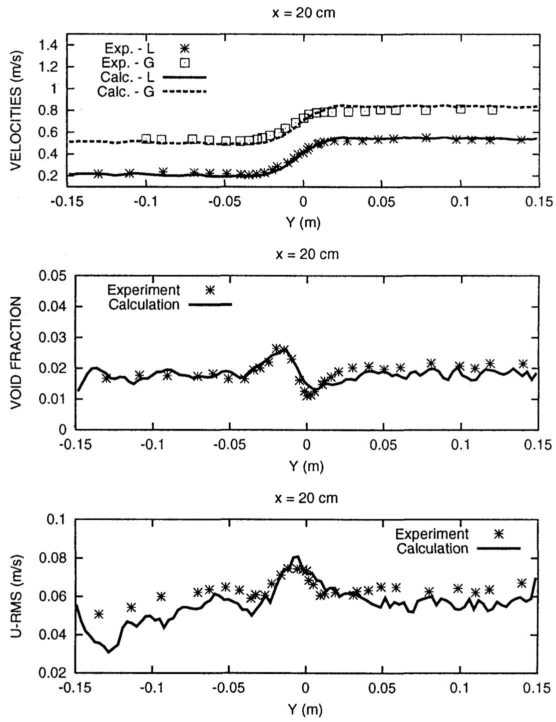

Figure 4: Distributions of mean velocities, void fraction and U_rms. Calculations with $\widetilde{\Delta}/D = 1.6$ and $C_s = 0.25$

The predictive performance of the employed SGS models is discussed within the context of Fig. 5 comparing the liquid velocity fluctuations with the experiments. Immediately after the injection location, at x=6cm, the standard SGS model with constant C_s delivers an overpredicted level of U_{rms} as compared with the DSM. The panel also shows that the performance of the DSM is better in the shear region, and in the rapid channel, than in the slow channel away from the mixing zone. The tendency is somewhat inverted at the next location, in the sense that the results of the standard SGS model are the closest to the experiment. The misrepresentation of the rms values away from the shear region is equally shared by both models; the possible explanations for this behaviour have already been described. The reason why

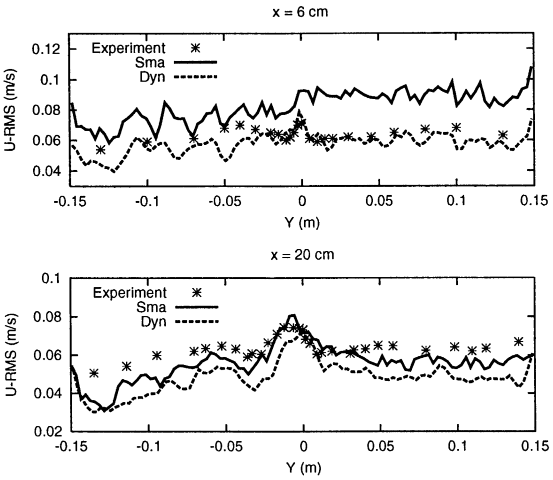

Figure 5: U_{rms} distributions obtained with both SGS models. Calculations with $\widetilde{\Delta}/D = 1.6$ and $C_L = 0.2$

at x=6cm the velocity fluctuations are more pronounced with the standard model is probably because there the flow is still affected by the proximity of the injection, in which case the actual C_s level is well above the value of 0.12 (it has a space average of 0.147). This naturally leads to a pronounced effective viscosity that attenuates further the fluctuating field, compared with the standard model. Looking at the next location (x=20cm) reveals that the level of U_{rms} is generally underpredicted, more with the DSM than with standard model. This is a misleading result since at that location the values of C_s were in average smaller than 0.12. At x=30cm where the averaged C_s value was found to converge towards 0.121, the standard and modified SGS models deliver almost the same result (results not shown). Figure 6 compares the time-averaged liquid and gas velocity distributions as delivered by the DSM approach combined with the Tran model (Eq. 4) and the present one for bubble-induced dissipation (Eq. 5). The results, which show an overall self-preserving behaviour, indicate that these quantities are not sensitive to any of the two models. The same remark holds for the mean void-fractions (results not included here). Overall, the predicted quantities agree very well with the experiment. The U_{rms} distributions plotted in Fig. 7 are in line with the observations made from the previous Figure; i.e. the fluctuating field feels the effect of promoting the eddy viscosity as a secondary effect only. When looking at Fig. 8, displaying

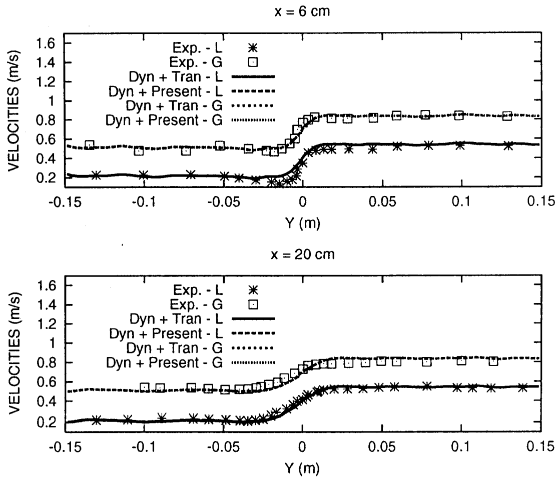

Figure 6: Velocity distributions obtained with various SGS models. Calculations with $\widetilde{\Delta}/D = 1.6$ and $C_L = 0.25$

the distributions of the ratio μ_t/μ, the previous remark may sound somewhat misleading, since the effective viscosity seems now to be enhanced (by a factor of 2 at x=20 and 30cm), in particular away from the mixing zone. But obviously, the shear stresses are important only in the regions where appreciable velocity gradients occur. In summary, always in the context of an idealized 2D simulation, it seems that casting the bubble-induced rms velocity field in terms of a pure shear-type model is not sufficient. This conforms with the observations of Roig *et al.* and Lance & Bataille who preferred to regard this effect as a superimposed kinetic energy induced by the the square of the slip velocity. A final conclusion cannot be drawn without looking at three-dimensional calculations. Also, a close inspection of Fig. 8 raises an additional point: at both elevations, around the mixing zone, the magnitude of μ_t/μ is comparable to those delivered by the DSM alone. This could *tentatively* be interpreted as the capacity of the proposed model to let the shear stress be solely dictated by the strength of the rate of strain $\widetilde{S}_{ij}$ where appropriate. What is finally worth noting from Fig. 8 is that the Tran (1997) model seems to have negligible impact when compared with the proposed one, which is not surprising (from a formulation point of view, at least) since the presence of the power 1/3 in Eq. (4) tends to smooth this term out.

CONCLUSIONS

Large-eddy simulation of a bubbly, turbulent, vertical shear flow has been performed. The test case was experimentally studied by Roig et al. (1997). The statistically two-

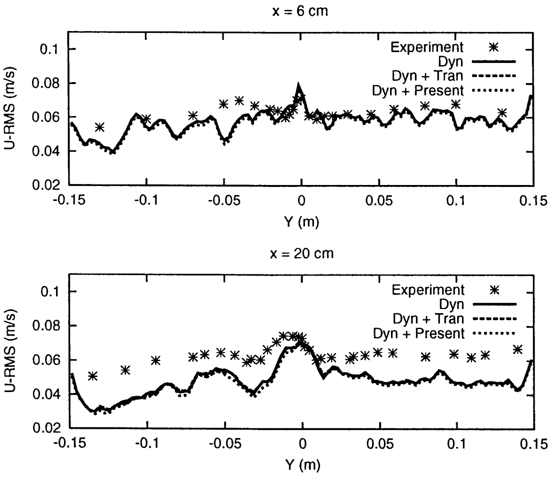

Figure 7: U_{rms} distributions obtained with various DSM-based models. Calculations with $\widetilde{\Delta}/D = 1.6$ and $C_L = 0.25$

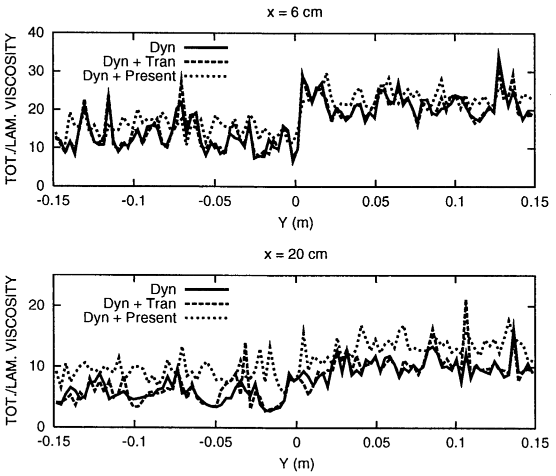

Figure 8: Distributions of μ_t/μ obtained with various DSM-based models. Calculations with $\widetilde{\Delta}/D = 1.6$ and $C_L = 0.25$

dimensional structure of the flow simplified appreciably the computations, enabling various sensitivity runs to be performed.

According to the simulations, the optimum grid concentration has to be such that the ratio of the cut-off filter to the bubble diameter $\widetilde{\Delta}/D$ converges towards 1.5. Forcing the cut-off length-scale to coincide with that of the dispersed phase permits the interaction with the smallest-resolved scale to be captured without additional approximation. This is in contradiction with Tran's (1997) arguments where the use of a computational grid much coarser than the diameter of the bubbles is advocated. The simulations have revealed that fixing the lift coefficient to $C_L = 0.5$ is probably exaggerated, and a good compromise can be obtained with $C_L = 0.25$. Without including this force, however, the calculations misrepresent the distribution of void fractions.

The investigation has also shown that the conventional Smagorinsky model performs quite well and gives results comparable to those with the dynamic procedure of Germano. An eventual backscatter effect of energy was not noticed in the simulations. A final conclusion in this respect could be made only in the light of three-dimensional simulations. Modifying the subgrid-scale model to account for bubble-induced turbulence modification did not bring the expected results. The only positive outcome of the proposed model is that it has a proven capacity of increasing or attenuating the eddy diffusivity where appropriate. It was also found that accounting for this effect only via a shear-type model is not sufficient. It would be intriguing to see how the proposed model would perform in a case where the overall motion is derived by the presence of bubbles, rather than by shear, e.g. a bubble plume. This investigation is presently underway, together with the three-dimensional LES of the flow considered in this contribution.

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