REYNOLDS-NUMBER DEPENDENCE OF STREAMWISE VELOCITY FLUCTUATIONS IN TURBULENT PIPE FLOW

Jonathan F. Morrison
Department of Aeronautics, Imperial College
London SW7 2BY, England

Weimin Jiang
CARDC, P. O. Box 211 Mainyang
Sichuan 621000, P. R. China

Beverley J. McKeon and Alexander J. Smits
Department of Mechanical & Aerospace Engineering, Princeton University
Princeton, NJ 08544, USA

ABSTRACT

Statistics of the streamwise velocity component in fully-developed pipe flow are examined for Reynolds numbers in the range $5.5 \times 10^4 \leq Re_D \leq 3.1 \times 10^6$. The second moment exhibits two maxima: one in the viscous sublayer is Reynolds-number dependent while the other, near the lower edge of the log region, is also Reynolds-number dependent and follows roughly the peak in Reynolds shear stress. The behaviour of both peaks is entirely consistent with the concept of inactive motion which increases with increasing Reynolds number and decreasing distance from the wall. No simple scaling is apparent. The second moment is compared with empirical and theoretical scaling laws and some anomalies are apparent. The scaling of spectra using y, R and u_τ is examined. It appears that even at the highest Reynolds number, they exhibit incomplete similarity only: while spectra do collapse with either inner or outer scales for limited ranges of wave number, these ranges do not overlap. Thus similarity may not be described as complete and any apparent k_1^{-1} range does not attract any special significance and does not involve universal constants. It is suggested that this is because of the influence of inactive motion. Spectra also show the presence of very long structures close to the wall.

INTRODUCTION

Recent results from the "superpipe" facility at Princeton University (Zagarola and Smits, 1998) have shown that the mean velocity profile outside the viscous sublayer in fully-developed pipe flow exhibits a power-law dependence in the region $60 < y^+ < 500$, and that outside this region, the mean velocity follows a log law provided $R^+ < 9000$. R^+ is the Kármán number based on pipe radius. The power-law dependence of the mean velocity on y^+ expresses the direct influence of viscosity outside the sublayer while the log-law dependence is an expression of a self-similar region in which the mean velocity scales on the wall-friction velocity, u_τ, and the distance from the wall, y, only. The literature abounds with data for near-wall turbulent flow showing that the higher moments do *not* scale simply, and this can be explained by Townsend's (1961) theory concerning "active" and "inactive" motion See also Bradshaw (1967) and Morrison *et al.* (1992). Of particular importance is the difference in the behaviour of wall-parallel (u, w) and wall-normal (v) velocity components as the wall is approached: the "impermeability constraint", which affects eddies out to a distance from the wall that is of the order of the eddy size is responsible for an increase of the former at the expense of the latter. Thus, to a first order, the inactive motion in the wall-parallel components carries no shear stress: by definition, the active motion is the shear-stress bearing motion. There are also many measurements that show that the active motion does not scale on inner variables either, a particular result being that the "constant stress" region $(-\overline{uv} \approx u_\tau^2)$ does not hold, except in the limit of very high Reynolds number. These effects may be traced to the direct influence of viscosity outside the sublayer. Given these difficulties, it is hardly surprising that simple arguments involving the overlap of scales (so giving rise to self-similarity) may well be inappropriate.

Since publication of Townsend's seminal work, considerable attention has been devoted to the deduction of spectral forms associated with the self-similar nature of "attached wall eddies". Such self-simlarity manifests itself at 'high' Reynolds numbers as a range of streamwise wave numbers, k_1, in which the spectrum of the streamwise velocity, $\phi_{11} \propto u_\tau^2 k_1^{-1}$. It should be noted that a prescribed slope over some region of wave number can usually be found in turbulence spectra on log-log axes. A simple theory for pipe flow was proposed by Perry and Abell (1977) and Perry *et al.* (1986), but it is equally appropriate for boundary layers (Perry and Li, 1990, Marušiç and Perry, 1995, Jones *et al.*, 2001). Given the prominence of the theory and its potential usefulness (so much so that its existence at practical Reynolds numbers is often taken for granted – Nikora, 1999), and given the uniqueness of the present results (in terms of the high Reynolds numbers) a careful reappraisal is clearly needed.

Scalings for 'large' scales (in which the direct effects of viscosity may be neglected) contributing to the streamwise velocity component may be scaled using either inner or outer scales. Outer scaling suggests that y is not important and that dimensional analysis therefore yields

$$\frac{\phi_{11}(k_1)}{Ru_\tau^2} = \frac{\phi_{11}(k_1R)}{u_\tau^2} = g_1(k_1R), \qquad (1)$$

while, alternatively, inner scaling suggests the exclusion of R as a relevant length scale so that, at higher wave numbers,

$$\frac{\phi_{11}(k_1)}{yu_\tau^2} = \frac{\phi_{11}(k_1y)}{u_\tau^2} = g_2(k_1y). \qquad (2)$$

The veracity of these scalings is usually judged by the degree of collapse of the spectra at wave numbers lower than that at which spectral transfer (at high Reynolds numbers, given by the mean dissipation rate) becomes important. In the range of wave numbers $R^{-1} < k_1 < y^{-1}$ over which both Eq. (1) and Eq. (2) are valid (that is collapse is evident with *both* scalings, as required by asymptotic matching), it then follows that

$$\phi_{11}(k_1) = Ru_\tau^2 g_1(k_1R) = yu_\tau^2 g_2(k_1y). \qquad (3)$$

Dimensional arguments and direct proportionality between g_1 and g_2 therefore imply

$$\frac{\phi_{11}(k_1R)}{u_\tau^2} = \frac{A_1}{k_1R} = g_1(k_1R), \qquad (4)$$

and

$$\frac{\phi_{11}(k_1y)}{u_\tau^2} = \frac{A_1}{k_1y} = g_2(k_1y), \qquad (5)$$

where A_1 is a universal constant. Collapse with both length scales therefore suggests a self-similar structure such that $\phi_{11}(k_1) \propto u_\tau^2 k_1^{-1}$. One could therefore call this situation "complete similarity". However, it is possible that, for example, while y and u_τ might form a complete parameter set to define the motion in the range of wave numbers over which collapse is apparent (Eq. (2)), these wave numbers might in fact be too high for collapse to be possible using R and u_τ (Eq. (1)). Thus *simultaneous* collapse is not possible. We shall refer to this situation as "incomplete similarity", in which case the constant A_1 in Eqs. (4) and (5) cannot be universal.

Note that this analysis is predicated on two principal assumptions. The first is that the kinematic viscosity, ν, does not enter the problem. This requires that $k_1 \ll u_\tau/\nu$. In turn, this requires the Reynolds number to be sufficiently high, or equivalently that y is sufficiently large such the energy-containing scales are not affected directly by viscosity. The second assumption is that u_τ is the correct velocity scale for both the inner and outer regions. In particular, in conformity with Townsend's theory, it supposes that inactive motion arises primarily through the influence of attached eddies and that therefore u_τ is the appropriate velocity scale. Note also that this analysis does not apply to the wall-normal velocity component which is blocked at wave numbers $k_1 \sim y^{-1}$.

Below, spectra are presented in premultiplied form on linear-log axes. A linear ordinate enables a closer scrutiny of scalings than that afforded by a logarithmic one. In addition, the use of non-dimensional axes ensures that not only the ordinate, but also the area under the spectra are directly proportional to energy. Thus integration of the spectra yields $\overline{u^2}^+ = \overline{u^2}/u_\tau^2$. Spectra are therefore in the form:

$$\frac{k_1R\phi_{11}(k_1R)}{u_\tau^2} = h_1(k_1R), \qquad (6)$$

for outer scaling, and for inner scaling, in the form:

$$\frac{k_1 y \phi_{11}(k_1 y)}{u_\tau^2} = h_2(k_1 y). \qquad (7)$$

In the context of assessing these scalings for data in the present experiment, it is useful to clarify precisely what the foregoing analysis indicates. Strictly, as long as $\nu/u_\tau \ll y \ll R$ (the Reynolds number is 'high'), Eqs. (4) and (5) should both show a k_1^{-1} range for $R^{-1} \ll k_1 \ll y^{-1}$. However, in order to remove the ambiguity concerning the relative values of y and R, one may fix alternately y in Eq. (4) and then R in Eq. (5). Eq. (6) invites us to retain only R and u_τ as independent variables. Thus while y is fixed, u_τ is varied by changing the pressure drop along the pipe. In practice, this involves a change of Reynolds number (strictly Kármán number) as changes of R are a little more problemmatical. This does not pose a problem as long as the Reynolds number is sufficiently high such that the wave-number range of interest is not directly affected by viscosity. Alternatively, Eq. (7) invites the use of y and u_τ only as independent variables for any fixed R. In this case, y can merely be varied (subject to $\nu/u_\tau \ll y \ll R$) at a fixed Reynolds number, although as long as ν can be neglected, a value of y at *any* Reynolds number might be chosen.

For brevity, in what follows we present spectra (obviously using both inner and outer scaling) at different y/R for the lowest and highest Reynolds number. The spectra at fixed y/R for several Reynolds numbers confirm the present conclusions and are presented elsewhere.

EXPERIMENTAL PROCEDURE

Details of the pressurised pipe and results from extensive pitot-tube measurements are provided in Zagarola and Smits (1998). Issues regarding the fully-developed nature of the flow, axisymmetry and temperature control have been dealt with exhaustively by them. Morrison and Smits (2001) have also fully addressed issues regarding roughness and conclude, as Zagarola and Smits (1998) do also, that the "superpipe" is smooth for all Reynolds numbers $Re_D < 35 \times 10^6$. Here, velocity measurements are made using standard hot-wire techniques using wires with length-to-diameter ratios, $l/d = 200$, and for $Re_D \geq 3 \times 10^6$, 100. Convergence of all moments is better than 1%, except for the third moment in the region $10 \leq y^+ \leq 30$. In this range the convergence of the third moment is better than 10% only because here its value is numerically close to zero. Positional accuracy as a fraction of R is better than 0.02%. Hot-wire calibration is achieved using a fourth-order polynomial, the signal being sampled using 12-bit A-D conversion. In order to maximise spatial resolution, the mean velocity is kept as low as possible, the sample rate being set such that the equivalent spatial resolution is slightly better than that set by the wire length. The signal was low-pass filtered at the Nyquist frequency and standard FFT algorithms (Hanning window) are used to calculate the spectra. No curve-fitting is used. $k_1 = 2\pi f/U(y)$.

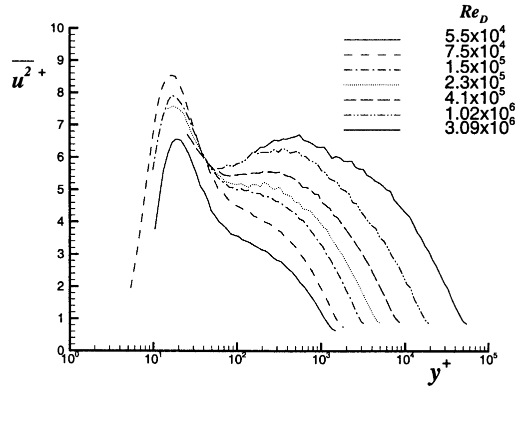

Figure 1: Second moment: wall scaling

RESULTS

Fig. 1 shows $\overline{u^2}^+$, which has *two* maxima: the first, prevalent at low Reynolds numbers, is well documented. With better resolution provided by LDA, the data of den Toonder and Nieuwstadt (1997) suggest that $\overline{u^2}^+$ reaches a maximum of about 7.3 that is constant with Reynolds number up to about 2.5×10^4. However, the present data show that this maximum is, in fact, Reynolds-number dependent, increasing with Reynolds number and reaching 8.6 at $Re_D = 7.5 \times 10^4$. At higher Reynolds numbers, the reduction in this peak with increasing Reynolds number is of course the result of poorer spatial resolution. Note that any resolution effects even at the lowest Re_D would reduce the maximum below that obtained using LDA. It has also been suggested by Mochizuki and Nieuwstadt (1996) that the position of this peak is also independent of Reynolds number at $y^+ \approx 15$. The present data appear to confirm this, although owing to the effects of probe resolution, no firm conclusions may be drawn. The near-wall peak indicates a principal feature of inactive motion, namely that it increases with Reynolds number. Interestingly, this peak coincides with the expected peak of turbulence energy production.

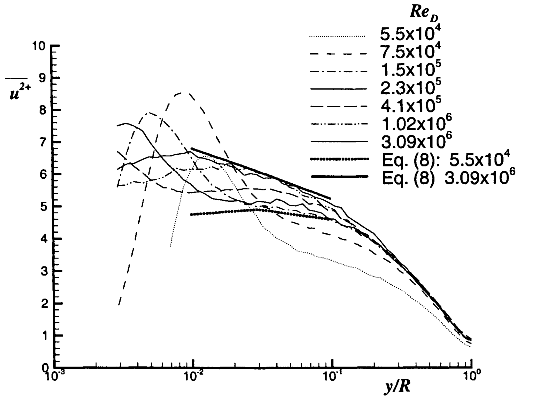

Figure 2: Second moment: outer scaling.

Fig. 2 shows the data of Fig. 1 replotted against y/R. There is a striking collapse in the outer region, $y/R > 0.4$, except at the lowest Reynolds number. There is no collapse in the overlap region, and, as Fig. 1 shows, no collapse anywhere using wall variables, except presumably very close to the wall. Note that the near-wall peak now depends on y/R: this provides a second principal conclusion, namely that inactive motion depends on distance from the wall.

Based on considerations of the self-similar structure of attached wall eddies suggested by Townsend (1976), Perry and Abell (1977) and Perry *et al.* (1986) have suggested logarithmic functional forms for the normal stresses of the surface-parallel velocities:

$$\overline{u^2}^+ = B_1 - A_1 \ln[\frac{y}{R}] - C(y^+)^{-0.5}. \quad (8)$$

The log term is obtained by integration of Eq. (4) or (5) for $R^{-1} < k_1 < y^{-1}$. For comparison with the data of Fig. 2, we take the constants suggested by Perry and Abell (1977): $B_1 = 3.53, A_1 = 0.8$ and $C = 9.54$. They are, respectively, 2.67, 0.9 and 6.06 in Perry *et al.* (1986) but this makes no difference to our conclusions concerning the proposed functional form.

Fig. 2 also shows a comparison with Eq. (8): at $Re_D = 3.09 \times 10^6$, the agreement is good, largely because the viscous deviation term is small. The changes with Reynolds number even at $y/R = 0.1$ derive from the viscous deviation term, which qualitatively predicts the direct influence of viscosity outside the sublayer correctly. At $Re_D = 5.5 \times 10^4$ however, the behaviour at small y/R is incorrect owing to the large inactive contribution. This is one of the two reasons for the change in gradient of the data (and therefore A_1) with Reynolds number. Moreover, for $0.02 < y/R < 0.1$, this change in gradient is not monotonic, first decreasing before increasing. This behaviour is therefore indicative of *two* effects: one the increase in the inactive contribution with increasing Reynolds number the other the reduction in direct viscous effects emanating from the sublayer as the Reynolds number increases. While the latter is estimated quite well (but only for $Re_D > 10^6$), no account of the former is taken in Eq. (8), which appears therefore to require an additional term, the form of which is strongly dependent on the choice of outer velocity scale.

If a spectral self-similar range exists, such that $\phi_{11}(k_1) \propto k_1^{-1}$ ("complete" similarity), the constant of proportionality (A_1 in Eq. (8)) is universal. However, the evidence of Fig. 2, is that the slope of the data in the vicinity of $y/R \approx 0.1$ (where the viscous deviation is negligible and where a k_1^{-1} range is most likely is still increasing at the highest Reynolds number. One should also bear in mind that $\overline{u^2}$, as the integral of ϕ_{11}, is less sensitive to Reynolds-number scalings than the integrand itself. It is possible that at even higher Reynolds numbers, the slope of $\overline{u^2}$ may asymptote to a constant value indicative of complete similarity in ϕ_{11}. Re-writing Eq. (8) and omitting the viscous deviation term

$$\overline{u^2}^+ = B_1 - A_1 \ln[y^+] + A_1 \ln[R^+] \quad (9)$$

shows that the outer peak in Fig. 1 will increase indefinitely with Reynolds number, regardless of considerations of the universality of A_1. One might suppose that, at some stage, the outer peak might become larger than the inner peak in the sublayer. However, this is unlikely as it is realistic to expect the inactive motion near the wall to continue to increase with Reynolds number as long as its source in the outer layer does.

Using inner scaling, Fig. 3 shows $\phi_{11}(k_1 y)$ in the form given by Eq. (7) for $Re_D = 5.50 \times 10^4$ over the range in y for which collapse might be expected. Fig. 4 shows equivalent data for $Re_D = 3.09 \times 10^6$ plotted in the same form. In Fig. 3, it is evident that the Reynolds number is simply too low for collapse to be possible. Note that $R^+ = 1500$ only and that the direct effects of viscosity permeate the whole layer, as evidenced by Fig. 2. At the highest Reynolds number (Fig. 4), there is some collapse for

$0.1 < k_1 y < 10$ approximately, the range increasing with Reynolds number. However, the collapse is not along a horizontal line, suggesting incomplete similarity.

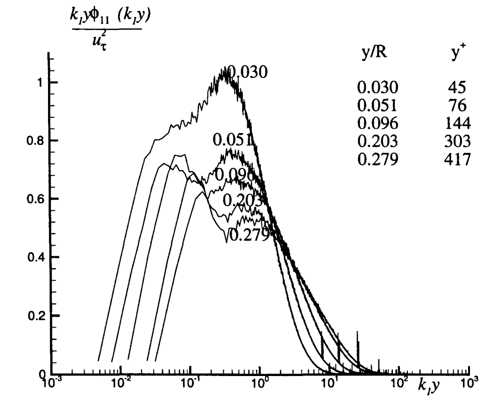

Figure 3: Inner scaling, $Re_D = 5.50 \times 10^4, R^+ = 1.50 \times 10^3$.

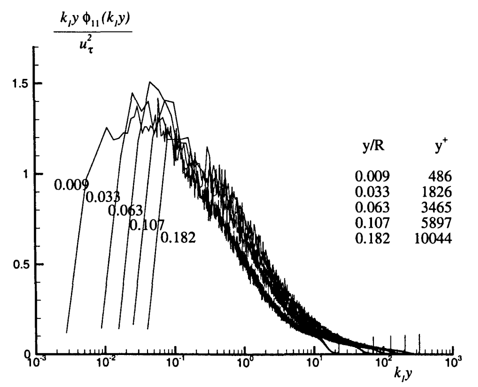

Figure 4: Inner scaling, $Re_D = 3.09 \times 10^6, R^+ = 5.52 \times 10^4$.

Fig. 5 shows the same data as in Fig. 4, but plotted using outer scaling. For $k_1 R \sim 1$, some collapse along a horizontal line is apparent for about half a decade in $k_1 R$, but for $0.033 \leq y/R \leq 0.107$ only. Inspection of Fig. 4 in the region of $k_1 y \sim 0.1$ shows that the same data ($0.033 \leq y/R \leq 0.107$) clearly do not collapse using inner scaling. Instead, spectra for $y/R = 0.033, 0.063$ and 0.107 show discrete peaks, appearing in a wave-number sequence determined by y^{-1}, equivalent to the collapse in Fig. 5 occurring at a point, $k_1 R \approx 0.75$. Since collapse only occurs with outer variables and not inner varuiables, this also suggests incomplete similarity.

Interestingly, as has been shown by Kim and Adrian (1999) and Jiménez (1998), the spectra show the presence of very long structures near the wall giving rise to a bimodal shape at low Reynolds numbers. Their wavelength increases as y increases, reaching a peak of about $10R$ at $y/R \approx 0.1$ before decreasing at larger y.

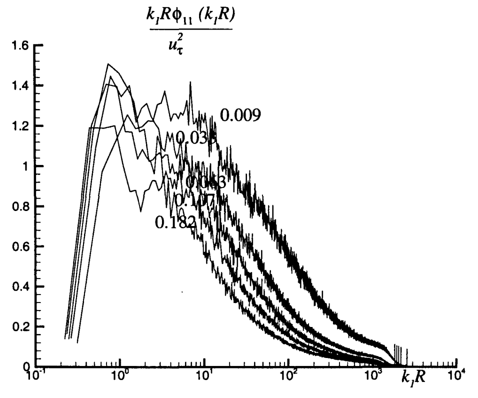

Figure 5: Outer scaling, $Re_D = 3.09 \times 10^6, R^+ = 5.52 \times 10^4$.

DISCUSSION AND CONCLUSIONS

On balance, it would appear that while collapse of the velocity spectra may be possible with either inner or outer scaling (incomplete similarity), it is unlikely that simultaneous collapse with both in the same wave-number range is possible (complete similarity), at least up to the maximum Reynolds number attained here. Thus spectra here do not exhibit a k_1^{-1} range indicative of self-similar structure. The behaviour of $\overline{u^2}$ is consistent with the notions that, (a), inactive motion increases with Reynolds number and that, (b), inactive motion increases as y/R decreases (down to the sublayer). On the basis of (a) and (b) alone, complete similarity as outlined above seems to be unlikely because, while the active motion scales on y and u_τ only (in the limit of infinite Reynolds number) as Townsend proposed, the inactive component always requires three scales, namely y, R, and a velocity scale, in compliance with (a) and (b) above. Whether u_τ is the correct choice of outer velocity scale or not has been questioned and it has been suggested that this should be $(U_{cl} - \overline{U})$ (Zagarola and Smits (1998). This issue has not yet been fully addressed.

It is possible that a k_1^{-1} range may appear at even higher Reynolds numbers typical of the atmospheric surface layer. Its often-published appearance (see, for instance, Kader and Yaglom, 1991) leads to the obvious question of why this might be so. Questions of interpretation aside, it has been suggested by Hunt and Morrison (2000) that the phenomenon of "shear-sheltering" makes possible a self-similar region which is sheltered from large-scale velocity fluctuations by the effects of strong shear. Thus the influence of inactive motion is mitigated, and can be represented by outer scales alone. However, this suggestion awaits futher investigation. Here we note merely that the term "inactive" may be somewhat of a misnomer, as shear induced by large eddies will undoubtedly be related to energy and shear-stress production. It is surely no coincidence that the near-wall peak in $\overline{u^2}$, which increases with Reynolds number, occurs at the same position as that of maximum energy production.

Consideration also needs to be given to the direct influence of viscosity outside the viscous sublayer, as evidenced by its effect on the mean flow (Zagarola and Smits, 1998), thus contravening one of the conditions necessary for complete similarity. It is interesting to note that, as suggested by Fig. 5, the lower limit to the region in which complete similarity is most likely to exist, $0.033 \leq y/R \leq 0.107$, is equivalent to $y^+ \approx 1800$ at $Re_D = 3.09 \times 10^6$, and that collapse of the spectra is significantly worse at lower Reynolds numbers. Thus it is very unlikely that complete similarity will be possible below this Reynolds number. Note that an equivalent boundary-layer Reynolds number is $Re_\theta \approx 150,000$! Fig. 2 shows that the direct effects of viscosity on $\overline{u^2}$ are apparent in the outer region for $Re_D < 10^5$.

The support of ONR under Grant Nos. N00014-98-1-0525 and N00014-99-1-0340 is gratefully acknowledged. JFM is indebted to both the Engineering and Physical Sciences Research Council (GR/M64536/01) and the Royal Academy of Engineering (England) for financial support.

References

Bradshaw, P. 1967 " 'Inactive' motion and pressure fluctuations in turbulent boundary layers," *J. Fluid Mechs.* **30**, 241–258.

Hunt, J. C. R. and Morrison, J. F. 2000 "Eddy structure in turbulent boundary layers," *Eur. J. Mechs./B (Fluids)*, **19** 673–694.

Jiménez, J. 1998 "The largest scales of turbulent wall flows," *Annual Research Briefs, Center for Turbulence Research* pp. 137–153.

Jones, M. B., Marušiç, I. and Perry, A. E. 2001 "Evolution and structure of sink-flow turbulent boundary layers," *J. Fluid Mechs.* **428** 1–27.

Kader, B. A. and Yaglom, A. M. 1991 "Spectra and correlation functions of surface layer atmospheric turbulence in unstable thermal stratification," *Turbulence and Coherent Structures*, (eds. O. Métais & M. Lesieur) Kluwer Academic Publishers, pp. 387–412.

Kim, K. C. and Adrian, R. A. 1999 "Very large-scale motion in the outer layer," *Phys. Fluids* **11**, 417–422.

Marušiç, I. and Perry, A. E. 1995 "A wall-wake model for the turbulence structure of boundary layers. Part 2. Further experimental support," *J. Fluid Mechs.* **298**, 389–407.

Mochizuki, S. and Nieuwstadt, F. T. M. 1996 "Reynolds-number-dependence of the maximum in the streamwise velocity fluctuations in wall turbulence," *Expts. Fluids* **21** 218–226.

Morrison, J. F. and Smits, A. J. 2001 "Comments on Colebrook's transitional roughness function and the implications for the Moody diagram," In preparation.

Morrison, J. F., Subramanian, S. C. and Bradshaw, P. 1992 "Bursts and the law of the wall in turbulent boundary layers," *J. Fluid Mechs.* **241**, 75–108.

Nikora, V. 1999 "Origin of the "-1" spectral law in wall-bounded turbulence," *Phys. Rev. Lett.* **83**, 734–736.

Perry, A. E. and Abell, C. J. 1977 "Asymptotic similarity of turbulence structures in smooth- and rough-walled pipes," *J.Fluid Mechs.* **79**, 785–799.

Perry, A. E., Henbest, S. and Chong, M. S. 1986 "A theoretical and experimental study of wall turbulence," *J. Fluid Mechs.* **165**, 163–199.

Perry, A. E. and Li, J. D. 1990 "Experimental support for the attached-eddy hypothesis in zero-pressure-gradient turbulent boundary layers," *J. Fluid Mechs.* **218**, 405–438 (1990).

Toonder, J. M. J. den and Nieuwstadt, F. T. M. 1997 "Reynolds number effects in a turbulent pipe flow for low to moderate Re," *Phys. Fluids* **9**, 3398–3409.

Townsend, A. A. 1961 "Equilibrium layers and wall turbulence," *J. Fluid Mechs.* **11**, 97–120.

Townsend, A. A. 1976 *The Structure of Turbulent Shear Flow.* Cambridge University Press, second edition.

Zagarola, M. V. and A. J. Smits, A. J. 1998 "Mean-flow scaling of turbulent pipe flow," J. Fluid Mechs. **373**, 33–79.