QUADRANT ANALYSIS OF ROUGH AND SMOOTH SURFACE CHANNEL FLOWS

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ABSTRACT

A comparison between the turbulent flows in a smooth and rough wall channel are reported. It is demonstrated that even though the outer layer may appear to be unaffected by the surface condition, since the mean velocity defect profiles are identical, significant modifications to the turbulent stresses as far out as $y/h \approx 0.4$ are found. Inside this limit the contributions from the fourth quadrant are found to be much more dominant in the rough case than over the smooth wall, confirming earlier findings that surface roughness tends to stimulate sweep type of motions.

INTRODUCTION

Flow over rough surfaces play an important role in industry. Townsend's (1976) similarity hypothesis implies that, outside the roughness sublayer ($\simeq$ 5 times the roughness height, k) the flow is independent of wall roughness at sufficiently high Re. An implication of this hypothesis is that the Reynolds stresses normalized by the friction velocity, u_τ, should be unaffected by the roughness outside the roughness sublayer. In a boundary layer, (Krogstad *et al.*, 1992, Krogstad and Antonia, 1999), this assumption has been questioned on the basis of differences observed in boundary layer measurements over smooth and rough walls. In that case u_τ was deduced from the velocity measurements, which may contain considerable uncertainty. In a channel flow u_τ may be estimated more accurately, since it is directly linked to the longitudinal static pressure gradient, which may be quite accurately measured.

The present study investigates the effects of surface roughness for $6500 < Re = h\overline{U}/\nu < 45000$ (h is the channel half height and $\overline{U}$ is the channel bulk velocity). Here only measurements for $Re \simeq 12000$ are shown. Measurements for the smooth wall channel are compared with DNS of Moser *et al.* 1999. Results from their $Re = 12400$ simulation suggest that this is sufficiently high to be free from most low-Re number effects.

EXPERIMENTAL DETAILS

The experiments where performed in a closed return wind tunnel with a working section made up of two parallel plates which formed a 2-D rectangular channel. The test section is 5 m long, with an inlet area of 1.35 m x 0.09 m. In the case of the rough surface, the roof and floor where covered with square bars 1.7 x 1.7 mm spanning the whole width of the section. (This corresponds to a ratio between the roughness height, k, and channel half height, h, of 0.038). The bar spacing was 8 times the width, making it a k-type roughness. According to Furuya *et al.* (1976) this roughness spacing creates the largest effect on the mean velocity profile. The flow along the roof and floor were both tripped at the inlet by a 3 mm diameter rod followed by a 12 cm strip of No. 40 grit sandpaper, both spanning the whole width of the section. The test section is fitted with pressure taps with 20 cm spacing along the centre line from which the pressure gradient was obtained. Additional taps are fitted off centre for rough two-dimensionality checks.

The measurements were taken at a bulk velocity of about $\overline{U} = 4$ m/s for both flows. Mean velocity and Reynolds stress data ($\overline{u^{+2}}, \overline{v^{+2}}, \overline{w^{+2}}, \overline{u^+v^+}, \overline{u^+w^+}$) were obtained using purpose made 2.5 μm diameter single- and X-wire probes. As expected for two-dimensional flows $\overline{u^+w^+}$ was very close to zero and will not be presented. For both experiments about 920 000 data were acquired to a PC at a sampling rate of 5kHz after the signal had been low-pass filtered at 2.5kHz and suitably amplified. The filter frequency

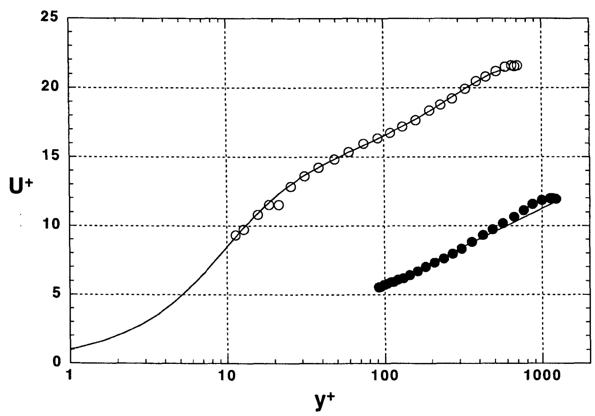

Figure 1: Mean velocity profiles, inner scaling. ● rough, ○ smooth, line: Moser *et al.* (1999).

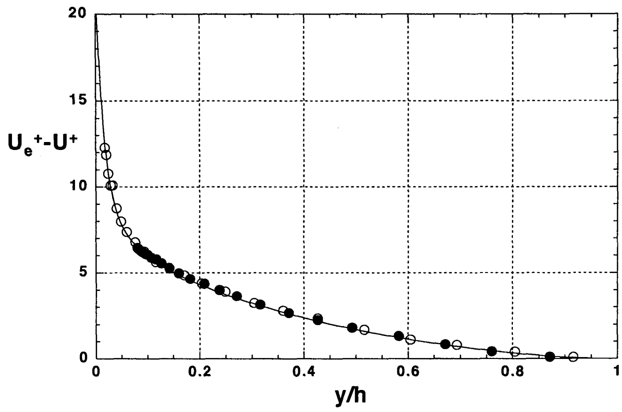

Figure 2: Velocity defect profiles. Symbols as in fig. 1.

closely matches the highest Kolmogorov frequency found in the flow.

RESULTS AND DISCUSSION

Mean velocity

$Re_\tau = hu_\tau/\nu$ for the smooth channel experiment was 630. The mean velocity profile for this case was found to match very closely the DNS data for $Re_\tau = 590$ computed by Moser *et al.* (1999) (Figure 1). Compared to the smooth reference case, the rough wall profile ($Re_\tau = 1140$) exhibits a downward shift in the log-law, ΔU^+, as expected from classical theory (e.g. Townsend, 1976). ΔU^+ was found to follow the characteristic logarithmic dependency with respect to k^+ , consistent with a k-type behaviour. At the Re for the measurements presented here, ΔU^+ was 5.6, corresponding to an effective sand roughness length of $k_s = 12.2$ mm. This is more than 7 times the physical roughness height, consistent with the observations of Furuya *et al.* (1976) that a spacing of about 8 times the height produces the largest roughness effects on the flow.

The error in origin ϵ, required in the case of a rough wall was determined using the semi-logarithmic form of the velocity distribution, as proposed by Moore (1951). Using the effective wall distance, $y_{eff} = y + \epsilon$, similarity between the smooth and rough surface flows is obtained in the velocity defect plot (Figure 2). Similarity in the outer layer suggests that the surface roughness effects are restricted to the inner wall layer.

Reynolds stresses

In contrast to the mean velocity similarity, the Reynolds stresses suggest differences between the two surfaces also in the outer layer. Although all stresses are similar for the smooth and rough surfaces for $y/h > 0.5$ (Figure 3-6), considerable differences are found further in. In the intermediate region, $0.15 < y/h < 0.5$, the rough wall $\overline{u^{+2}}$ data lie slightly above the smooth wall data, while the rough data is quickly reduced further in. For $y/h < 0.5$, the other non-zero stresses, $\overline{v^{+2}}$, $\overline{w^{+2}}$ and $-\overline{u^+v^+}$, are all considerably reduced by the surface roughness. This corroborates the findings of Krogstad and Antonia (1999), who attributed this effect to a substantial change in turbulent diffusion due to the roughness. This is studied further in the next section using quadrant analysis.

One of the basic problems of rough wall experiments is that the origin of the velocity profile is not known. Therefore some degree of uncertainty exists in the determination of the appropriate wall distance, which may affect the comparison with smooth wall data in the near wall region. However, the data shows that roughness effects extend well beyond this uncertainty. Roughness effects up to $y/h \approx 0.4$ correspond to distances of more that 10 times the roughness height. This shows that surface roughness effects are not limited to the near wall region, as assumed in classical theories, but are likely to affect the entire channel flow. Mazouz *et al.* (1998) and Sabot *et al.* (1977), reported the same trend and found considerable differences across the entire channel for $\overline{v^{+2}}$ and $\overline{w^{+2}}$. However, their roughness elements were more than twice as large as those used in the present experiment, but the results confirm roughness effects extending to more than 10 times the element height.

$-\overline{u^+v^+}$ was found to be considerably reduced close to the rough surface. This is

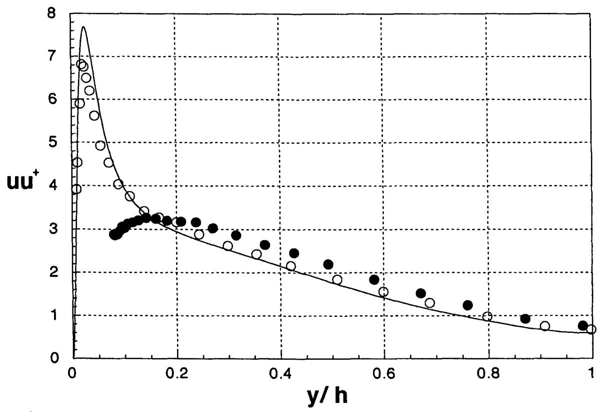

Figure 3: Normal stress $\overline{u^2}^+$. Symbols as in fig. 1.

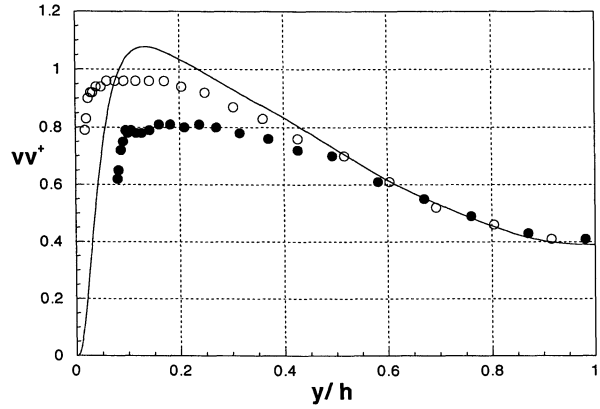

Figure 4: Normal stress $\overline{v^2}^+$. Symbols as in fig. 1.

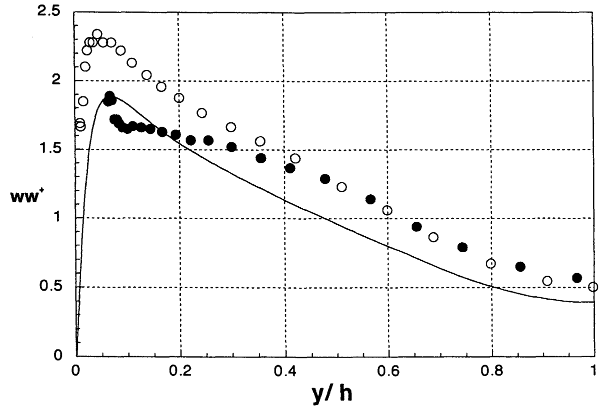

Figure 5: Normal stress $\overline{w^2}^+$. Symbols as in fig. 1.

consistent with the strong reduction in $\overline{v^{+2}}$. In contrast to the present study, Labraga *et al.* (1997) (experiments done under identical conditions as Mazouz *et al.*, 1998) found that $-\overline{u^+v^+}$ was only slightly (about 10 %) lower than for the smooth wall. They attributed the differences to insufficient angular response of the X-wire probes. Krogstad *et al.* (1992) has questioned these possible measurement errors in a rough boundary layer, claiming they should only affect measurements very close to the wall and may therefore not explain differences observed outside y/δ=0.2.

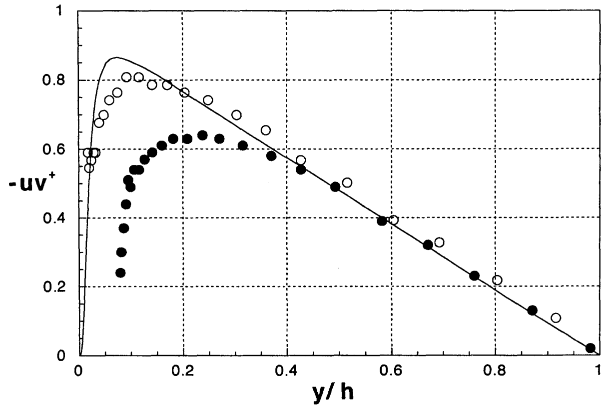

Figure 6: Shear stress $-\overline{uv}^+$. Symbols as in fig. 1.

The strong effect of roughness up to $y/h \simeq 0.5$ in the present study and even beyond in other experiments (Mazouz *et al.*, 1998, Sabot *et al.*, 1977) suggest that the size of the roughness elements may be too large to consider these experiments as simple perturbations to smooth channel flows. Therefore the experiments will be repeated with significantly lower relative roughness heights to see if the effects of the surface roughness still extend into the outer layer.

Despite the differences in the stresses, the turbulent mixing length distribution (Figure 7) seems to be quite unaffected by the roughness. Close to the wall, where the differences in shear stresses between the smooth and rough surfaces are most distinct, there is very good similarity in the mixing length. This suggests that the mixing length is not a very sensitive indicator of the roughness effect.

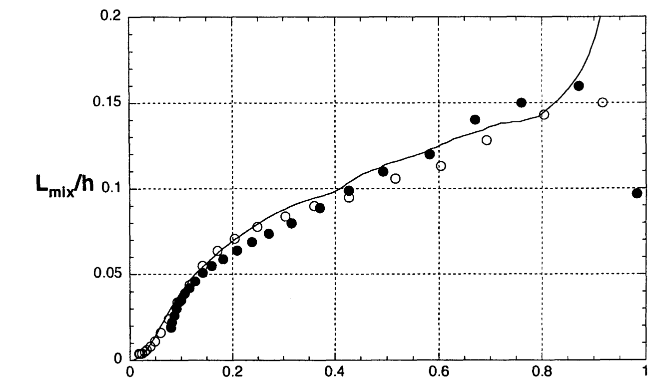

Figure 7: Mixing length distributions. Symbols as in fig. 1.

Quadrant analysis

Quadrant analysis (see Lu and Wilmarth, 1973) has proven to be a useful tool in assessing structural changes in turbulence. In the $u-v$ plane, the shear stress $uv = const.$ defines a hyperbola in two antisymmetric quadrants. Hence a triggering function $\mid uv \mid \geq Hu'v'$, where the prime denotes a r.m.s. value and H is a threshold level, excludes shear stresses within a "hyperbolic hole". The contribution to $\overline{uv}$ from a particular quadrant, Q, may then be written

$$(\overline{uv})_Q = \lim_{T\to\infty} \frac{1}{T} \int_0^T uv(t)\; I_Q(t)\; dt\,. \qquad (1)$$

Here $I_Q(t)$ is the trigger function defined as

$$I_Q = \begin{cases} 1 & \text{when } \mid uv \mid_Q \geq Hu'v' \\ 0 & \text{otherwise.} \end{cases} \qquad (2)$$

It is worth noticing that an instantaneous motion with a large value of u and small v may contribute to $(\overline{uv})_Q$ in the same way as an event with small u and high v, although the types of motion will be very different. Hence the quadrant method alone can not be used to uniquely distinguish contributions from a particular type of motion.

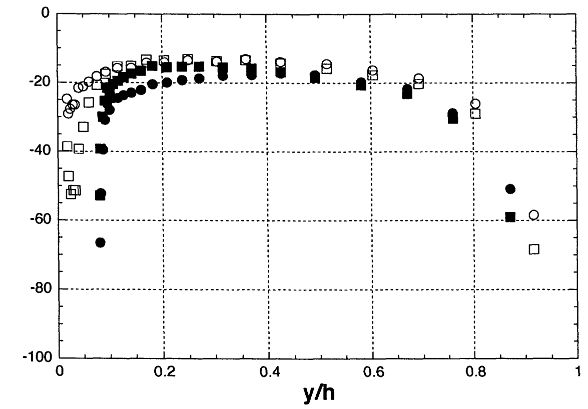

Figure 8: Contributions from $Q1$ and $Q3$ for H = 0. ○: $Q1$, □ : $Q3$. Open symbols: Smooth surface; filled symbols: Rough surface.

The shear stress signal is characterised by strong intermittency, where the main contributions to the averaged stresses come from large excursions in the second and fourth quadrants. When all events are included (H = 0), Figures 8 and 9 show that $Q2$ and $Q4$ both contribute between 60 to 80 % of $\overline{uv}$ throughout most of the channel flow. The overshoot is compensated by the negative contributions

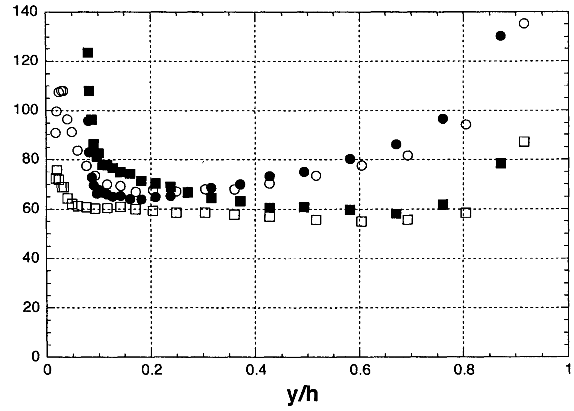

Figure 9: Contributions from $Q2$ and $Q4$ for H = 0. ○: $Q2$, □ : $Q4$. Open symbols: Smooth surface; filled symbols: Rough surface.

from $Q1$ and $Q3$ which are also very similar in magnitude. The distributions for the two surfaces are very similar for most of the channel except for the rise in all contributions near the wall, which occurs somewhat further out in the rough case. This is because the measurements in the rough case were all taken above the roughness element. A closer inspection does however reveal that the $Q4$ contribution becomes signifcantly more important for the rough surface compared to the smooth case as the wall is approached. This corroborates the findings of Krogstad and Antonia (1999) that the main effect of the surface roughness is a reduction of the damping of v motions as the wall is approached. Figure 10 shows $\alpha = Q2/Q4$, the ratio between the $Q2$ and $Q4$ contributions. This ratio is the same for the two flows above $y/h \approx 0.4$, but near the surface the $Q4$ events dominate in the rough case, while $Q2$ events dominate for the smooth surface.

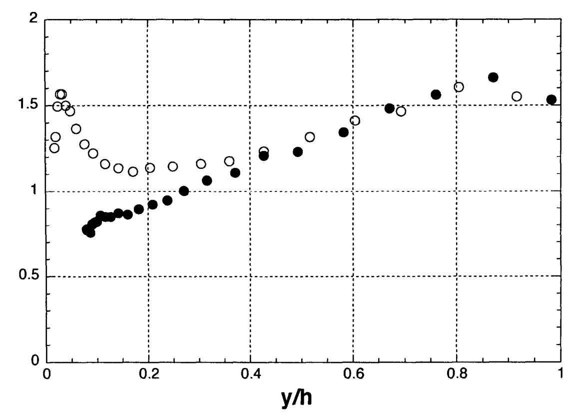

Figure 10: Ratio between $Q2$ and $Q4$ contributions for H = 0. Open symbols: Smooth surface; filled symbols: Rough surface.

This trend becomes even more obvious as

the threshold level is increased (Figures 11). For $H = 1.0$ and 2.5 (which corresponds to contributions only from events which are about 2.5 and 6 times stronger than $\overline{uv}$ respectively), the $Q2$ events are seen to dominate the entire flow field for the smooth wall, while $Q4$ becomes increasingly important for the rough surface for $y/h < 0.3$.

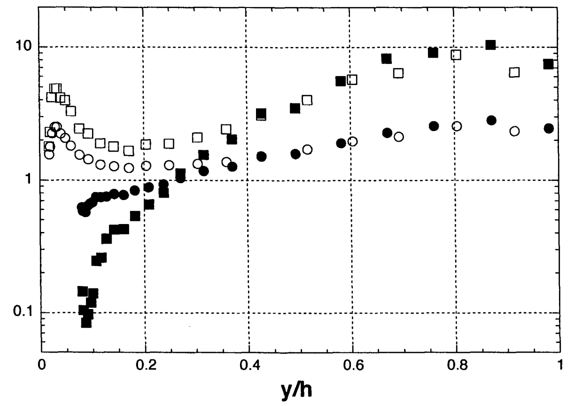

Figure 11: Ratio between $Q2$ and $Q4$ contributions. ○: $H = 1$, □ : $H = 2.5$. Open symbols: Smooth surface; filled symbols: Rough surface.

The quadrant analysis therefore confirms the speculations of Krogstad *et al.* (1992) that surface roughness does not simply cause a modification of the wall boundary condition, but affects the turbulence structure over a considerable part of the outer layer as well.

For the smooth surface, the time between consecutive $Q2$ detections was found to be about $T_2^+ = T_2 u_\tau^2/\nu \approx 100$ across most of the channel when $H = 0$ (Figure 13). This is after detections occuring within a window of $\tau_{group}^+ = 20$ have been grouped as one event. The time between $Q4$ events was found to be slightly longer. For the rough surface the time between the events show a similar distribution as for the smooth case, but the times are about 40 % longer. The same trend is also observed for $Q1$ and $Q3$ events (Figure 12). The data were also scaled using outer variables ($\overline{T_2} = T_2\overline{U}/h$), but neither scaling method managed to collapse the smooth and rough data.

When H was increased the number of detections decreases, although it was always high enough to be statistically significant. (For $H = 1$ between 2500 and 3000 detections were found for both surfaces for $Q2$ and $Q4$; about half as many for the other two quadrants.) The differences between the $Q2$ and $Q4$ detection times were reduced as H was increased, as exemplified for $H = 1$ in Figure 14. For the smooth surface the $Q2$ and $Q4$ times are now virtually indistinguishable and the rough wall data follow them closely.

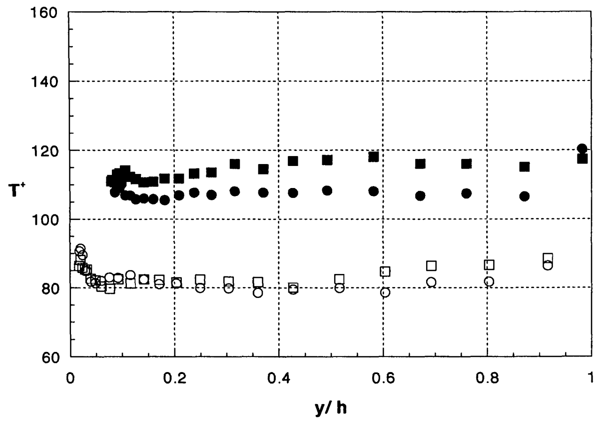

Figure 12: Time between $Q1$ and $Q3$ events. $H = 0$. Symbols as in fig. 8.

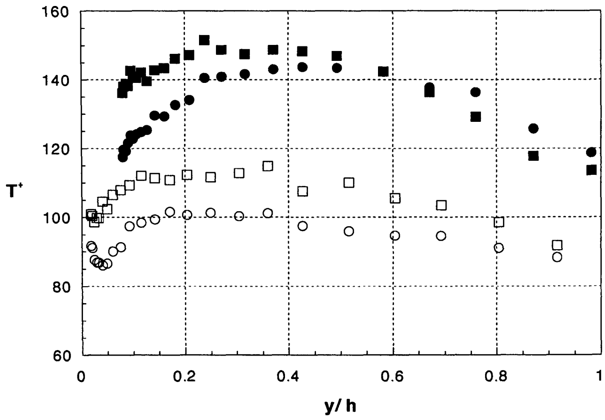

Figure 13: Time between $Q2$ and $Q4$ events. $H = 0$. Symbols as in fig. 9.

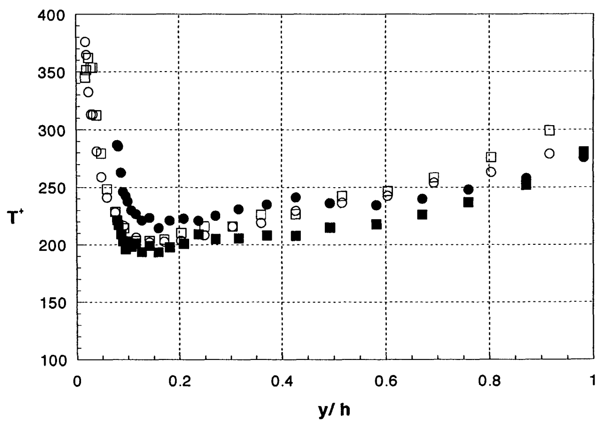

Figure 14: Time between $Q2$ and $Q4$ events. $H = 1$. Symbols as in fig. 9.

The duration of the grouped $Q2$ and $Q4$ events are shown in Figure 15. It is apparent that the duration of the events scale very well with inner variables for both cases. The fractional time $\Delta T_i^+/T_i^+$ is a rough estimate of the relative time during which shear stress in quadrant i is being produced. Since this ratio is similar for the two surfaces throughout most of the layer, it may be speculated that the events triggered above the rough surface on average are weaker than in the smooth wall case.

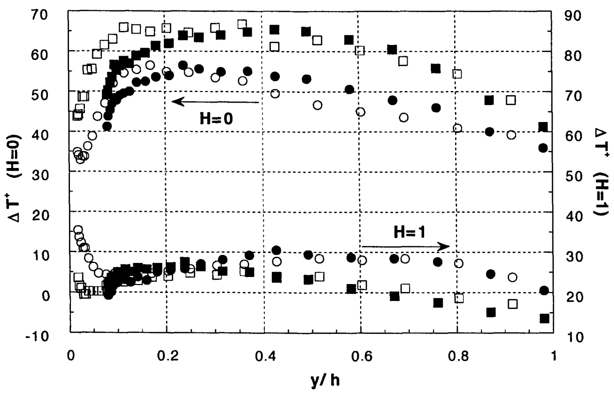

Figure 15: Durations of $Q2$ and $Q4$ events. Symbols as in fig. 8.

CONCLUSIONS

Measurements of 5 of the Reynolds stresses in a channel flow where the surfaces were roughened by means of transverse bars showed that the stresses are affected far beyond the immediate wall region. Although the mean velocity defect for the rough wall and reference data taken in the same channel under smooth wall conditions were found to collapse exactly, considerable differences in the stresses were found over almost half the channel height. This is at variance with conventional rough wall hypotheses which assume that the roughness effect is primarily a modification in the wall boundary condition which primarily affects the flow in the immediate vicinity of the roughness elements.

A quadrant analysis showed that it is primarily the fourth quadrant or sweep type events which are affected by the roughness. This is consistent with previous speculations (Krogstad *et al.*, 1992) that a rough wall primarily affects the turbulent structure by breaking up the streamwise vortices and reducing the damping of the wall perpendicular motion. The first effect reduces the streamwise length scales and therefore makes the flow more isotropic, while the second effect mainly affects the intensity of the sweeps near the wall.

For very weak events over the rough surface, the time between the shear stress producing events were found to be increased compared to its smooth counterpart, but the differences were found to be reduced when only stronger events were considered. The durations of the events was found to be very little affected by the surface condition and was found to scale very well with inner variables.

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