ON THE EFFECT OF FINITE REYNOLDS NUMBER AND INITIAL CONDITIONS ON THE AXISYMMETRIC WAKE

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ABSTRACT

Equilibrium similarity considerations are applied to the axisymmetric turbulent wake. Two solutions are found; one for infinite Reynolds number, one for small Reynolds number, and both dependent on the upstream conditions. Neither agrees particularly well with the data. For both solutions, the local Reynolds number of the flow diminishes with increasing downstream distance. As a consequence, even when the initial Reynolds number is large, the flow evolves from one state to the other. Intermediate asymptotics is used to provide a bridge between the two solutions, which is in excellent agreement with the experimental data.

INTRODUCTION

The axisymmetric wake is of fundamental importance since it is one of few flows where the local Reynolds number decreases as the flow evolves. Also, the equations of motion governing the axisymmetric wake contain all of the important dynamical terms for turbulent flow away from surfaces. Hence the data from this flow form an important data base for developing turbulence models of all types, as well as for validating DNS and LES simulations.

The following observations can be made from various experiments:

- Different initial conditions affect the growth rates, contrary to the classical theory which states that all wakes should depend only on the downstream distance, x, and the drag, $\pi U_\infty^2 \theta^2$. This is most strikingly illustrated by the photographs of Cannon et al. (1993). Data from Cannon (1991) (for several different wake generators that all have the same drag) and Menut et al. (2000) are plotted in Figure 1. These show the variation with x of the transverse length scale defined by

$$\delta_*^2 = \lim_{r\to\infty} \frac{1}{U_o} \int_0^r (U_\infty - U)\, r\, dr \qquad (1)$$

where U_o is the centerline velocity deficit. The data do not collapse to a single curve, nor do these source dependent effects vanish, even for large Reynolds numbers or large downstream distance. The curves denoted "model" will be explained in a later section.

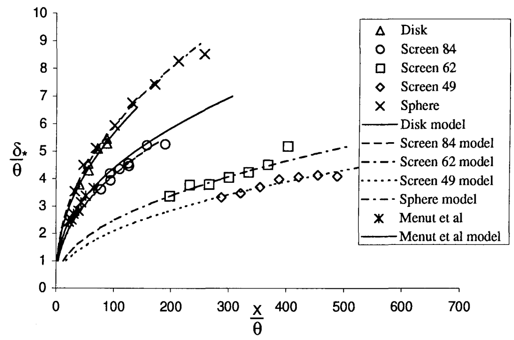

Figure 1: Cross-stream length scale, δ_*/θ versus x/θ, Cannon (1991) and Menut et al. (2000).

- The wake does not in general grow as the 1/3-power of downstream distance, x, as predicted by the classical similarity theory (c. f., Tennekes and Lumley, 1972). As noted previously by Cannon, curve fits to the data agree equally well with both a cube root and a square root variation, and not particularly well with either.

- In apparent contradiction, the mean velocity profiles from all experiments collapse on a single curve when scaled with centerline velocity deficit and δ_*, as illustrated in Figure 2a.

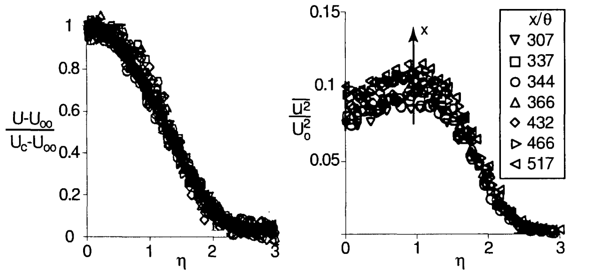

Figure 2: a) Mean velocities, b) Turbulence intensities, Cannon (1991)

- And to further confuse the issue, the turbulence intensities do not appear to collapse at all, even for fixed upstream conditions. Figure 2b shows profiles of $\overline{u^2}/U_o^2$ for various downstream distances for one of the screen wakes (the most porous) in Cannon (1991). In fact, it seems like the there are two branches; one for the near wake and one for the far wake.

These observations cannot be explained by classical similarity analysis, which argues that the asymptotic wake is independent of its initial conditions and depends only on the distance downstream and the drag. Nor can they be explained by measurement errors. While the low turbulence intensities of the axisymmetric wake make measurements far downstream difficult, they also insure that the hot-wire techniques utilized are highly accurate. At very least the problems presented by wake measurements are no more difficult than for grid turbulence, for which hot-wire measurements have long been accepted.

The classical self-preservation approach to free shear flows was questioned by George (1989) and (1995), who argued that it was based on assumptions that were not in general valid. He proposed a new methodology called *equilibrium similarity analysis*, and demonstrated with it that solutions were possible which depended *uniquely* on the upstream conditions. The new theory was in striking agreement with the nearly concurrent experiments of Wygnanski et al. (1986) for two-dimensional wakes. These showed dramatic differences between spreading rates and eddy structure which depended on the wake generator.

George (1989) also argued that the axisymmetric wake would behave similarly. He predicted that the mean velocity profiles from the different experiments would be the same, if scaled by the centerline deficit velocity and velocity deficit half-width, even if the wakes grew at different rates. This is consistent with the observations shown in Figure 2a. This result is very important, since previous researchers have often used such collapse to argue that wakes are independent of upstream conditions. The whole point of George's analysis, however, is that the source-dependent differences only show up in the spreading rate, and the higher turbulence moments.

In this paper, the analysis of George (1989) is re-visited, corrected, and extended. It will be shown that two different equilibrium similarity solutions for the axisymmetric wake are possible: one for very high local Reynolds numbers, and another for very low. Most importantly, because the local Reynolds number decreases with distance downstream, the flow will evolve from one state to the other, no matter how high the initial Reynolds number of the flow. The available experimental data is analyzed, especially addressing the particular points listed above. Not surprisingly (given the state of confusion regarding it), most of the experiments are shown to take place in the evolution region where neither limit applies exactly. An intermediate asymptotics solution for this region is shown to be in excellent agreement with the data.

GOVERNING EQUATIONS

The Reynolds averaged x-momentum equation for the axisymmetric far wake without swirl reduces to second order to:

$$U_\infty \frac{\partial}{\partial x}(U-U_\infty) = -\frac{1}{r}\frac{\partial}{\partial r}(r\overline{uv}) + \left\{\frac{\partial}{\partial x}\left(\overline{v^2}-\overline{u^2}\right) + \nu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(U-U_\infty)\right)\right\} \qquad (2)$$

Here, the r-momentum equation has been used to integrate out the pressure. The terms in curly brackets are usually neglected, but are retained here.

The momentum equation can be integrated over a cross-section to yield an integral constraint for the conservation of momentum:

$$U_\infty \int_0^\infty (U_\infty - U)\, 2\pi r\, dr \cong \pi\theta^2 U_\infty^2 \qquad (3)$$

where θ is the momentum thickness.

As noted by George (1995), the momentum equation alone is not sufficient to determine the similarity constraints. Even the inclusion of the kinetic energy equation is not enough to close the system so that the x-dependence can be determined. Instead, the individual Reynolds stress equations have to be investigated. These, together with the constraint of continuity on the pressure-strain rate terms provide the necessary conditions. The component Reynolds stress equations for the far wake are:

$\overline{u^2}$ balance

$$U_\infty \frac{\partial}{\partial x}\left(\frac{1}{2}\overline{u^2}\right) = -\overline{uv}\frac{\partial}{\partial r}(U - U_\infty) - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{1}{2}\overline{u^2 v}\right) + \overline{\frac{p}{\rho}\frac{\partial u}{\partial x}} - \frac{1}{\rho}\frac{\partial}{\partial x}\overline{pu} + \nu\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial}{\partial r}\left(\frac{1}{2}\overline{u^2}\right)\right\} - \varepsilon_u \quad (4)$$

$\overline{v^2}$ balance

$$U_\infty \frac{\partial}{\partial x}\left(\frac{1}{2}\overline{v^2}\right) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{1}{2}\overline{v^3}\right) + \frac{\overline{vw^2}}{r} + \overline{\frac{p}{\rho}\frac{\partial v}{\partial r}} - \frac{1}{\rho}\frac{\partial}{\partial r}\overline{pv} + \nu\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{1}{2}\overline{v^2}\right)\right\} - \varepsilon_v \quad (5)$$

$\overline{w^2}$ balance

$$U_\infty \frac{\partial}{\partial x}\left(\frac{1}{2}\overline{w^2}\right) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{1}{2}\overline{uw^2}\right) - \frac{\overline{vw^2}}{r} + \nu\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{1}{2}\overline{w^2}\right)\right\} - \varepsilon_w \quad (6)$$

$\overline{uv}$ balance

$$U_\infty \frac{\partial}{\partial x}(\overline{uv}) = -\overline{v^2}\frac{\partial}{\partial r}(U - U_\infty) - \frac{1}{r}\frac{\partial}{\partial r}\left(r\overline{uv^2}\right) + \frac{\overline{uw^2}}{r} + \overline{\frac{p}{\rho}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right)} - \frac{1}{\rho}\left(\frac{\partial}{\partial r}\overline{pu} + \frac{\partial}{\partial x}\overline{pv}\right) + \nu\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{1}{2}\overline{uv}\right)\right\} - \varepsilon_{uv} \quad (7)$$

where ε_u, ε_v, ε_w, and ε_{uv} are the components of the homogenous dissipation.

The Similarity Transformations

We seek solutions on the form (written here for the momentum equation and $\overline{u^2}$ equations only — the others are treated similarly):

$$U - U_\infty = U_s(x) f(\eta, *) \quad (8a)$$

$$-\overline{uv} = R_s(x) g(\eta, *) \quad (8b)$$

$$\frac{1}{2}\overline{u^2} = K_u(x) k_u(\eta, *) \quad (8c)$$

$$\frac{1}{2}\overline{u^2 v} = T_{u^2 v} t_{u^2 v}(\eta, *) \quad (8d)$$

$$\overline{\frac{p}{\rho}\frac{\partial u}{\partial x}} = P_u(x) p_u(\eta, *) \quad (8e)$$

$$\frac{1}{\rho}\overline{pu} = P_u^D(x) p_u^D(\eta, *) \quad (8f)$$

$$\varepsilon_u = D_u(x) d_u(\eta, *) \quad (8g)$$

where $\eta = r/\delta(x)$ and $*$ denotes a possible (unknown) dependence on initial conditions.

The Momentum Integral

Substitution of eq. (8) into eq. (3) yields:

$$U_s \delta^2 \int_0^\infty 2 f \eta \, d\eta = U_\infty \theta^2 \quad (9)$$

It follows immediately that if $\delta \equiv \delta_*$ and $U_s \equiv U_o$:

$$\frac{U_s}{U_\infty} = \left[\frac{\theta}{\delta_*}\right]^2 \quad (10)$$

The Transformed Mean Momentum Equation

Substituting eq. (8) into the momentum equation, eq. (2), and rearranging the terms yields:

$$\left[\frac{\delta}{U_s}\frac{dU_s}{dx}\right] f - \left[\frac{d\delta}{dx}\right]\eta f' = \left[\frac{R_s}{U_\infty U_s}\right]\frac{(\eta g)'}{\eta} + \left[\frac{\nu}{U_\infty \delta}\right]\frac{(\eta f')'}{\eta} \quad (11)$$

where $'$ denotes derivation with respect to η. Note that the second order term could have been retained. To this point the mean momentum equations have simply been transformed by the separation of variables in eq. (8) so that all of the explicit x-dependence is in the bracketed terms. Thus the results are completely general and no similarity assumptions have been made (although the form of the solutions has been restricted).

The Transformed Reynolds Stress Equations

Substituting eq. (8) into the transport equations for Reynolds stresses yields:

$\overline{u^2}$-equation

$$\left[U_\infty \frac{dK_u}{dx}\right] k_u - \left[\frac{U_\infty K_u}{\delta}\frac{d\delta}{dx}\right] \eta k_u' = - \left[\frac{R_s U_s}{\delta}\right] f' - \left[\frac{T_{u^2v}}{\delta}\right] \frac{(\eta t_{u^2v})'}{\eta} + [P_u] p_u + \left[\frac{dP_u^D}{dx}\right] p_u^D - \left[\frac{P_u^D}{\delta}\frac{d\delta}{dx}\right] \eta (p_u^D)' + \left[\frac{\nu K_u}{\delta^2}\right] \frac{(\eta k_u')'}{\eta} - [D_u] d_u \quad (12)$$

$\overline{v^2}$-equation

$$\left[U_\infty \frac{dK_v}{dx}\right] k_v - \left[\frac{U_\infty K_v}{\delta}\frac{d\delta}{dx}\right] \eta k_v' = - \left[\frac{T_{v^3}}{\delta}\right] \frac{(\eta t_{v^3})'}{\eta} + \left[\frac{T_{vw^2}}{\delta}\right] \frac{t_{vw^2}}{\eta} + [P_v] p_v + \left[\frac{P_v^D}{\delta}\right] (p_v^D)' + \left[\frac{\nu K_v}{\delta^2}\right] \frac{(\eta k_v')'}{\eta} - [D_v] d_v \quad (13)$$

$\overline{w^2}$-equation

$$\left[U_\infty \frac{dK_w}{dx}\right] k_w - \left[\frac{U_\infty K_w}{\delta}\frac{d\delta}{dx}\right] \eta k_w' = - \left[\frac{T_{uw^2}}{\delta}\right] \frac{(\eta t_{uw^2})'}{\eta} - \left[\frac{T_{vw^2}}{\delta}\right] \frac{t_{vw^2}}{\eta} + \left[\frac{\nu K_w}{\delta^2}\right] \frac{(\eta k_w')'}{\eta} - [D_w] d_w \quad (14)$$

$\overline{uv}$-equation

$$- \left[U_\infty \frac{dR_s}{dx}\right] g - \left[\frac{U_\infty R_s}{\delta}\frac{d\delta}{dx}\right] \eta g' = \left[\frac{K_v U_s}{\delta}\right] f' k_v - \left[\frac{T_{uv^2}}{\delta}\right] \frac{(\eta t_{uv^2})'}{\eta} + \left[\frac{T_{uw^2}}{\delta}\right] \frac{t_{uw^2}}{\eta} + [P_{uv}] p_{uv} - \left[\frac{P_u^D}{\delta}\right] (p_u^D)' - \left[\frac{dP_v^D}{dx}\right] (p_v^D) - \left[\frac{P_v^D}{\delta}\frac{d\delta}{dx}\right] \eta (p_v^D)' - \left[\frac{\nu R_s}{\delta^2}\right] \left(\frac{(\eta g)'}{\eta}\right)' - [D_{uv}] d_{uv} \quad (15)$$

As before, the equations have simply been transformed by the similarity transformations so that all the explicit x-dependence is in the bracketed terms.

EQUILIBRIUM SIMILARITY SOLUTIONS TO THE TRANSFORMED EQUATIONS

For the particular type of "equilibrium" similarity solutions suggested in George (1995), *the terms in the governing equations must maintain the same relative balance as the flow evolves.* These "equilibrium" similarity solutions exist only if the terms within square brackets have the same x-dependence, and are independent of the similarity variable, η. Thus, the bracketed terms must remain proportional to each other as the flow evolves. This is denoted by the symbol $\sim$ which should be interpreted as "has the same x-dependence as"[1].

For the mean momentum equation, these equilibrium similarity constraints can be written as:

$$\left[\frac{\delta}{U_s}\frac{dU_s}{dx}\right] \sim \left[\frac{d\delta}{dx}\right] \sim \left[\frac{R_s}{U_\infty U_S}\right] \sim \left[\frac{\nu}{U_\infty \delta}\right] \quad (16)$$

Note that there is nothing in the equations or the theory which suggests that the constants of proportionality are independent of source conditions, nor in fact do they appear to be. This is contrary to the usual assumptions in self-preservation analysis (c. f., Tennekes and Lumley, 1972). It is trivial to show that the relation between the first and second terms of eq. (16) is satisfied by the momentum integral result of eq. (10).

The proper scale for $-\overline{uv}$ is obtained by using the second and third terms, which yields:

$$R_s \sim U_\infty U_s \frac{d\delta}{dx} \quad (17)$$

It is immediately obvious how the equilibrium similarity approach yields a different and more general result than the classical approach, where it is *assumed* without justification that $R_s = U_s^2$ (c. f., Tennekes and Lumley, 1972).

The same equilibrium similarity hypothesis can be applied to the component Reynolds stress equations; namely that all of the bracketed terms should remain proportional (i.e., have the same x-dependence). For example, inserting eq. (8) into eq. (4) yields after some elementary calculus that equilibrium similarity can be maintained only if:

$$\frac{\delta}{K_u}\frac{dK_u}{dx} \sim \frac{d\delta}{dx} \sim \frac{T_u \delta}{U_\infty K_u} \sim \frac{D_u \delta}{U_\infty K_u} \sim \frac{\nu}{U_\infty \delta} \quad (18)$$

Similar relations arise from the other component equations.

All of these relations *cannot* simultaneously be satisfied given the constraints already placed on U_s, δ, and R_s from the mean momentum equation. On the other hand a solution is

[1]Note that the symbol $\sim$ has nothing to do with order of magnitude in this paper.

possible if the viscosity is identically zero, since then all terms involving the viscosity fall out of the problem. And also a solution for finite viscosity is possible if it can be shown that the production term in the Reynolds shear stress equation, $\overline{v^2}\partial U/\partial y$, is negligible relative to the leading terms in the equation.

It will be demonstrated below that these are in fact limiting solutions for very large turbulence Reynolds number, and for very low turbulence Reynolds number. Note that the latter solution should *not* be confused with the laminar solution, but instead identified with turbulent flow for which the velocity spectra do not have a developed $k^{-5/3}$ range. And by contrast, the high Reynolds number limit will be a flow which does have an easily apparent inertial subrange in the spectra. Further it will be demonstrated that no matter how high the Reynolds number of the drag-producing device, say $R_\theta = U_\infty \theta/\nu$, the diminishing local Reynolds number will move the equations (and the solutions as well) from one regime to the other.

The Infinite Reynolds Number Solution and its Limitations

A solution having the same x-variation as the classical solution can be derived by setting the viscous terms in eq. (11) to (15) exactly equal to zero, which corresponds to the limiting solutions at infinite Reynolds numbers. It is straightforward to show (in the same manner as George, 1995) that all of the remaining constraints can be satisfied. Of particular interest are the following:

$$\frac{d\delta}{dx} \sim \frac{D_u \delta}{U_\infty K_u} \tag{19}$$

$$K_u \sim K_v \sim K_w \sim U_s^2 \tag{20}$$

$$D_u \sim D_v \sim D_w \sim U_s^3/\delta \tag{21}$$

The scaling for the dissipation is just what one should expect for an infinite Reynolds number solution where the dissipation is completely controlled by the energetic turbulence (i. e., $\epsilon \propto u^3/l$ in the usual notation of texts).

It follows immediately after some manipulation that:

$$\frac{\delta_*}{\theta} = a\left[\frac{x - x_o}{\theta}\right]^{1/3} \tag{22}$$

$$\frac{U_s}{U_\infty} = b\left[\frac{x - x_o}{\theta}\right]^{-2/3} \tag{23}$$

where $a = a(*)$, $b = b(*)$, and $x_o = x_o(*)$ is a virtual origin. This is, of course, the classical solution with but a single difference – the dependence of the coefficients on upstream conditions, $*$. This possible dependence must be acknowledged, since there is nothing in the equations themselves to suggest independence of upstream conditions. The mean velocity profile, on the other hand, can be shown to be independent of upstream conditions. This is achieved by incorporating a factor of $[R_s/(U_\infty U_s)\, d\delta/dx]$ into the definition of g so that there are no parameters at all in eq. (11).

Now it was noted in the introduction that the cube root solutions simply do not account for most of the data, and especially the careful data of Cannon (1991). So where might the problem be? It is easy to show that, unlike most other free shear flows, this infinite Reynolds number solution contains the seeds of its own destruction. The *local* Reynolds number, $R = U_s\delta/\nu$, controls the relative importance of the viscous terms in the mean moment and Reynolds shear stress equations. Substitution of eq. (22) and (23) into the definition of R yields:

$$R = \frac{U_s \delta_*}{\nu} = \frac{U_\infty \theta}{\nu}\left[\frac{x - x_o}{\theta}\right]^{-1/3} \tag{24}$$

Thus, no matter how large the initial Reynolds number, R_θ, eventually far enough downstream it is diminished until the viscous terms can no longer be neglected. And if the viscous terms are not negligible, the infinite Reynolds number similarity solution cannot be even approximately true.

The Low Re Solution

As noted above, there is another equilbrium similarity solution to the same set of equations. The difference is that this time the terms involving viscosity are kept. This produces one additional constraint:

$$\frac{d\delta}{dx} \sim \frac{\nu}{U_\infty \delta} \tag{25}$$

It is extremely important to note that even though some of the relations are the same (e.g., K_u/U_s^2 = constant), the constants of proportionality (or more properly, the parameters of proportionality since they all depend on $*$) are most likely different from those governing the infinite Reynolds number solution.

There is one problem which at first glance appears to be quite serious. All of the constraints in the Reynolds shear stress equation cannot be met, in particular the one arising from the production term, $\overline{v^2}\partial U/\partial y$. These offending terms in fact die off with distance downstream faster than the remaining terms in the equation. Therefore, they can also be neglected in the analysis.

It is straightforward to show that eq. (25) can be integrated to obtain:

$$\frac{\delta_*}{\theta} = c\, R_\theta^{1/2} \left[\frac{x - x_{oo}}{\theta}\right]^{1/2} \tag{26}$$

$$\frac{U_s}{U_\infty} = d\, R_\theta \left[\frac{x - x_{oo}}{\theta}\right]^{-1} \tag{27}$$

where as before $c = c(*)$, $d = d(*)$, and $x_{oo} = x_{oo}(*)$ is a virtual origin which may be different than the one obtained above. And as for the infinite Reynolds number solutions, the mean velocity profile can be shown to be independent of upstream conditions. It is easy to show that the local Reynolds number continues to fall with increasing distance downstream; hence the approximations improve with distance downstream.

A solution for moderate Reynolds numbers

Unfortunately, as is clear from the data presented earlier, most of the experimental data is between the two limiting solutions. Hence neither alone describes the data well. Intermediate asympotics, however, offers the possibility of bridging the gap. The easiest way to understand what is required is to note the appearance of the similarity scaling functions for the dissipation; namely D_u, D_v and D_w. As noted above, for the high Reynolds number solutions, $D_u \sim U_s^3/\delta \sim U_s K_u/\delta$, while for the low Reynolds number solutions, $D_u \sim \nu U_s^2/\delta^2 \sim \nu K_u/\delta^2$. This is exactly the kind of behavior that has long been accounted for by turbulence modelers.

The simplest intermediate asymptotics solution which accounts for both the high and low Reynolds number dissipation limits is simply their sum. Assume then that:

$$D_u = \alpha \frac{U_s K_s}{\delta} + \beta \frac{\nu K_u}{\delta^2} \tag{28}$$

Then applying the similarity constraints yields:

$$\frac{d\delta}{dx} = \left\{ \alpha \frac{U_s K_s}{\delta} + \beta \frac{\nu K_u}{\delta} \right\} \frac{\delta}{U_\infty K_u} \tag{29}$$

After some manipulation this reduces to:

$$\frac{d\delta}{dx} = \alpha \left(\frac{\theta}{\delta}\right)^{-2} + \beta \left(\frac{\theta}{\delta}\right)^{-1} \tag{30}$$

This can be integrated directly to obtain δ/θ as a function of x/θ.

Figure 1 shows the Cannon (1991) and Menut et al. (2000) data and the integral of eq. (30) where the coeffients have been determined by optimization techniques. The theory describes the data remarkably well, and also makes understandable Cannon's difficulties in making sense of it.

CONCLUSIONS

The conclusions that can be drawn are that the initial conditions dominate the axisymmetric wake. The effect of initial conditions shows up in growth rate and higher moments, see George (1989). Local Reynolds number effects are also very important since it goes down as the flow evolves. This accounts for deviations from simple power law behaviour of the growth rate. Simple power laws are only reached in the limits $Re \to \infty$ and $Re \to 0$.

REFERENCES

Cannon, S. C., 1991, "Large-scale structures and the spatial evolution of wakes behind axisymmetric bluff bodies," Ph.D. Thesis, Dept. of Aerosp. and Mech. Engr., Univ. of Arizona.

Cannon, S., Champagne, F., and Glezer, A., 1993, "Observations of large-scale structures in wakes behind axisymmetric bodies," *Experiments in Fluids*, Vol. 14, pp. 447–450.

George, W. K., 1989, "The self-preservation of turbulent flows and its relation to initial conditions and coherent structures," *Advances in Turbulence*, George, W. K. and Arndt, R., ed., pp. 39–73.

George, W. K., 1995, "Some new ideas for similarity of turbulent shear flows," *Turbulence, Heat and Mass Transfer* Hanjalic, K. and Pereira, J. C. F., ed., Begell House, N.Y.

Menut, P., Delville, J., and Charnay, G., 2000, "Drag transition effect on the turbulent wake of an axisymmetric bluff body," *Submitted for publication.*

Tennekes, H. and Lumley, J. L., 1972, *"A First Course in Turbulence,"* The MIT Press.

Wygnanski, I. J., Champagne, F. H., and Marasli, B., 1986, "On the large scale structures in two-dimensional, small-deficit, turbulent wakes," *Journal of Fluid Mechanics*, Vol. 168, pp. 31–71, 1986.