

# CIRCULATION TIME HISTORY AND ONSET OF THREE-DIMENSIONAL VORTEX SHEDDING IN THE WAKE BEHIND A DISK

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## ABSTRACT

The development of the wake vortex behind a disk started near-impulsively from rest was investigated with flow visualization and the particle image velocimetry. The circulation growth behind the near-impulsively started disk was found to scale with the non-dimensional time  $U_0 t/D$  ( $U_0$  constant disk velocity,  $t$  time,  $D$  diameter) for Reynolds numbers ranging from 3,100 to 12,400. The normalized circulation reached a maximum value of  $\Gamma/U_0 D=2.5$  at a time approximately equal to 4, after which the vortex ring deformed and subsequently shed. The results are compared with the two-dimensional counterpart as well as computations.

## INTRODUCTION

The behavior of the vortex rings generated by a piston-cylinder setup depends on the ratio of piston stroke to cylinder inside diameter ( $L/D$ ), and for large ratios  $L/D$ , the vortex ring is followed by a trailing jet. According to Gharib et al.(1998) the circulation in the vortex ring reached a maximum value. The ratio  $L/D$  was approximately 4 and was referred to as "formation number". Earlier, Taylor (1953) made an analysis on circulation to be generated by a disk that is impulsively started (and dissolved). However, the transitional stage of the growth of the axisymmetric bluff-body wake has not been studied in detail. The present paper addresses experimentally the issue of the circulation growth in the wake behind a disk and the process in which the axisymmetric vortex structure transitions into a steady-state three-dimensional wake. Some earlier results on vortex roll-up and transition to 3D flow were presented in Balligand et al, 1998. This paper incorporates new results to provide a more integrated view of the subject.

## EXPERIMENTAL SET-UP

The experiment was carried out in a computer-controlled water towing tank and a water channel. The towing tank measured 1.22m in length, 0.46m in width and 0.76m in height. The test section of the recirculating water channel measured 0.61m x 0.61m x 2.44m. These facilities are described in detail in Balligand (2000) and Higuchi et al (1996).

The disk model used for dye flow visualization was 10.16 cm in diameter and 1.5 mm in thickness with a circumferential gap to introduce fluorescent dye. For the digital particle image velocimetry, disks made of anodized aluminum or transparent acrylic were used. Disk was cantilever-supported from upstream with a streamlined vertical strut. In some experiments, a disk of 5.08cm diameter was used and denoted by disk "B" in the legends.

## RESULTS

Figures 1 show the cross-sectional views of the initial wake development behind an impulsively started disk. The disk model in the towing tank was ramped up linearly at a specific value of the acceleration parameter,  $A_p = D(dU/dt)/U_0^2$  to a constant velocity  $U_0$ . Time is normalized as  $T = U_0 t / D$  where  $D$  is the disk diameter. The K-H instability waves were visible in the periphery of the primary vortex. Individual waves carried small vorticity as measured by the PIV (Balligand, et al. 1998.) These are possibly triggered by the minute carriage vibrations though no direct correlation was detected. Similar instability waves were found behind a near-impulsively started flat plate. It is of interest to note that Koumoutsakos and Shiels (1996) found similar instability waves in their numerical calculations behind a uniformly accelerated plate but not behind an impulsively started one, possibly underlining difference in triggering mechanism between computation and experiment.

The time-dependent velocity vector and vorticity fields within the vortex wake were measured with the PIV technique. The technique utilizes the multi-layer adaptive scheme and is described in Zhang (1998). The measured circulation time history is shown in Fig. 2. The contour integral and the integral of vorticity are carried out in the wake extending approximately 1 diameter downstream containing the primary vortex. The growth of the wake size and circulation behind the impulsively started disk scaled with the velocity  $U_0$  and the disk diameter  $D$  over a wide range of the Reynolds numbers tested ( $Re_D=3,100-12,400$ ).

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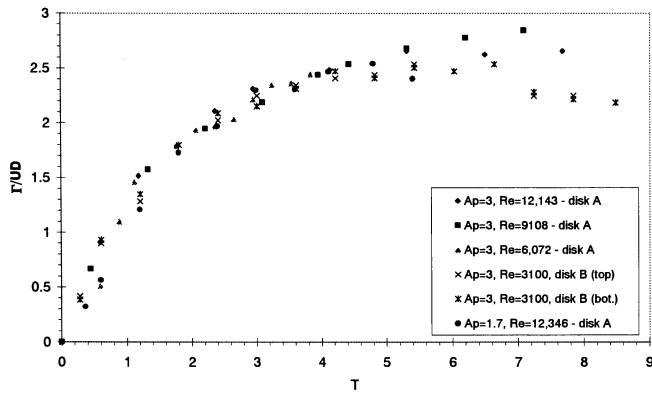
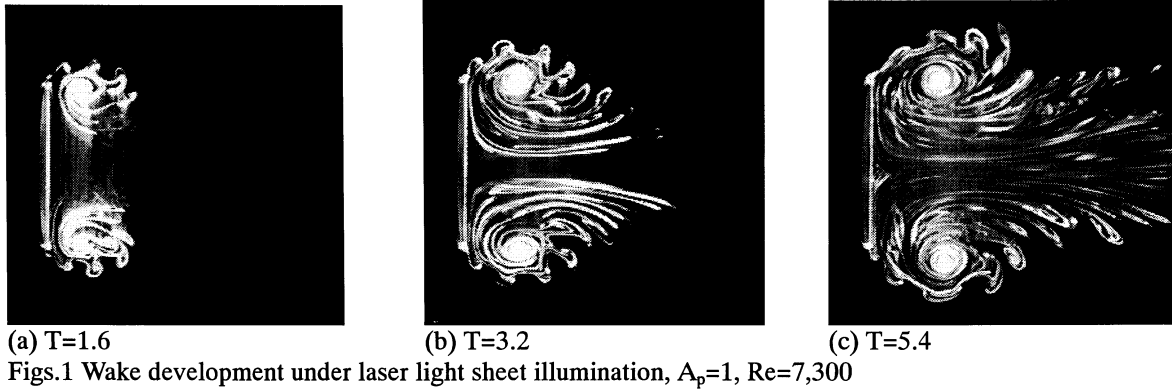


Fig. 2 Circulation growth: impulsively started disk at various Reynolds numbers

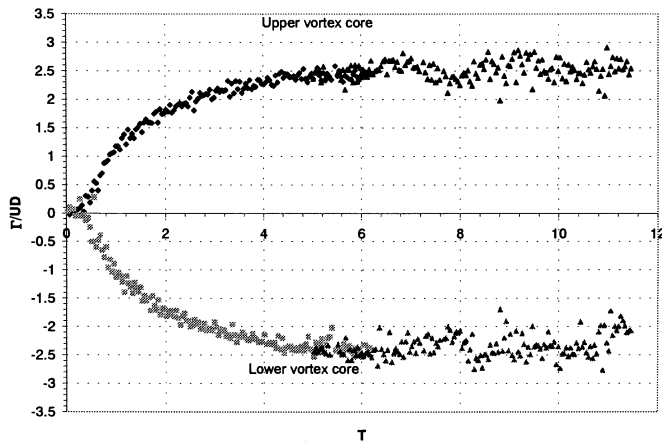


Fig. 3 Circulation time history,  $A_p=0.6$ ,  $Re=3140$

The acceleration parameter  $A_p$  was also varied but the wake could be deemed impulsively started for sufficiently large acceleration parameter. The circulation follows a similar pattern until it reaches  $\Gamma/U_0D=2.5$  approximately at  $T=4$ . Subsequent decrease in the circulation in some cases is likely due to vorticity carrying shear layer extending beyond the interrogation region.

To study the time variation further, the circulation was traced both at the top and bottom sides in the cross section. The transparent disk was used to include the vortices close to the edge of the disk. The vortex

asymmetry sets in at  $T=5$  as evidenced in the location of the vortex center location (Balligand, 2000). Nonetheless, the circulation remains symmetric indicating a contiguous vortex ring.

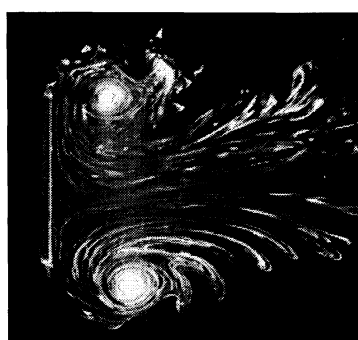
In general, even though the vortex ring has grown in size axisymmetrically, an asymmetry eventually sets in as the circulation reaches this asymptotic value. The maximum circulation contained in the single vortex is analogous to the observation by Gharib et al (ibid) for the vortex ring, though the subsequent vortex shedding in the jet was axisymmetric unlike in the wake.

Figure 4a shows the view subsequent to the sequence shown in Figs. 1. The stagnation behind the disk is shifted and the tilting of the vortex is visible. Figures 4bc are from another run where uniform illumination is used simultaneously with the laser sheet. The deformation of the vortex ring is clearly visible. The circulation in the initial asymmetric wake measured on different radial planes was constant, indicating that the vortex ring was merely deformed. Subsequently, the vortex eventually shed downstream asymmetrically. The result in Fig. 4d was taken in the water channel as the free stream was ramped up from rest to a constant speed. The vortex ring stretched into a hairpin shape and the induced velocity moved the downstream part of the vortex ring onto one side. A newly formed vortex is visible on the bottom side of the disk in Fig. 4d. The newly formed vortex is non-axisymmetric and is likely to set the plane of

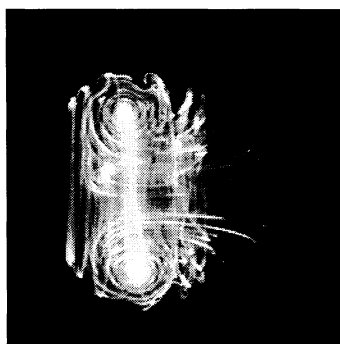
symmetry for subsequent vortex sheddings. If the vortex peels off gradually around the edge, the helical vortex shedding is possible, but typically a preferred direction is encountered, though its direction may vary with time. The multi-scale coherent structure in the turbulent wake behind the disk (see, e.g., Berger, et al. 1990) can be seen emerging at this early stage.

Computation with the axisymmetric vortex method predicts the behavior of the unsteady wake well until the computed circulation reaches the present maximum value.

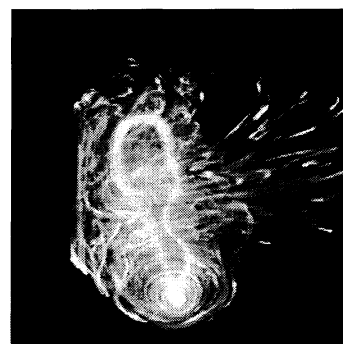
The time variation of the vortex asymmetry can be viewed in a sequence of end-view photos taken in the water channel in Figs. 5. The free stream velocity could be ramped up in repeatable manner while at a much slower rate than in a towing tank. The initial



(a)  $A_p=1$ ,  $Re=7,300$ ,  $T=6.7$



(b)  $A_p=3$ ,  $Re=4,200$ ,  $T=3.5$

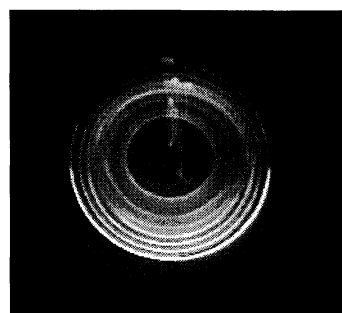


(c)  $A_p=3$ ,  $Re=4,200$ ,  $T=5.9$

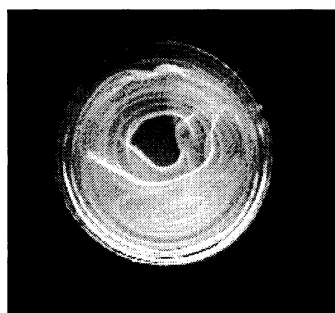


(d)  $A_p=0.35$ ,  $T=10.2$ ,  $L/D=7.7$ ,  $Re=3,600$

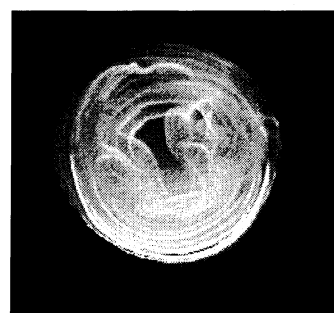
Figs. 4 Onset of three-dimensional wake structure



(a)  $T=9.4$ ,  $L/D=5.4$



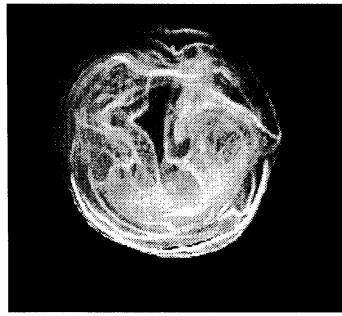
(b)  $T=15.8$ ,  $L/D=10.8$



(c)  $T=17$ ,  $L/D=12$



(d)  $T=17.8, L/D=12.8$



(e)  $T=18.2, L/D=13.2$

Fig. 5 End cross sectional view  $ReD=3070, A_p \approx 0.4$

acceleration corresponds to  $A_p=0.4$ , but the final free stream speed is reached approximately at  $T=8$  (Balligand, 2000). Due to the slow acceleration, the free stream has moved 5.5 diameters at this point (i.e.,  $L/D=5.5$ ). The wake was illuminated by a laser sheet 0.5 diameter downstream of the disk. Axisymmetric vortex roll-up is visible in Fig. 5a. At  $T=9.4$  the downstream end of the tilted vortex loop starts to cross the laser sheet at the bottom, and by  $T=17.8$  a cross section of the counter-rotating vortex pair is clearly seen.

## DISCUSSION

The initial growth of the vortex wake is similar after near-impulsive start. The two-dimensional counterpart of the present experiment was carried out using a flat plate. The time variations of the recirculation region (i.e., measured position of the rear stagnation point) are compared in Fig. 6. For the 2D case, the bubble size computed by Koumoutsakos and Shiels (1996) using Navier-Stokes calculations are also listed. Present experimental results follow Taneda's data while the Reynolds number dependency predicted by Koumoutsakos and Shiels were not reproduced. The circulation growths with time are shown in Fig. 7. Here 2D and axisymmetric computations of circulations are by discrete vortex methods by Spalart (Higuchi et al. 1994) and by Strickland (Higuchi et al,

1996). For the latter, the circulation within 1 diameter downstream is included.

In case of a piston-generated vortex ring, the vortex ring grows till it becomes axis-touching. Subsequent movement of the piston results in axisymmetric pinched-off vortex ring. (Gharib et al. 1998). The circulation growth is linear with time as approximated by a slug flow model that has been adopted by many including Glezer (1988). Behind a disk, on the other hand, the circulation within the vortex ring grows more rapidly with time, corresponding to a large initial drag and rise in impulse. Following growth is limited unlike a piston motion. Unlike axisymmetric pinched-off vortex rings in jet, the vortex wake behind the disk in the present Reynolds number range typically first goes through instability and non-axisymmetric deformation. Subsequent vortex shedding has a preferred direction, depending on the initial vortex. Discrete vortex computation by Strickland (1994) showed pinched-off vortex rings but these were results of inherent axisymmetry of the computer code. Shirayama (1992) computed unsteady three-dimensional initial vortex formation and shedding in detail behind a sphere up to Reynolds number of 500. The maximum circulation strength, however, is not stated in the article.

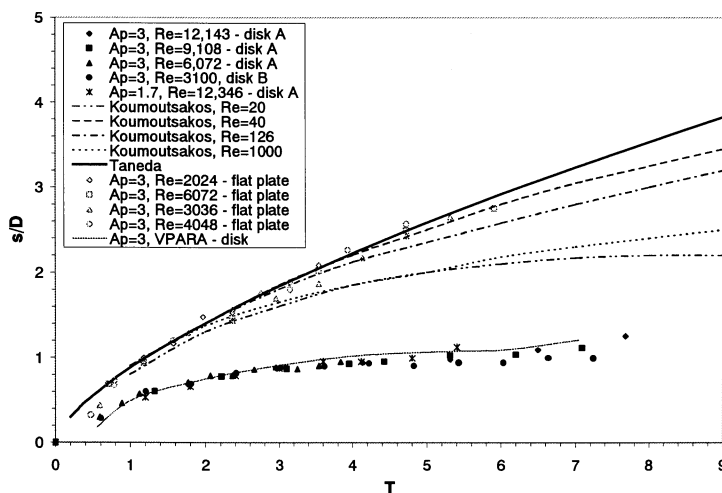


Fig. 6 Wake bubble length: impulsively started flat plate and disk

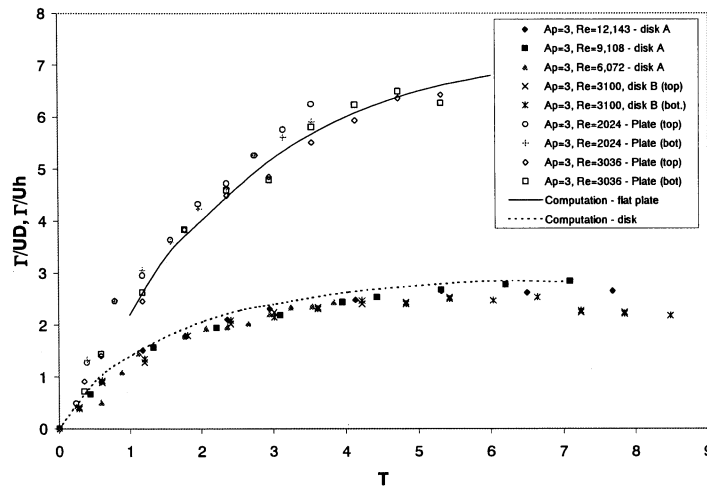


Fig. 7. Circulation growth: impulsively started solid disk and flat plate

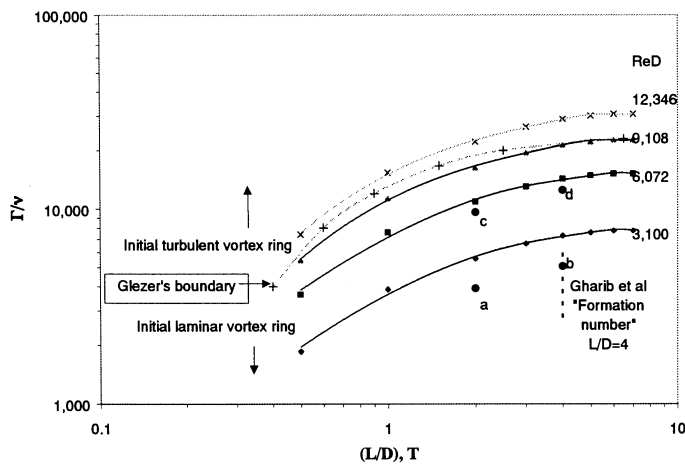


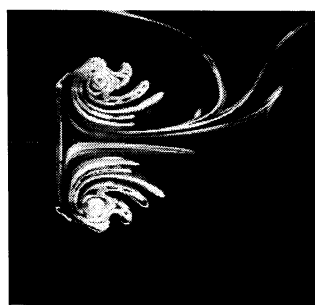
Fig. 8 Map of circulation Reynolds number vs  $L/D$  of  $T$ , for disk B accelerated at  $A_p=3$  to different Reynolds number (see Fig. 8 for a-d)

Figure 8 compares the circulation Reynolds number of the present disk wake and Glezer (1988) and Gharib et al.'s (1998) vortex rings. Primary vortex structure behind the disk remained unaffected with Reynolds number while differences in secondary structures such as K-H waves are discernible as shown in Figs. 9. Gharib noted a likelihood of variation in the formation number  $L/D$  depending on piston velocity and the velocity profile of the jet. The circulation or impulse is not modeled in terms of piston (or model) velocity in the wake as in the case of a piston generated vortex ring. However, when the disk was accelerated in slow acceleration, the onset of the asymmetry was delayed as the growth of the vortex and its circulation. The PIV measurements were not carried out in the present water channel experiment. The uniformly accelerated wake did not reach the asymmetrical stage due to the limited length of the tank. Here, the acceleration and the diameter are used to scale the circulation instead of the constant velocity.

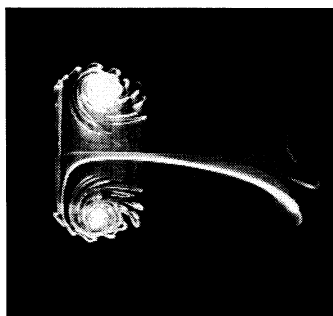
A new long towing tank is readied and the kinetic energy in the wake will be evaluated.

## CONCLUSIONS

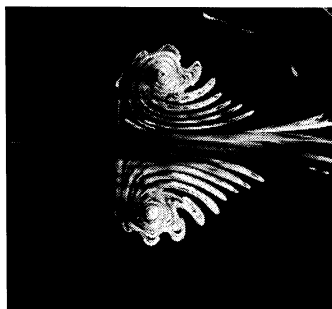
Time history of the circulation contained in the wake of axisymmetric bluff-body has been investigated. The circulation growth at various rapid start-up from the rest to a constant speed follows a similar scaling law, and reaches an asymptote. ( $\Gamma/UD \approx 2.5$ ,  $T \approx 4$ ). The value is naturally larger than Taylor's  $2/\pi$  but growth rate is smaller than the slug flow model. The process is analogous to what has been reported for a piston-driven round jet (formation number for the vortex ring.) However, the notable differences are in the initial vortex growth and the subsequent vortex dynamics after maximum circulation is achieved. The slowly or uniformly accelerated disk experienced delayed asymmetry with more gradual circulation growth.



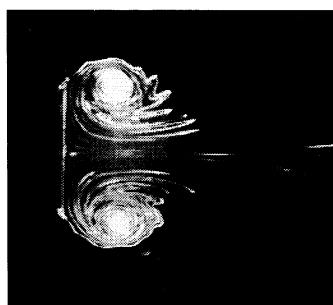
(a)  $T=2$ ,  $Re_D=2,100$ ,  $\Gamma/\nu=3,930$



(b)  $T=4$ ,  $Re_D=2100$ ,  $\Gamma/\nu=5,100$



(c)  $T=2$ ,  $Re_D=5,200$ ,  $\Gamma/\nu=9,640$



(d)  $T=4$ ,  $Re_D=5,200$ ,  $\Gamma/\nu=12,510$

Fig. 9 Comparison of vortex roll-up for two different Reynolds numbers  $A_p=3$ , disk B

Onset of asymmetry and subsequent vortex shedding have been addressed in connection with numerical simulations and 2D counterpart.

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