# EXPERIMENTAL STUDIES AND MODELLING OF FOUR-WAY COUPLING IN PARTICLE-LADEN HORIZONTAL CHANNEL FLOW

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#### **ABSTRACT**

Detailed measurements in a horizontal channel flow laden with solid particles with different size and loading ratio were performed using phase-Doppler anemometry. The data were required for the validation of numerical calculations based on the Euler/Lagrange approach. In this approach the Reynolds-averaged continuity and Navier-Stokes equations in connection with a full Reynolds-stress model constitute the basis. The conservation equations of course include appropriate terms for two- way coupling. For modelling the particle phase in the Lagrangian frame all relevant effects are accounted for, such as, transverse lift forces, turbulent dispersion, wall collisions with roughness, and inter-particle collisions. Comparison of measurement and numerical calculation is presented for different particle diameters and mass loading. The agreement was found to be reasonable good for both mean and fluctuating velocities.

## INTRODUCTION

Confined gas-solid flows are frequently found in industrial and chemical process technology. As a result of the complex micro-physical phenomena affecting the particle motion, such as turbulent dispersion, wall collisions, inter-particle collisions, and flow modulation by the particles, a reliable numerical prediction is rather sophisticated. An essential requirement are reliable experiments which may be used as a basis for model development and refinement and additionally for the validation of the numerical calculations. A number of experiments were performed in the past aiming at a detailed analysis of particulate flows in pipes and channels. A very detailed set of experiments was also provided by Tsuji et al. (1982, 1984) for a gas-solid flow in a horizontal and vertical pipe using different types of relatively large polystyrene spheres. Kulick et al. (1994) experimentally analysed a downward directed gas-solid flow in a channel of 40 mm height. The channel had a length of 5.2 m and a sophisticated feeding system was used in order to insure a homogeneous dispersion of the particles. The particles used

in the experiment were Lycopodium, glass beads with different diameter, and copper beads. The main objective in this study was the analysis of turbulence modulation considering particle mass loadings up to about 0.5 (kg particles)/(kg air). Unfortunately, the experiments were not done carefully enough which has been demonstrated by a number of studies using LES (lage eddy simulations) and DNS (direct numerical simulations). Detailed experiments in different elements of pipes with 80 and 150 mm diameter were performed using phase-Doppler anemometry (PDA) and a laser light sheet technique for particle concentration measurements by Huber and Sommerfeld (1998). The particles were again spherical glass beads with number mean diameters of 40 and 100  $\mu$ m, respectively. A mass loading up to about 2 could be analysed in these studies. The experiments revealed detailed information about the development of the cross-sectional particle concentration in different pipe elements. It was clearly demonstrated that wall roughness plays a significant role in the development of the particle concentration in different cross-sections of the pipe system. In extension of the experimental studies previously performed in different pipes the present measurements are related to a detailed analysis of a gas-solid flow in a horizontal channel by considering also different degrees wall roughness and varying the other parameters, such as, conveying velocity, particle mass loading, particle mean diameter, and size distribution (Kussin and Sommerfeld 2001). These data are being used for the validation and improvement of the Euler/Lagrange approach using recently developed models on wall collisions (Sommerfeld and Huber 1999) and inter-particle collisions (Sommerfeld 2001).

## **TEST FACILITY**

The entire test facility is shown in Fig. 1. The main component of the test facility is a horizontal channel of 6 m length which has a height of 35 and a width of 350 mm, so almost two-dimensional flow conditions can be established. The upper and lower channel walls were made of stainless steel plates which could be exchanged in order to study the effect of wall material and

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wall roughness on the particle behaviour. The measurements were performed close to end of the channel at a distance of 5.8 m from the entrance. In order to allow optical access for the applied phase-Doppler anemometry (PDA), the side walls were made of glass plates and a glass window of 35 by 460 mm was inserted at the top wall. The required air flow rate was provide by two roots blowers mounted in parallel with nominal flow rates of 1002  $m^3/h$  and 507  $m^3/h$ , respectively. The blowers are connected with the test section using a 130 mm-pipe. Just prior to the channel a mixing chamber for injecting the particles and a flow conditioning section where the cross-section changes from circular to rectangular are mounted. Additionally, several sieves are inserted in this section in order to ensure rather homogeneous flow conditions at the entrance of the channel. In a straight section of 2 m before the mixing chamber a flow meter, and temperature, humidity, and pressure sensors are installed. For feeding the particle material into the mixing chamber a screw feeder is used, where the particle mass flow rate can be adjusted accordingly. In order to ensure a continuous particle feeding the air is injected into the mixing chamber through a converging nozzle, whereby a lower pressure is established. The resulting jet enters the exit pipe of the mixing chamber on the opposite side. At the end of the channel a 90°-bend is mounted which is connected to a flow passage where the crosssection changes from rectangular to circular. A flexible pipe is used for conveying the gas-particle mixture to a cyclone separator. The separated particles are re-injected into the reservoir of the particle feeder through a bucket wheel. Finally, the air from the cyclone passes through a bagfilter in order to remove also very fine particles (i.e. the tracer particles) and is released into the environment. The test facility described above allows for reach conveying velocities of up to 30 m/s and mass loadings up to 2 (kg dust)/(kg air) could be established.

The particles used in the experiment were spherical glass beads with different mean diameter  $D_p = 60$ , 100, 195, 625 and 1000  $\mu m$  ( $\rho_p = 2500kg/m^3$ ). For allowing simultaneous measurements of the air and particle velocities, spherical tracer particles with a nominal size of 4  $\mu m$  were added to the flow. This was done by mixing the tracers with the solid particles in the reservoire on the feeder. Hence, a discrimination between dispersed phase particles and tracer was possible using the method of Qiu et al (1991). For the present experiments stainless steel walls with a mean roughness height of 7  $\mu m$  were used. Additional experimental results are presented by

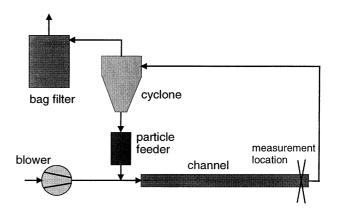


Figure 1: Schematics of experimental facility. Kussin and Sommerfeld (2001).

### SUMMARY OF NUMERICAL APPROACH

The simulation of the particle-laden gas flow in an horizontal channel has been performed using the Euler/Lagrange methodology. The fluid flow was calculated based on the Euler approach by solving the full Reynolds stress turbulence model equations extended in order to account for the effects of the dispersed phase (Kohnen and Sommerfeld, 1997).

The time-dependent conservation equations for the fluid may be written in the general form (in tensorial notation):

$$(\rho\phi)_{,t} + (\rho U_i \phi)_{,i} = (\Gamma_{ik}\phi_{,k})_{,i} + S_\phi + S_{\phi p}$$
 (1)

where  $\rho$  is the liquid density,  $U_i$  are the Reynolds-averaged velocity components, and  $\Gamma_{ik}$  is an effective transport tensor. The usual source terms within the continuous phase are summarised in  $S_{\phi}$ , while  $S_{\phi p}$  represents the additional source term due to phase interaction. Table 1 summarises the meaning of this quantities for the different variables  $\phi$ , being P the mean pressure,  $\mu$  the gas viscosity and  $R_{jl} = \overline{u'_j u'_l}$  the components of the Reynolds stress tensor.

The simulation of the particle phase by the Lagrangian approach requires the solution of the equation of the motion for each computational particle. This equation includes the particle inertia, drag, gravity-buoyancy, slip-shear lift force and slip-rotational lift force. Other forces such as Basset history term, added mass and fluid inertia are negligible for high ratios of particle to gas densities. The change of the angular velocity along the particle trajectory results from the viscous interaction with the fluid (i.e., the torque  $\vec{T}$ ). Hence, the equations of motion for the particles are given by:

$$\frac{dx_{p\,i}}{dt} = u_{p\,i} \tag{2}$$

φ	$\Gamma_{ik}$	$S_{\phi}$
1	0	0
$U_j$	$\mu \delta_{ik}$	$-P_{,j} + (\Gamma_{jk}U_{i,k})_{,i}$
		$-\rho R_{ij,i} + \rho g_j$
$R_{jl}$	$c_S \rho R_{ik} k / \varepsilon$	$\mathcal{P}_{jl} - \varepsilon_{jl} + \Pi_{jl}$
ε	$c_{\varepsilon}\rho R_{ik}k/\varepsilon$	$c_{\varepsilon 1} \mathcal{P}_{kk} \varepsilon / k - \rho c_{\varepsilon 2} \varepsilon^2 / k$
$\mathcal{P}_{jl} = -\rho(R_{jk}U_{l,k} + R_{lk}U_{j,k})$		
$arepsilon_{jl} = rac{2}{3} ho\delta_{jl}arepsilon$		
$\Pi_{jl} = -c_1  ho rac{arepsilon}{k} (R_{jl} - rac{1}{3} \delta_{jl} R_{kk})$		
$-c_2 ho(\mathcal{P}_{jl}-rac{1}{3}\delta_{jl}\mathcal{P}_{kk})$		
$c_S = 0.22 \ c_{\varepsilon} = 0.18 \ c_{\varepsilon 1} = 1.45$		
$c_{\varepsilon 2} = 1.9 \ c_1 = 1.8 \ c_2 = 0.6$		

Table 1: Summary of terms in the general equation for the different variables that describe the gas phase.

$$m_p \frac{du_{p\,i}}{dt} = \frac{3}{4} \frac{\rho}{\rho_p D_p} m_p c_D(u_i - u_{p\,i}) |\vec{u} - \vec{u}_B|$$

$$+ m_p g_i \left( 1 - \frac{\rho}{\rho_p} \right) + F_{ls\,i} + F_{lr\,i}$$

$$I_p \frac{d\omega_{p\,i}}{dt} = T_i$$

$$(4)$$

Here,  $x_{p\,i}$  are the coordinates of the particle position,  $u_{p\,i}$  are its velocity components,  $u_i = U_i + u_i'$  is the instantaneous velocity of the gas,  $D_p$  is the particle diameter and  $\rho_p$  is the density of the solids.  $m_p = (\pi/6)\rho_p D_p^3$  is the particle mass and  $I_p = 0.1 m_p D_p^2$  is the moment of inertia for a sphere. The drag coefficient is obtained using the standard correlation:

$$c_D = \begin{cases} 24 Re_p^{-1} (1 + Re_p^{0.687}) & Re_p \le 1000\\ 0.44 & Re_p > 1000 \end{cases}$$
 (5)

where  $Re_p = \rho D_p |\vec{u} - \vec{u}_p|/\mu$  is the particle Reynolds number.

The slip-shear force is based on the analytical result of Saffman (1965) and extended for higher particle Reynolds numbers according to Mei (1992):

$$\vec{F}_{ls} = 1.615 D_p \mu Re_s^{1/2} c_{ls} [(\vec{u} - \vec{u}_p) \times \vec{w}]$$
 (6)

where  $\vec{\omega} = \nabla \times \vec{u}$  is the fluid rotation,  $Re_s = \rho D_p^2 |\vec{\omega}|/\mu$  is the particle Reynolds number of the shear flow and  $c_{ls} = F_{ls}/F_{ls,Saff}$  represents the ratio of the extended lift force to the Saffman force:

$$c_{ls} = \begin{cases} (1 - 0.3314\beta^{0.5})e^{-Re_p/10} \\ +0.3314\beta^{0.5} & Re_p \le 40 \\ 0.0524(\beta Re_p)^{0.5} & Re_p > 40 \end{cases}$$
(7)

and  $\beta$  is a parameter given by  $\beta = 0.5 Re_s/Re_p$ .

The applied slip-rotational lift force is based on the relation given by Rubinow and Keller (1961), which was extended to account for the relative motion between particle and fluid. Moreover, recent measuments by Oesterlé and Bui Dinh (1998) allowed an extension of this lift force to higher particle Reynolds numbers. Hence, the following form of the slip-rotation lift force has been used:

$$\vec{F}_{lr} = \frac{\pi}{8} D_p^3 \rho \frac{Re_p}{Re_r} c_{lr} [\vec{\Omega} \times (\vec{u} - \vec{u}_p)]$$
 (8)

with  $\vec{\Omega} = 0.5\nabla \times \vec{u} - \vec{\omega}_p$  and the Reynolds number of particle rotation is given by  $Re_r = \rho D_p^2 |\vec{\Omega}|/\mu$ . The lift coefficient according to Oesterlé and Bui Dinh (1998) is given for  $Re_p < 140$  by:

$$c_{lr} = 0.45 + \left(\frac{Re_r}{Re_p} - 0.45\right)e^{-0.05684Re_r^{0.4}Re_p^{0.3}} \tag{9}$$

For the torque acting on a rotating particle the expression of Rubinow and Keller (1961) was extende to account for the relative motion between fluid and particle and higher Reynolds numbers:

$$\vec{T} = \frac{\rho}{2} \left( \frac{D_p}{2} \right)^5 c_R |\vec{\Omega}| \vec{\Omega} \tag{10}$$

where the coefficient of rotation is obtained from Rubinow and Keller (1961) and direct numerical simulations of Dennis et al. (1980) in the following way:

$$c_R = \begin{cases} \frac{64\pi}{Re_r} & Re_r \le 32\\ \frac{12.9}{Re_r^{0.5}} + \frac{128.4}{Re_r} & 32 < Re_r < 1000 \end{cases}$$
(11)

The equations to calculate the particle motion are solved by integration of the differential equations (Eqs. 2-4). For sufficiently small time steps and assuming that the forces remain constant during this time step, the new particle location, the linear and angular velocities are calculated.

The instantaneous fluid velocity components at the particle location occurring in (3) are determined from the local mean fluid velocity interpolated from the neighbouring grid points and a fluctuating component generated by the Langevin model described by Sommerfeld et al. (1993). In this model the fluctuation velocity is composed of a correlated part from the previous time step and a random component sampled from a Gaussian distribution function. The correlated part is calculated using appropriate time and length scales of the turbulence form the Reynolds stress turbulence model.

When a particle crosses a wall, the wall collision model provides the new particle linear and

angular velocities and the new location in the computational domain after rebound. The applied wall collision model, accounting for wall roughness, is described in Sommerfeld and Huber (1999). Inter-particle collisions are modelled by the stochastic approach described in detail in Sommerfeld (2001). This model relies on the generation of a fictitious collision partner and accounts for a possible correlation of the velocities of colliding particles in turbulent flows.

#### EFFECT OF PARTICLES ON GAS FLOW

The standard expression for the momentum equation source term due to the particles has been used. It is obtained by time and ensemble averaging for each control volume in the following form:

$$\overline{S_{U_ip}} = -\frac{1}{V_{cv}} \sum_k m_k N_k \times$$

$$\sum_{n} \left\{ \left( [u_{p i}]_{k}^{n+1} - [u_{p i}]_{k}^{n} \right) - g_{i} \left( 1 - \frac{\rho}{\rho_{p}} \right) \Delta t_{L} \right\}$$
(12)

where the sum over n indicates averaging along the particle trajectory (time averaging) and the sum over k is related to the number of computational particles passing the considered control volume with the volume  $V_{cv}$ . The mass of an individual particle is  $m_k$  and  $N_k$  is the number of real particles in one computational particle.  $\Delta t_L$  is the Lagrangian time step which is used in the solution of (3).

The source terms in the conservation equations of the Reynolds stress components,  $R_{jl}$ , are expressed in the Reynolds average procedure as:

$$S_{R_{il}p} = \overline{u_i S_{U_l p}} + \overline{u_l S_{U_i p}} - (U_j \overline{S_{U_l p}} + U_l \overline{S_{U_i p}})$$
(13)

while the source term in the  $\varepsilon$ -equation is modelled in the standard way:

$$S_{\varepsilon p} = C_{\varepsilon 3} \frac{1}{2} \frac{\varepsilon}{k} S_{R_{jj}p} \tag{14}$$

with  $C_{\varepsilon 3} = 1.0$  and the sum is implicit in the repeated subindex j.

#### RESULTS

The numerical simulations have been compared with experimental data obtained in the horizontal channel facility described in the first section. The following three cases have been considered:

- 1.  $D_p = 195 \mu m$ , mass loading ratio  $\eta = 0.1$ .
- 2.  $D_p = 195 \mu m$ , mass loading ratio  $\eta = 1.0$ .
- 3.  $D_p = 60 \mu m$ , mass loading ratio  $\eta = 0.2$ .

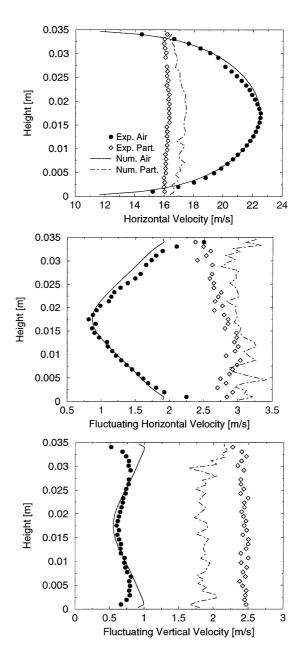


Figure 2: Comparison of measurements and calculation for Case 1, i.e.  $D_p = 195 \mu m$  and  $\eta = 0.1$ . Horizontal mean velocity (top), rms horizontal velocity (middle) and rms vertical velocity(bottom).

The simulations have been performed using the full Reynolds stress turbulence model described in the previous section and accounting for particle-wall and inter-particle collisions, i.e. considering the so called four-way coupling.

Figure 2 shows the results for Case 1, where due to the small mass loading the effect of inter-particle collisions is small compared to the particle-wall collisions. The fluid variables show a good agreement with experimental data. The particles horizontal mean velocity, on the other hand, is overpredicted regarding the measurements at maximum 6 % (top); the particle horizontal rms values compare reasonably well with

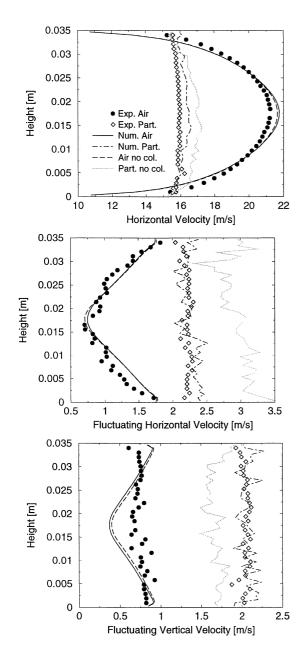


Figure 3: Comparison of measurements and calculation for Case 2, i.e.  $D_p=195\mu m$  and  $\eta=1.0$ . Horizontal mean velocity (top), rms horizontal velocity (middle) and rms vertical velocity (bottom). Also the simulations without considering inter-particle collision are included.

experiments (middle), but a problem exists in the prediction of the vertical component of the particles fluctuating velocity (bottom), which is found to be considerably higher in the PDA measurements. This is supposed to be the result of bias effects in the measurements since this large difference is not observed for other cases.

In the same arrangement as in the previous figure, the results for the Case 2 are presented in Figure 3. In this case the mass loading is ten times larger than in Case 1, so the interparticle collisions are expected to play a remarkable role. To point out this effect, also results without considering inter-particle collisions are

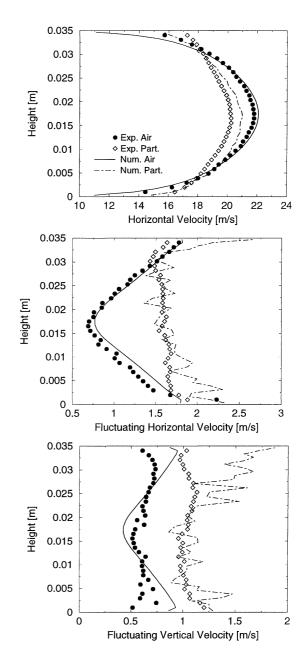


Figure 4: Comparison of measurements and calculation for Case 3, i.e.  $D_p=60\mu m$  and  $\eta=0.2$ . Horizontal mean velocity (top), rms horizontal velocity (middle) and rms vertical velocity(bottom).

shown in Fig. 3. The agreement with the measured gas properties is again good enough, and this time also the values for the particle phase are quite close to the experimental data when the four-way coupling is considered. If only the two coupling is taken into account, i.e. neglecting inter-particle collisions, an overprection of the particles horizontal fluctuating and mean velocities is observed as well as an underestimation of the vertical rms velocity values. All this behaviour agrees with that pointed out in Huber and Sommerfeld (1998), where the inter-particle collisions tend to isotropize the particle normal fluctuating stresses.

Figure 4 shows the comparison for the Case 3 of smaller particles. Here, in order to get a better agreement with the measurements, a size distribution, suggested by the real particles injected, has been implemented. As it is usual, the predictions of the gas phase variables are close enough to the experiments. The particle mean horizontal velocity (top) is slightly overestimated regarding the measurements, a fact that has also been found in Case 1, while the corresponding rms values (middle) are reasonably well captured. The bottom graphic in Fig. 4 shows the comparison for the vertical velocity rms values, where the simulations provide similar results to the experiments for the particles in the lower two thirds of the channel, while there is an overprediction in the upper third. This effect can be explained because in the upper regions of the channel the number density of particles is smaller than in the lower parts leading to a larger values of the rms components of the velocity.

#### CONCLUSION

This paper shows that the use of the Reynolds stress turbulence model in conjunction with a wall roughness model and a stochastic interparticle model is appropriate enough to predict the behaviour of particle-laden turbulent gas flow in channels. Not only the mean but also rms particle velocities have been taken into account showing reasonably good agreement with detailed experimental measurements carried out using PDA.

The future work will be devoted to deal with larger particles as well as the evaluation of pressure drop in the channel.

#### Acknowledgement

The research project is financially supported by the Deutsche Forschungsgemeinschaft under contract number So 204/12-1 and 2, which is gratefully acknowledged. Also, we gratefully acknowledge to Dr. G. Kohnen his useful comments and suggestions.

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