MODELING OF UNSTEADY TURBULENT WALL FLOWS WITH AND WITHOUT ADVERSE PRESSURE GRADIENT BY A k-ω /RAPID DISTORTION MODEL

Sedat Tardu
Laboratoire des Ecoulements Géophysiques et Industriels (LEGI)
BP 53 X 38041 Grenoble-Cédex France
email: Sedat.Tardu@hmg.inpg.fr

Paulo Da Costa
Laboratoire des Ecoulements Géophysiques et Industriels (LEGI)
BP 53 X 38041 Grenoble-Cédex France
email: Da_Costa@hmg.inpg.fr

ABSTRACT

The k-ω model is combined with the rapid distortion scheme to develop effective unsteady closures in non-equilibrium wall flows subjected to oscillating shear. The phase averaged eddy viscosity is related to the modulation of the effective strain parameter whose distribution is obtained by making use of the bench data of either steady turbulent channel or boundary layer flow. The model is free of any assumption concerning the modulation of production nor the dissipation. The results are compared with the experiments in details, including the modulation characteristics of the turbulent wall shear stress intensity. The model predicts the time mean and phase averaged flow quantities in a satisfactory way. The same procedure is applied to a turbulent boundary layer with adverse pressure gradient.

INTRODUCTION

Background and aim of the study

The aim of this study is to develop effective real unsteady closures to be applied to wall flows subjected to imposed unsteadiness and that are consequently in temporal non-equilibrium. Such flows occur on helicopter blades, in turbomachinery, in internal combustion systems etc… The forcing may simply be achieved by a time-varying flow rate or pressure gradient. Considerable experimental efforts have been made in the past to explore the different physical aspects of unsteady turbulent flows. Several groups involved in this subject. Tu and Ramaprian (1983), Mao and Hanratty (1986) , Brereton et al. (1990), Tardu et al. (1994) are only a few of them. The unsteady wall flows have also attracted the attention of the modelers. Most of the researchers have directly applied the phase averaged version of existing closures, by taking more or less into account the real non-equilibrium character of the problem. We may quote, Fan et al. (1993), and Chernobrovkin and Laksminarayana (1999) who applied the k-ε scheme, Ekaterinaris et al. (1994) who used three different versions of the k-ω method combined with the baseline and shear stress transport models, and finally Srinivasan et al. (1995) who chose algebraic models and renormalization group analysis. The majority of the models found in the literature use quasi-steady assumptions, with the exception of Mankbadi and Liu (1992), and to some extends Mao and Hanratty (1986) and Greenblatt (1998).

New closure

The direct application of the k-ω closure to the unsteady turbulent channel flow is:

$$\frac{\partial\langle\omega\rangle}{\partial t}=-\langle\gamma\rangle\frac{\langle\omega\rangle}{\langle k\rangle}\langle u'v'\rangle\frac{\partial\langle u\rangle}{\partial y}-\beta\langle\omega\rangle^2+\frac{\partial}{\partial y}\left[\left(\nu+\sigma\langle\nu_t\rangle\right)\frac{\partial\langle\omega\rangle}{\partial y}\right] \quad (1)$$

$$\frac{\partial\langle k\rangle}{\partial t}=-\langle u'v'\rangle\frac{\partial\langle u\rangle}{\partial y}-\langle\beta'\rangle\langle\omega\rangle\langle k\rangle+\frac{\partial}{\partial y}\left[\left(\nu+\sigma\langle\nu_t\rangle\right)\frac{\partial\langle k\rangle}{\partial y}\right] \quad (2)$$

where $-\langle u'v'\rangle$ is the *phase average* of the Reynolds shear stress, $\langle\gamma\rangle$, σ, β and $\langle\beta'\rangle$ are numerical constants depending upon the turbulent Reynolds number $\langle Re_t\rangle=\frac{\langle k\rangle}{\langle\omega\rangle}$, $\langle\;\rangle$ denotes the phase average and ν is the cinematic viscosity. The rest of the symbols are usual, with y standing for the wall normal distance, u the streamwise velocity, k the kinetic energy, ν_t the eddy viscosity, and t the time. According to Wilcox (1998) the effect of the turbulent Reynolds number the coefficients appearing in (1) and (2) are modeled as:

$$\langle\gamma\rangle=\frac{5}{9}\frac{1/10+\langle Re_t\rangle/R_\omega}{1+\langle Re_t\rangle/R_\omega}\left(\langle\chi\rangle\right)^{-1} \quad (3a)$$

$$\langle \beta' \rangle = \frac{9}{100} \frac{5/18 + \left(\langle Re_t \rangle / R_\beta \right)^4}{1 + \left(\langle Re_t \rangle / R_\beta \right)^4} \quad (3b)$$

with:

$$\langle \chi \rangle = \frac{1/40 + \langle Re_t \rangle / R_k}{1 + \langle Re_t \rangle / R_k} \quad (3c)$$

The ensemble of the numerical constants appearing in (1), (2) and (3) are, $\sigma = 0.5$, $\beta = 3/40$, $R_\omega = 2.7$, $R_\beta = 8$ and $R_k = 6$ (Wilcox, 1998). The adjustment of these coefficients is based on the modeling of the transition in a turbulent boundary layer. The effect of $\langle Re_t \rangle$ is mainly confined in the viscous and low buffer sublayers.

The equations (1) and (2) are combined with the Boussinesq relationship:

$$-\langle u'v' \rangle = \langle \nu_t \rangle \frac{\partial \langle u \rangle}{\partial y} \quad (4a)$$

and the entire closure is achieved by relating the eddy viscosity to $\langle k \rangle$ and $\langle \omega \rangle$ by

$$\langle \nu_t \rangle = \langle \chi \rangle \frac{\langle k \rangle}{\langle \omega \rangle} \quad (4b)$$

Clearly, this approach is *quasi-steady*, and therefore, its validity is restricted to low frequency imposed oscillations. The equations (1) and (2) are "exact" but (4) supposes time equilibrium.

The rapid distortion model is therefore introduced in a way similar, but not identical to Mankbadi and Liu (1992, MK hereafter). To this end, the effective strain parameter $\langle \alpha_{eff} \rangle$ governed by:

$$\frac{\partial \langle \alpha_{eff} \rangle}{\partial t} = -\frac{\langle \alpha_{eff} \rangle}{T_d(y)} + \frac{\partial \langle u \rangle}{\partial y} + \frac{\partial}{\partial y}\left[D(y) \frac{\partial \langle \alpha_{eff} \rangle}{\partial y} \right] \quad (5)$$

is used, according to Maxey (1982) and where T_d stands for the distortion time scale. Note that the diffusion of the effective strain is also taken into account through the last term as suggested by Townsend (1970). Subsequently, the cyclic modulations of the Reynolds shear stress and of the eddy diffusivity are computed according to:

$$\frac{-\langle u'v' \rangle}{\langle k \rangle} = F\left(\langle \alpha_{eff} \rangle \right) \quad (6a)$$

$$\langle \nu_t \rangle = \langle \chi \rangle F\left(\langle \alpha_{eff} \rangle \right) \frac{\langle k \rangle}{\frac{\partial \langle u \rangle}{\partial y}} \quad (6b)$$

where the function $F\left(\langle \alpha_{eff} \rangle \right)$ together with the distortion time scale $T_d(y)$ are obtained by the bench data of the steady turbulent channel -or boundary layer flow. As already pointed out, this unsteady closure scheme is similar to that of MK. There are however important differences:

i- There is any hypothesis on the effect of the imposed unsteadiness on the time mean flow characteristics here, while MK suppose that the time mean flow is unaffected.

ii- MK use a linearized version of the rapid distortion equation (5), while the relationships (1), (2) and (6) are "exact" and can be used for large imposed amplitudes here.

iii- MK suppose that there is time equilibrium between the turbulence energy and dissipation near the wall, while any hypothesis of this kind is made here.

iv- Finally we also introduce the gradient-type diffusion term in the effective strain equation 2, and take into account the transport of $\langle \alpha_{eff} \rangle$ by small-scale turbulence.

We numerically have shown that the $k - \omega$ equation (1) and (2) are *compatible* with the rapid distortion equation (5) and (6) in unsteady turbulent channel flow and boundary layers with adverse pressure gradient as well.

It has to be noted that the turbulent Reynolds number correction is quasi-steady in the closure we used. That gave acceptable agreement with the experimental data, as we will discuss hereafter. The rapid distortion equations do only indirectly take into account the presence of the wall. The rapid distortion model used here results from the match of the response of an axially symmetric homogeneous turbulence, with the near wall flow, through the similarity of the structure parameters, such as $\frac{-\overline{u'v'}}{\overline{u'u'}}$ distribution, versus the effective strain parameter (Maxey, 1982).

Experiments

The experiments have been conducted in the unsteady water channel of LEGI. A detailed description of the test facility, acquisition and processing of the data may be found in different publications of the group (Tardu et al., 1994, Tardu et al., 1995).

RESULTS

The classical triple decomposition is used. A quantity q is decomposed into a mean value $\bar{q}$, oscillating $\tilde{q}$, and fluctuating q' component. The phase average of q is defined as $\langle q \rangle = \bar{q} + \tilde{q}$. The modulation characteristics of $\langle q \rangle$ are defined as the amplitude $A_{\tilde{q}}$ and phase $\phi_{\tilde{q}}$ of its first Fourier mode. The relative amplitude $a_{\tilde{q}} = \frac{A_{\tilde{q}}}{\bar{q}}$ is introduced for convenience.

There is no effect of the imposed unsteadiness on the time mean flow characteristics, even when the imposed amplitude at the centerline is as large as

30% and the imposed frequency is near the bursting frequency of the base flow (not shown here, see Da-Costa, 2000). This is in perfect agreement with the experiments (Tardu et al., 1994).

Fig. 1 shows the ratio of the wall shear stress modulation amplitude to the amplitude $A_{\tilde{\tau}\,Stokes}$ that a purely viscous Stokes flow would have. The frequency parameter denoted by $l_s^+ = \sqrt{\dfrac{1}{\pi f^+}}$ in the abscissa, is the Stokes length in wall units, with f^+ standing for the imposed frequency ($^+$ denotes values non-dimensionalized by viscosity and shear velocity). The results inferring from the modeling are compared both with the measurements (Tardu and Binder, 1993) and with MK. It is seen that there is good agreement between the measurements and the predictions, and that the MK model fails in the low frequency (quasi steady) regime. It is also interesting to note that the numerical results are below one for $l_s^+ \le 15$, exactly as in the experiments. The phase shift (not shown) predicted by the model indicates a value of 45° between the wall shear stress and the centerline velocity, accordingly to the viscous Stokes model. There is a coexistence of the unaffected time mean flow with a purely oscillating viscous flow when the frequency is large enough. This behavior is explained with the confinement of the oscillating shear into the low buffer layer at $l_s^+ \le 10$. Consequently, the oscillating wall shear stress does not interact with the turbulence producing eddies in the high frequency regime. The amplitude and phase shift distributions of the measured and computed streamwise velocity oscillations $\langle U \rangle$ are also in close agreement in the whole imposed frequency regime investigated here.

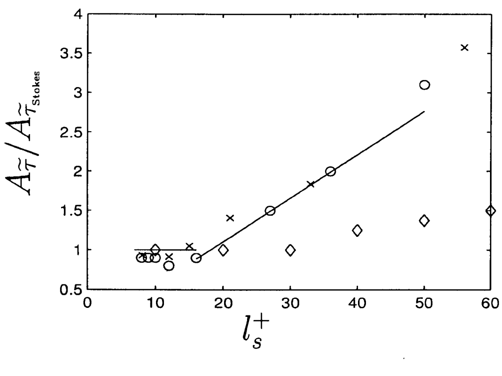
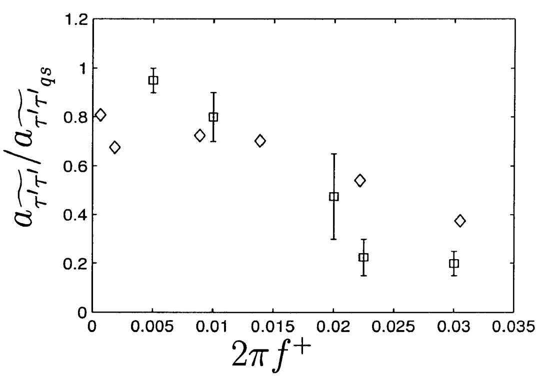


Figure 1: Amplitude of the wall shear stress versus the frequency parameter. This model: x, Mankbadi and Liu (1992):◇ Measurements Tardu and Binder (1993): o. The imposed amplitude is 20 % of the centerline velocity.

The response of turbulence is particularly difficult to model in unsteady wall flows. Fig. 2a shows the ratio of the relative amplitude of the turbulent wall shear stress intensity $\langle \tau' \tau' \rangle$ to the relative amplitude in the quasi-steady regime. The rapid decrease of the turbulence intensity modulation with the imposed frequency from the quasi-steady limit is clearly seen in this figure. This is due to the finite response time of the near wall turbulence that can not follow the imposed unsteadiness when the imposed time scale is smaller than its relaxation time. The decrease in $a_{\widetilde{\tau'\tau'}}$ is accompanied with the apparition of large phase shifts $\Phi_{\widetilde{\tau'\tau'}} - \Phi_{\tilde{U}_c}$ as shown in Fig. 2b. There is a good agreement with the experiments, although the predictions are slightly larger at the highest imposed frequency investigated here. The agreement between the experiments and the model is also acceptable as long as the phase shifts are concerned. The time lag between $\langle \tau' \tau' \rangle$ and the centerline velocity oscillations is about 90 wall units. Such detailed comparisons concerning the response of turbulence at the wall are not common in the literature.

The high frequency imposed oscillations are particularly interesting regarding the rapid distortion theory. Fig. 4a shows the relative amplitude of the Reynolds shear stress modulation at the imposed

a-



b-

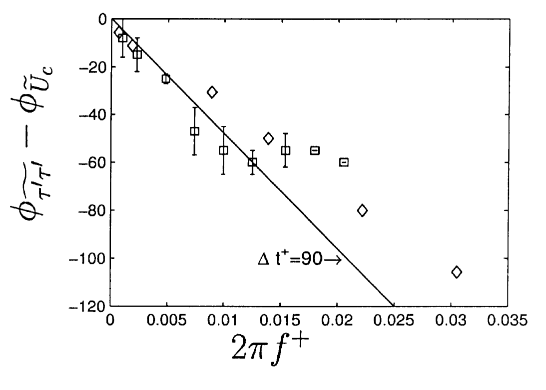


Figure 2: Modulation characteristics of the wall shear stress turbulent intensity versus the imposed frequency in wall units. Relative amplitude (a) and phase shift (b). This model: ◇ , Experiments (Tardu and Binder , 1993) with dispersion symbol: □.

frequency $f^+ = 0.005$ where the modulations of the turbulent quantities decrease drastically, due, once more, to the finite response time of the near wall turbulence. This fact can clearly be understood by the direct comparisons of Fig. 4 with Fig. 3 that shows results in the quasi-steady regime (please be aware that the scale in the abscissas of these figures are different). The model agrees satisfactorily well with the experimental data, not only for the amplitudes, but also for the phase shifts (Fig. 4b). The agreement is almost perfect for a large range of imposed frequencies in the whole layer. We show in Fig. 5 the modulation characteristics of the Reynolds shear stress at $y^+ = 12$ to illustrate the success of the closure. Similar agreement has been found for the amplitude and phase shift of the production that has been experimentally determined before (Tardu et al., 1995). It has to be recalled that the direct application of k-ε and Reynolds shear stresses transport models are not as successful as the model investigated here. The success of k-ω/ Rapid distortion in the whole layer is surprisingly satisfactory.

The model has been extended to a turbulent boundary layer with a time mean adverse pressure gradient $\frac{d\overline{P}}{dx} > 0$, without major modification. The function $F\left(\left\langle \alpha_{eff} \right\rangle\right)$ and the distortion time scale distribution were determined by making use of the DNS data of Spalart (1988). The Fig. 6 shows some numerical data and measurements performed by Jarayaman et al. (1982) in a turbulent boundary layer, at a mild Clauser parameter $\beta = \frac{d\overline{P}}{dx}\frac{\delta^*}{\overline{\tau}} = 0.5$ where δ^* is the local displacement thickness, and at a local frequency parameter $l_s^+ = 11.8$. We compare the relative

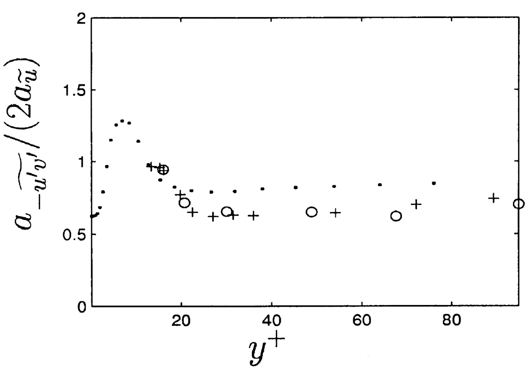
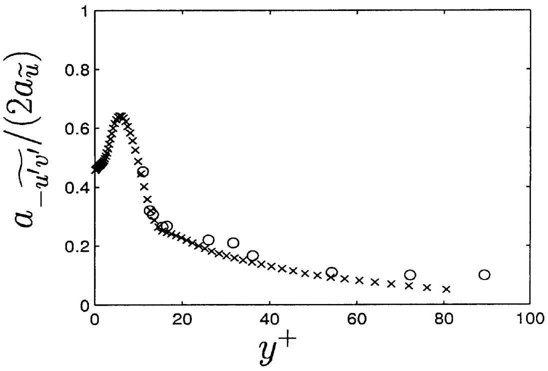


Figure 3: Amplitude of the Reynolds shear stress versus the wall normal distance at $l_s^+ = 56$ (in the quasi-steady regime. The symbols used are: .... : This model, o: Experiments at $l_s^+ = 42$, +: Experiments at $l_s^+ = 30$.

a-



b-

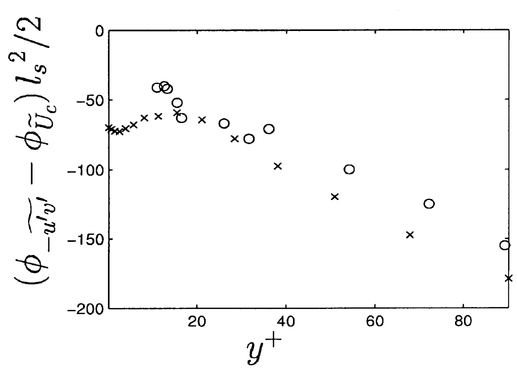
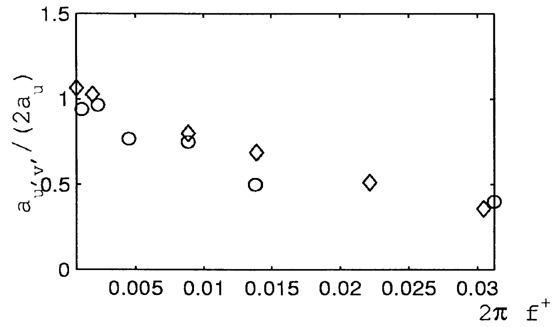


Figure 4: Modulation characteristics in the relaxation regime at $l_s^+ = 8$. Comparison between the model (**x**), and the experiments (o). a- Amplitude, b-Time lag.

amplitude and phase shift of the turbulent kinetic energy $\langle k \rangle$ inferred from the modeling, with experimental values related to the measured streamwise turbulent intensity $\langle u' u' \rangle$. Thus, perfect agreement can not be expected, at least near the wall. Yet, it is seen, that the predictions are quite acceptable.

The computations have shown that there is no serious effect of the adverse pressure gradient on the time mean flow up to $\beta = 5$, $l_s^+ = 7$, when the imposed amplitude is 10% of the free stream velocity. The oscillating Stokes regime is still present under $\frac{d\overline{P}}{dx} > 0$. The wall shear stress is more modulated in the presence of strong adverse pressure gradient, in the range of mild and low imposed frequency regime. The ratio, $\frac{a_{\tilde{\tau}\tau'}}{a_{\tilde{\tau}}}$ that goes to 2 in the low frequency regime in channel flow, is found to be systematically smaller at $l_s^+ \geq 20$ in the presence of adverse pressure gradient.

a-



b-

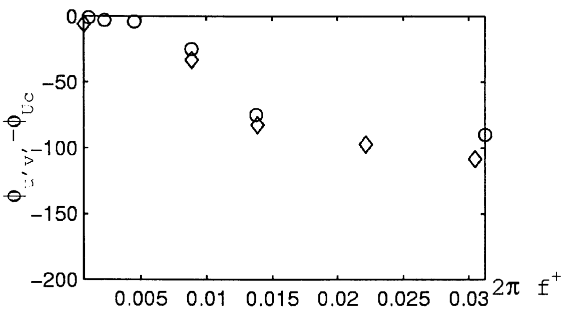
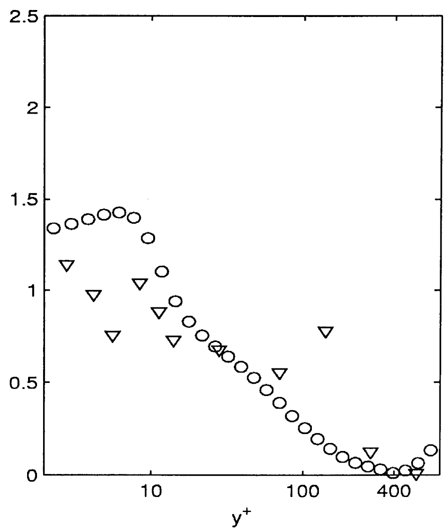


Figure 5: Comparison of the modulation characteristics (a: amplitude, b: phase shift) between the model (◇) and experiments (o) versus the imposed frequency at $y^+ = 12$.

Some remarks are necessary with regards to the effect of the rapid distortion over the k-ω model alone with and without wall correction. We conducted several numerical experiments, by separately considering the simple quasi-steady k-ω model (without rapid distortion and through equation 4), together with models with and without $\langle Re_t \rangle$ correction. The following points have been noticed:

- The quasi-steady k-ω closure compares well with the full model at $y^+ \geq 20$ and in the whole relaxation regime $f^+ \leq 0.005$.
- In return, the rapid distortion plays an essential role in the low buffer and viscous sublayers, wherein the low Reynolds effect through $\langle Re_t \rangle$ is also primordial. The k-ω closure, alone, overestimates the amplitude of the shear stress up to 60% in the high frequency range near the wall. It gives considerably different modulation characteristics of the wall shear stress turbulent intensity, compared with the experiments.
- The effect of the diffusion term in the transport equation of the effective strain parameter $\langle \alpha_{eff} \rangle$ is negligible.

$\frac{a_{\tilde{k}}}{a_{\tilde{u}}}$ (*Model*) $\frac{a_{\tilde{u}'u'}}{a_{\tilde{u}}}$ (*Experiments*)



$\phi_{\tilde{k}} - \phi_{\tilde{U}_c}$ (*Model*) $\phi_{\tilde{u}'u'} - \phi_{\tilde{U}_c}$ (*Experiments*)

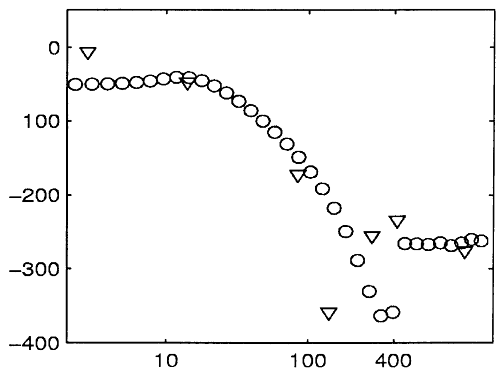


Figure 6: Modeling of an unsteady turbulent boundary layer with time mean adverse pressure gradient at a Clauser parameter equal to 0.5, and $l_s^+ = 11.8$. Comparisons are related to the turbulent kinetic energy and streamwise turbulent intensity. ∇: Experiments (Jarayaman, 1982), o: Model.

CONCLUDING REMARKS

We carefully and in some details compared the results inferred from the model with the majority of available experimental data. The study is complete in that sense and the model is successful in the relaxation regime for $f^+ \leq 0.005$. There are few experimental data set for $f^+ \geq 0.005$ that indicates a fully different reaction of the near wall turbulence. In that regime, the experimental data shows that fluctuating shear stresses become *modulated again*, contrarily to the relaxation regime (Finnicum and Hanratty, Tardu and Binder, 1993). There is no known model that gives satisfactory prediction, even qualitatively at $f^+ \geq 0.005$. We could obtain fairly good agreement between the predictions and the measurements by both forcing and phase shifting the near wall, low turbulent Reynolds number correction. Although this procedure can be somewhat considered artificial, it has the merit to point at the primordial role played by the wall in the regeneration of the turbulence activity modulation. It is finally asked here, if the future development of the model should not contain a supplementary term in the transport equation of the effective strain:

$$\frac{\partial\langle \alpha_{eff}\rangle}{\partial t} = -\frac{\langle \alpha_{eff}\rangle}{T_d(y)} + \frac{\partial\langle u\rangle}{\partial y} + \frac{\partial}{\partial y}\left[D(y)\frac{\partial\langle \alpha_{eff}\rangle}{\partial y}\right] + \frac{\partial}{\partial y}\left[\langle V\alpha_{eff}\rangle\right] \quad (7)$$

where both a gradient type diffusion and convection by large-scale $\langle V\rangle$ motions are included. The modeling of the later could, eventually, give better insight into the mechanism at very high-imposed frequencies.

References

Brereton G.J., Reynolds W.C., Jarayaman R., 1990 "Response of a turbulent boundary layer to sinusoidal free stream unsteadiness" J. Fluid Mech., 221, 131.

Da_Costa, P., 2000 "Application de la distorsion rapide à des écoulements turbulents instationnaires", PhD Thesis, Université Joseph Fourier, Grenoble, France.

Fan S., Laksminarayana B., Barnett M., 1993 "A low Reynolds number k-ε model for unsteady turbulent boundary layer flows" AAIA Journal, 31, 1777-1784.

Chernobrovkin A., Lakshminarayana B., 1999 "Development and validation of Navier Stokes procedure for turbomachinery flow" AAIA Journal, 37, 557-563.

Ekaterinaris J.A., Menter F.R., 1994 "Computation of oscillating airfoil flows with one and two equations turbulence models" AAIA Journal, 32, 2359-2053.

Finnicum D.S., Hanratty T.J., 1988 "Influence of imposed flow oscillations on turbulence" Phys. Chem. Hydrodynamics, 10, 585.

Greenblatt D., 1998 "Hybrid turbulence model for unsteady boundary layers" AAIA Journal, 36, 481-483.

Jarayaman R., Parikh P., Reynolds W.C., 1982 "An experimental study of the dynamics of an unsteady turbulent boundary layer" Technical Report N° Tf-18, Dept. Mech. Eng., Stanford Un., CA.

Mankbadi R.R., Liu J.T.C., 1992 "Near-wall response in turbulent shear flows subjected to imposed unsteadiness" J. Fluid Mech., 238, 55-71.

Mao X., Hanratty T.J., 1986 "Studies of wall stress in a turbulent pulsating pipe flow" J. Fluid Mech., 194, 15-44.

Maxey M.R., 1982 "Distortion of turbulence in flows with parallel streamlines" J. Fluid Mech., 124, 261-282.

Spalart P.R., 1988 " Direct numerical simulation of a turbulent boundary layer up to $Re_\theta = 1410$" J. Fluid Mech., 187, 61-98.

Srinivasan G.R., Ekaterinaris J.A., McCroskey W.J., 1995 "Evaluation of turbulence models for unteady flows of an oscillating airfoil" Computers & Fluids, 7, 833-861.

Tardu S., Binder G., 1994 "Response of turbulence to imposed oscillations of high frequency" Phys. Fluids, 5(8), 2028-2037.

Tardu S., Binder G., Blackwelder R., 1994 "Turbulent channel flow subjected to large imposed velocity oscillations" J. Fluid Mech., 267, 109-151.

Tardu S., Feng M.Q., Binder G., 1995 "Fine structure of Reynolds shear stress in an unsteady turbulent channel flow" Turbulent Shear Flows-9, Ed. Durst et al., 301-322.

Townsend A.A. 1970 "Entrainment and the structure of turbulent flow" J. Fluid Mech., 137, 31-58.

Tu S., Ramaprian B.R., 1983 "Fully developed periodic turbulent pipe flow, part 1, main experimental results and comparison with predictions" J. Fluid Mech., 137, 31-58

Wilcox, D.C., 1998 "Turbulence Modeling for CFD", KNI inc., Anaheim.