TRANSPORT AND ALGEBRAIC MODELS OF PARTICLE KINETIC STRESSES IN TURBULENT FLOWS

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ABSTRACT

The objective of this paper is twofold: first, present full transport and explicit algebraic models for predicting kinetic stresses of particles suspended in turbulent flows, and, second, perform an analysis of particle motion and its stability in equilibrium flows.

INTRODUCTION

A variety of two-fluid models within the framework of the Eulerian continuum-modeling manner have been proposed for the predictions of particle-laden turbulent flows. The majority of these models have been based on Boussinesq type eddy viscosity approximations for the particle kinetic stresses. At the same time, despite their computational efficiency, the eddy viscosity models are unable to properly describe essentially anisotropic turbulent flows, especially two-phase flows, because the anisotropy of the particle kinetic stresses may be considerably greater than that of the fluid Reynolds stresses. Concurrently, to improve the continuum approach, several authors (e.g., Simonin, 1991; Zaichik and Vinberg, 1991; Reeks, 1993; Lain and Kohnen, 1999; Taulbee et al., 1999) proposed the second-order models including the transport equations for the particle kinetic stress tensor components.

Although turbulence models based on the transport equations for individual stresses have unassailable advantages over the eddy viscosity models, they result in very long computational times and thus reduce drastically the efficiency of the two-fluid continuum approach for industrial applications. Therefore, it is worth to develop algebraic stress models for the particulate phase by analogy with the familiar ones for the turbulent fluid. A methodology of obtaining algebraic models implies to invoke the concept of equilibrium turbulence states, and moreover the equilibrium state flows are very useful as benchmark flows for validating the transport second-order closure models. That is why we consider the particle transport in equilibrium turbulent flows. As is known, the equilibrium states can be realized in both homogeneous flows and inhomogeneous layers if the turbulence production equals to the turbulence dissipation. In these cases the convection and diffusion effects are negligible, and the fluid, fluid-particle as well as particle anisotropy tensor components in the final stage of evolution achieve some equilibrium constant values.

In this paper, the concentration of the dispersed phase is assumed to be small enough that the 'back-effect' of particles on the fluid turbulence and inter-particle collisions may be not taken into consideration.

GOVERNING EQUATIONS

The particle transport model under development is based on a kinetic equation for the probability density function (PDF) $P(\boldsymbol{x},\boldsymbol{v},t)$ which is defined as a density of particles located in a spatial position $\boldsymbol{x}$, with a velocity $\boldsymbol{v}$, at time t. The main advantage of statistical methods can be attributed to the fact that a closure performed at the PDF level allows the generation of some consistent constitutive relations for the particle turbulent stresses as well as the inter-phase fluid-particle correlations instead of employing apart closure approximations of these terms directly in the moment equations. The kinetic equation for the PDF of the velocity distribution in a turbulent fluid flow field, which is modeled by a Gaussian random process with a known autocorrelation function, is written in the form (Zaichik, 1999)

$$\frac{\partial P}{\partial t}+v_k\frac{\partial P}{\partial x_k}+\frac{\partial}{\partial v_k}\left[\left(\frac{U_k-v_k}{\tau_p}+F_k\right)P\right]$$
$$=\langle u_i'u_k'\rangle\left[\frac{f_u}{\tau_p}\frac{\partial^2 P}{\partial v_i\partial v_k}+g_u\frac{\partial^2 P}{\partial x_i\partial v_k}+\ell_u\frac{\partial U_n}{\partial x_k}\frac{\partial^2 P}{\partial v_i\partial v_n}\right.$$
$$\left.+\tau_p h_u\frac{\partial U_n}{\partial x_k}\left(\frac{\partial^2 P}{\partial x_n\partial v_i}+\frac{\partial V_j}{\partial x_n}\frac{\partial^2 P}{\partial v_i\partial v_j}\right)\right] \quad (1)$$

Here U_i and V_i are the averaged velocities of the fluid and particulate phases, F_i is an external force (e.g., gravity) acceleration, τ_p is the particle response time, and $\langle u'_i u'_j \rangle$ are the fluid Reynolds stresses. The left-hand side of (1) describes evolution in time and convection in phase space, whereas the terms on the right side characterize the interactions between particles and turbulent fluid eddies. Equation (1) is valid for heavy particles, the density of which is much greater than that of the fluid (in this case, the drag force acting on a particle by the surrounding fluid flow is only of importance).

Equation (1) generates a set of governing conservation equations representing the particle fraction, momentum, and turbulent stresses as the appropriate statistical moments of the PDF

$$\frac{\partial \Phi}{\partial t} + \frac{\partial \Phi V_k}{\partial x_k} = 0 \qquad (2)$$

$$\frac{\partial V_i}{\partial t} + V_k \frac{\partial V_i}{\partial x_k} = -\frac{\partial \langle v'_i v'_k \rangle}{\partial x_k} + \frac{U_i - V_i}{\tau_p} + F_i - \left(\langle v'_i v'_k \rangle + g_u \langle u'_i u'_k \rangle + \tau_p h_u \langle u'_i u'_n \rangle \frac{\partial U_k}{\partial x_n} \right) \frac{\partial \ln \Phi}{\partial x_k} \qquad (3)$$

$$\frac{\partial \langle v'_i v'_j \rangle}{\partial t} + V_k \frac{\partial \langle v'_i v'_j \rangle}{\partial x_k} + \frac{1}{\Phi} \frac{\partial \Phi \langle v'_i v'_j v'_k \rangle}{\partial x_k} = -\langle v'_i v'_k \rangle \frac{\partial V_j}{\partial x_k} - \langle v'_j v'_k \rangle \frac{\partial V_i}{\partial x_k} + \langle u'_i u'_k \rangle \left(\ell_u \frac{\partial U_j}{\partial x_k} - g_u \frac{\partial V_j}{\partial x_k} \right) + \langle u'_j u'_k \rangle \left(\ell_u \frac{\partial U_i}{\partial x_k} - g_u \frac{\partial V_i}{\partial x_k} \right) + \frac{2}{\tau_p} \left(f_p \langle u'_i u'_j \rangle - \langle v'_i v'_j \rangle \right) \qquad (4)$$

In these equations, the coefficients f_u, g_u, l_u, and h_u characterize an entrainment of particles into the fluctuating motion of the carrier fluid, namely, they indicate whether particles respond to turbulent velocity fluctuations and determine the degree of coupling between the fluid and particulate phases. To calculate these coefficients one has to know the velocity correlation of fluid motion along a particle trajectory. If the velocity auto-correlation function is described by the frequently used exponential approximation $\Psi_{Lp}(t) = \exp(-t/T_{Lp})$ where T_{Lp} is the eddy-particle interaction time, the entrainment coefficients take the form (Zaichik, 1999)

$$f_u = (1+\Omega)^{-1}, g_u = [\Omega(1+\Omega)]^{-1}, l_u = [\Omega(1+\Omega)^2]^{-1}$$

$$h_u = [\Omega^2 (1+\Omega)^2]^{-1}, \; \Omega = \tau_p / T_{Lp}$$

EQUILIBRIUM TURBULENT FLOWS

In uniformly sheared and strained as well as other single-phase turbulent flows when convective and transport effects can be neglected, there is a wealth of evidence from physical and numerical experiments which suggest that an equilibrium state is ultimately reached. This equilibrium state is characterized by constant values of b_{ij} and all appropriate normalized higher-order correlations. By analogy, the turbulent motion of particles is regarded as equilibrium one if the convection and diffusion effects are negligible, and consequently $b_{p\,ij}$ as well as other normalized correlations do not change in time. Here $b_{ij} = \langle u'_i u'_j \rangle / 2k - \delta_{ij}/3$ and $b_{p\,ij} = \langle v'_i v'_j \rangle / 2k_p - \delta_{ij}/3$ are the fluid and particle anisotropy tensors, where $k = \langle u'_k u'_k \rangle / 2$ and $k_p = \langle v'_k v'_k \rangle / 2$ denote the fluid and particle turbulent kinetic energies. Thus, in the equilibrium state, the triple moments of velocity fluctuations in (4), which characterize the diffusion transfer, turn into zero, the set of the conservation equations for the statistical moments of the PDF is broken, and the second-moment equations describing particle kinetic stresses become closed. Moreover, in this case, time evolutions of the second-order fluid and particle velocity fluctuations obey an exponential law

$$\langle u'_i u'_j \rangle \sim \langle v'_i v'_j \rangle \sim \exp(\chi t), \; \chi = \frac{\varepsilon}{k} \left(\frac{P}{\varepsilon} - 1 \right) \qquad (5)$$

where ε is the dissipation rate of the fluid turbulence energy, and $P = -\langle u'_i u'_j \rangle U_{i,j}$ is the turbulence production.

With accounting for (5), equation (4) takes the form

$$\chi \langle v'_i v'_j \rangle + \langle v'_i v'_k \rangle V_{j,k} + \langle v'_j v'_k \rangle V_{i,k} - \langle u'_i u'_k \rangle \left(l_u U_{j,k} - g_u V_{j,k} \right) - \langle u'_j u'_k \rangle \left(l_u U_{i,k} - g_u V_{i,k} \right) - 2 \left(f_u \langle u'_i u'_j \rangle - \langle v'_i v'_j \rangle \right) / \tau_p = 0 \qquad (6)$$

It is evident that (6) represents a closed system of algebraic equations, from which one can find the second-moment velocity fluctuations of particles if the mean velocity gradients of the continuous and dispersed phases as well as the turbulent stresses of the fluid are known.

Let us consider the particle transport in uniformly sheared and strained turbulent flow when the mean velocity gradient of the fluid can be given by

$$U_{i,j} = \frac{\partial U_i}{\partial x_j} = \begin{pmatrix} S_{11} & S_{12} & 0 \\ 0 & -\frac{1+\alpha}{2} S_{11} & 0 \\ 0 & 0 & \frac{1+\alpha}{2} S_{11} \end{pmatrix} \qquad (7)$$

where $\alpha = 0$ corresponds to the axis-symmetric strain flow, and $\alpha = 1$ relates to the plane one.

If an external force, F_i, is homogeneous, substitution of (7) into (3) yields the following relation for the mean velocity gradient of the dispersed phase

$$V_{i,j} = \frac{\partial V_i}{\partial x_j} = \begin{pmatrix} \frac{A-1}{2\tau_p} & \frac{2S_{12}}{A+B} & 0 \\ 0 & \frac{B-1}{2\tau_p} & 0 \\ 0 & 0 & \frac{C-1}{2\tau_p} \end{pmatrix} \quad (8)$$

$$A = \sqrt{1+4\tau_p S_{11}}\,,\ B = \sqrt{1-2(1+\alpha)\tau_p S_{11}}$$
$$C = \sqrt{1-2(1-\alpha)\tau_p S_{11}}$$

The steady-state solution (8) may be realized with the proviso that $-(4\tau_p)^{-1} \le S_{11} \le [2(1+\alpha)\tau_p]^{-1}$. As is seen from (8), the mean velocity shear, S_{12}, does not affect the strain rate of particle velocities. Conversely, the mean strain rate, S_{11}, influences on the particle velocity shear increasing it. Therefore, in spite of the fact that the shear and the strain effect independently on the fluid velocity field, a superposition of these two effects does not take place in respect to the mean velocity field of the dispersed phase.

Consider first the behavior of particle fluctuating velocities in uniformly sheared homogeneous turbulence with no strain ($S_{11} = 0$). The properties of fluid-phase turbulence have been chosen according to experimental data by Tavoularis and Karkin (1989): $b_{11} = 0.21, b_{22} = -0.13, b_{33} = -0.08$, $b_{12} = -0.16$, $S_{12}(k/\varepsilon) = 5$, and $P/\varepsilon = 1.6$. The eddy-particle interaction time is assumed to be the same recommended in Simonin et al. (1995), $T_{Lp} = 0.482k/\varepsilon$. Figure 1 shows the behavior of the particle velocity anisotropy tensor components, $b_{p\,ij}$, predicted by (6) versus the product of the particle response time, τ_p, and the shear rate, S_{12}. The magnitudes of $b_{p\,ij}$ at $\tau_p S_{12} = 0$ correspond to the components of the fluid anisotropy tensor, b_{ij}. As is seen, the particle velocity anisotropy increases with an increase in both the particle inertia and the mean velocity gradient, and the streamwise velocity fluctuations of high-inertia particles are essentially more than the ones in the normal and spanwise directions. The results predicted are in good agreement with LES (Simonin et al., 1995) and DNS (Taulbee et al., 1999). Moreover, the model predictions are consistent with aforementioned computations and theoretical results by Reeks (1993) and Liljegren (1993) regarding the effect of a mean velocity gradient on the particle streamwise velocity variance in shear flows. This variance is strongly affected by velocity gradients and, due to the lack of small-scale dissipation of velocity fluctuations, can exceed the fluid one. Figure 2 displays the dependencies of the particle kinetic energy normalized by the fluid turbulent energy on $\tau_p S_{12}$. It is interesting to note that this dependence for the equilibrium solution has a non-monotonous form and passes through a small maximum before diminishing. In Fig. 2, the predictions of Fevrier and Simonin (1998) are demonstrated as well. There is quite good agreement about tendencies given by both models.

In Fig. 1 and 2 are also depicted the particle turbulent properties corresponding to the steady-state solution of (6) at $\chi = 0$ (Zaichik, 1999). As is evident from Fig. 1, the equilibrium and steady-state values of the anisotropy tensor components are found to be close. However, the behavior of the relative turbulent energy of particles in both cases is entirely different because k_p/k increases with $\tau_p S_{12}$ after passing a weakly expressed minimum.

Consider next the effect of strain on the second moments of particle velocity fluctuations in a homogeneous flow with plane expansion ($S_{11} > 0$) or contraction ($S_{11} < 0$) in the absence of shear ($S_{12} = 0$). The turbulence parameters of the fluid are taken as: $b_{11} = -0.2$ and $b_{22} = 0.2$ for $S_{11} > 0$, $b_{11} = 0.2$ and $b_{22} = -0.2$ for $S_{11} < 0$, $b_{33} = 0$, $S_{12}(k/\varepsilon) = 3$, and $P/\varepsilon = 1.6$. Figure 3 demonstrates the equilibrium particle fluctuating velocity intensities related to corresponding values of the fluid as functions of $\tau_p S_{11}$. A growth of anisotropy of velocity fluctuations with increasing particle inertia in both expansion and contraction engages our attention. In the case of expansion, the fluctuating energy of high-inertia particles can considerable exceed the turbulent energy of the fluid. However, the particle velocity variance in the stretched (x_1) direction is smaller than the fluid one and decreases with increasing τ_p, whereas the behaviour of that in the squeezed (x_2) direction is of opposite tendency. In the spanwise direction, the particle velocity variance remains below the fluid stress and reduces with τ_p. These features are in general agreement with DNS computations of Mashayek et al. (1999).

The near-wall turbulent layer with logarithmic velocity profile is not a homogeneous flow since the mean shear rate, S_{12}, increases when the distance from the wall decreases, however, it is equilibrium one because the condition $P/\varepsilon = 1$ is fulfilled. In this case, according to (5), $\chi = 0$, and hence the equilibrium solution of (6) will represent simultaneously the steady-state one. The parameters of fluid turbulence are given in accordance with experimental data by Laufer (1951) as: $b_{11} = 0.22$, $b_{22} = -0.15$, $b_{33} = -0.07$, $b_{12} = -0.16$, and $S_{12}(k/\varepsilon) = 3.1$. Figure 2 shows that the behavior of the particle turbulent energy in the near-wall layer is like as it takes place in the steady-state homogeneous shear flow. Moreover, as is obvious from Fig. 4, the effect of particle inertia on the anisotropy tensor components is identical to that in the uniformly sheared flow field. In Fig. 2 and 4, the results of LES computations by Fevrier and Simonin (1998) for a logarithmic layer and by Wang and Squires (1996) for a channel flow (at $y_+ \approx 100$) are also displayed.

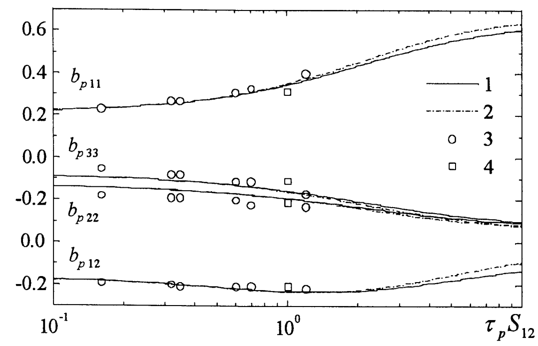

Figure 1. Streamwise (b_{p11}), normal (b_{p22}), spanwise (b_{p33}), and shear (b_{p12}) components of the particle velocity anisotropy tensor in the uniformly sheared flow: 1 - equilibrium solution, 2 - steady-state solution, 3- LES by Simonin et al. (1995), 4 - DNS by Taulbee et al. (1999).

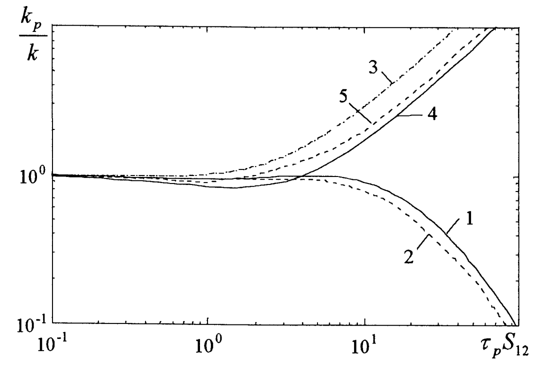

Figure 2. Particle-to-fluid kinetic energy in the uniformly sheared (1,2,3) and near-wall (4,5) flows: 1,3,4 - authors' model; 1,2 - equilibrium solution; 3 - steady-state solution; 2,5 - model by Fevrier and Simonin (1998).

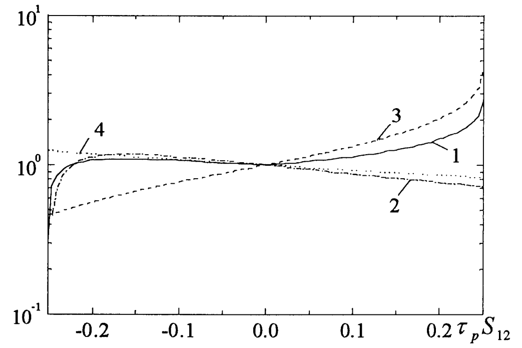

Figure 3. Normalized particle fluctuating velocity intensities in the homogeneous plane strain flow: 1-k_p/k, 2-$\langle v_1'^2\rangle/\langle u_1'^2\rangle$, 3-$\langle v_2'^2\rangle/\langle u_2'^2\rangle$, 4-$\langle v_3'^2\rangle/\langle u_3'^2\rangle$.

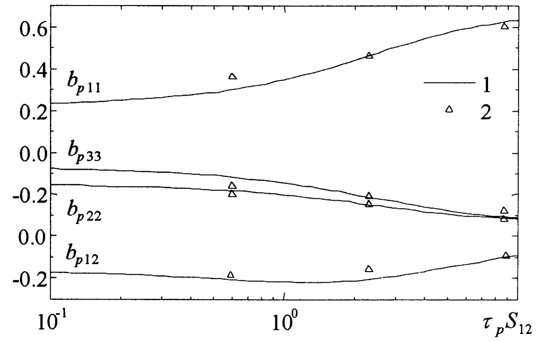

Figure 4. Streamwise, normal, spanwise, and shear components of the particle velocity anisotropy tensor in the near-wall flow: 1 - predictions, 2 - LES by Wang and Squires (1996).

STABILITY OF EQUILIBRIUM STATES

In what follows an analysis is conducted to examine the stability of the particle transport in the equilibrium turbulent flows considered. To do this, a system comprising equations for the particle anisotropy tensor, $b_{p\,ij}$, and the normalized fluctuating energy, $\bar{k}_p = k_p/k$, is invoked. This equation system follows from (4) and has the form

$$\frac{db_{p\,ij}}{dt} = -\left(b_{p\,ik} + \frac{\delta_{ik}}{3}\right)V_{j,k} - \left(b_{p\,jk} + \frac{\delta_{jk}}{3}\right)V_{i,k}$$

$$-\frac{1}{\bar{k}_p}\left[\left(b_{ik} + \frac{\delta_{ik}}{3}\right)G_{jk} + \left(b_{jk} + \frac{\delta_{jk}}{3}\right)G_{ik}\right]$$

$$+2\left(b_{p\,ij} + \frac{\delta_{ij}}{3}\right)\left[\left(b_{kn} + \frac{\delta_{kn}}{3}\right)V_{k,n} + \left(b_{kn} + \frac{\delta_{kn}}{3}\right)\frac{G_{kn}}{\bar{k}_p}\right]$$

$$+\frac{2f_u}{\tau_p \bar{k}_p}\left(b_{ij} - b_{p\,ij}\right), \quad G_{ij} = g_u V_{i,j} - l_u U_{i,j}$$

$$\frac{d\bar{k}_p}{dt} = -2\bar{k}_p\left(b_{p\,ij} + \frac{\delta_{ij}}{3}\right)V_{i,j} - 2\left(b_{ij} + \frac{\delta_{ij}}{3}\right)G_{ij}$$

$$+\frac{2}{\tau_p}\left(f_u - \bar{k}_p\right) - \bar{k}_p \frac{\varepsilon}{k}\left(\frac{P}{\varepsilon} - 1\right) \qquad (9)$$

Equations (9) are linearized about equilibrium solution, for which purpose the variables are written as the sums of the steady equilibrium and unsteady disturbed terms

$$b_{p\,ij} = b^{eq}_{p\,ij} + b'_{pij},\ \bar{k}_p = \bar{k}^{eq}_p + \bar{k}'_p$$

where $b'_{p\,ij}$ and $\bar{k}'_p$ are assumed to be small compared respectively to $b^{eq}_{p\,ij}$ and $\bar{k}^{eq}_p$. The time dependence of the perturbation quantities is taken as $\exp(\omega t/\tau_p)$, and by this means the stability analysis of the problem reduces to the evaluation of the eigenvalues of appropriate matrix. These eigenvalues may be complex. A complex eigenvalue with a positive real part ω_r indicates that a perturbation will grow and hence the equilibrium solution is unstable.

The eigenvalues obtained for the uniformly sheared and near-wall equilibrium flows are exhibited in Fig. 5 and 6. As a result it is found that, in the simple homogeneous shear flow, there are three eigenvalues one of which changes the sign of its real part at $\tau_p S_{12} \cong 7.5$, and, hence, the equilibrium state of particle fluctuating motion is stable if $\tau_p S_{12} < 7.5$ and unstable if $\tau_p S_{12} > 7.5$. The existence of a critical value of $\tau_p S_{12}$ testifies that the equilibrium stable particle transport in the homogeneous flow field with a constant mean velocity gradient takes place only at relatively small values of the particle response time and is not physically realizable for high-inertia particles. In contrast to this, the equilibrium motion of particles in the near-wall shear flow is stable over the entire range of $\tau_p S_{12}$. Furthermore, the linear stability analysis shows that the equilibrium statistics of particle fluctuating velocities in homogeneous flows with contraction and expansion are stable for all cases when the averaged particle momentum equations possess steady-state solutions.

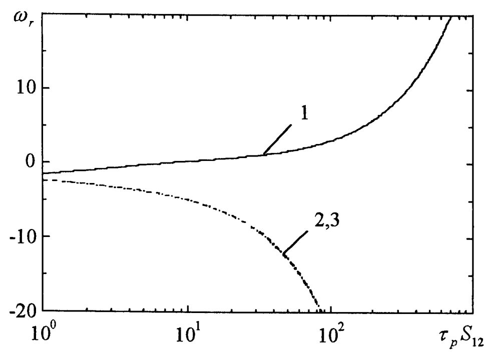

Figure 5. Real parts of the eigenvalues for the uniformly sheared flow: 1 – real eigenvalue; 2,3 – complex-conjugated eigenvalues.

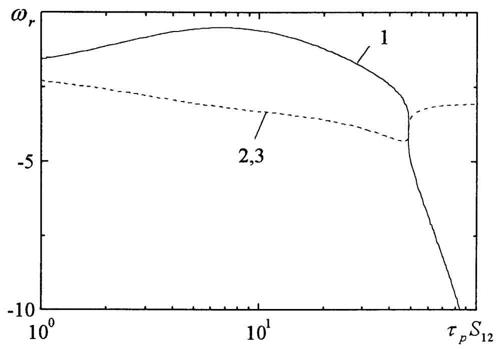

Figure 6. Real parts of the eigenvalues for the near-wall flow: 1 – real eigenvalue; 2,3 – complex-conjugated eigenvalues.

ALGEBRAIC MODELS OF STRESSES

By means of the basis equilibrium hypothesis that convective and transport effects can be neglected, the second-moment transport equations (4) yield a closed system of algebraic equations for the determination of the particle stress anisotropy in terms of the mean velocity gradients

$$b_{pij} = \frac{P_{p\,ij} - (\langle u'_i u'_k\rangle G_{jk} + \langle u'_j u'_k\rangle G_{ik}) + 2 f_u \langle u'_i u'_j\rangle/\tau_p}{P_{p\,kk} - 2\langle u'_k u'_n\rangle U_{k,n} + 4 f_u k/\tau_p} - \frac{\delta_{ij}}{3}$$

$$P_{p\,ij} = -\left(\langle v'_i v'_k\rangle V_{j,k} + \langle v'_j v'_k\rangle V_{i,k}\right) \qquad (10)$$

It is clear that (10) is an implicit algebraic stress model since the anisotropy tensor, $b_{p\,ij}$, appears on both sides of the equation. As is well known, the computational efforts of using implicit model in complex turbulent flows may be excessively large and even be of the same order as by employing the full transport stress model. With a view to construct a more numerically beneficial algebraic model which provides an explicit relationship between the particle stress tensor and the mean velocity gradients, we employ an iteration procedure to resolve the implicit algebraic equations. This procedure is based on the following iteration expression of the particle stress production in the right-hand side of (10)

$$P^{n+1}_{p\,ij} = -\frac{2}{3} k_p \left(V_{i,j} + V_{j,i}\right) - 2k_p \left(b^n_{p\,ik} V_{j,k} + b^n_{pjk} V_{i,k}\right) \quad (11)$$

where n is the number of iteration. Equations (10) and (11) generate a hierarchy of explicit algebraic models for the kinetic particle stresses. The first term of the iteration procedure results from the isotropic approach in (11), namely, $b^0_{p\,ij} = 0$, and gives a linear relationship between the stress and mean velocity gradient tensors in accord with Boussinesq's hypothesis. The second term of the iteration expansion yields a quadratic approximation for the particle stresses in terms of the mean velocity gradients. In Fig. 7, the first linear and second quadratic approximations are compared with the exact solution of (6) for the homogeneous shear flow (results for the near-wall flow are similar to those presented in Fig. 7). It is obvious that the quadratic approximation transcends substantially the linear one and gives a sufficiently correct description of stress anisotropy. Although algebraic models are strictly justified only for equilibrium flows, it is hoped that, by analogy with single-phase turbulence, the quadratic explicit algebraic model developed may be useful in the calculations of complex non-equilibrium two-phase turbulent flows.

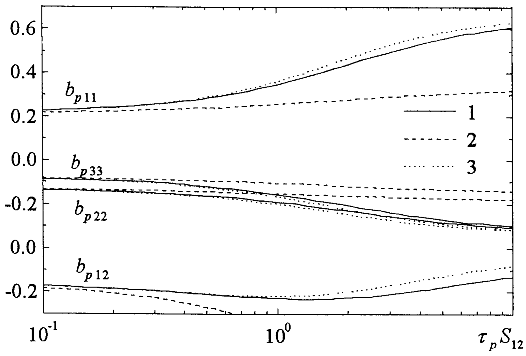

Figure 7. Particle stress anisotropy tensor components in the uniformly sheared flow: 1 - exact solution, 2 - linear approximation, 3 - quadratic approximation.

SUMMARY

On the basis of results presented one can draw the following conclusions:

1. The particle transport model based on a kinetic equation for the PDF of the velocity distribution yields the equilibrium fluctuating velocity anisotropy which is in good agreement with LES and DNS computations for homogeneous shear, near-wall as well as strain turbulent flows.
2. The stability analysis reveals i) the existence of a critical particle response time beyond which the equilibrium particle fluctuating motion in the uniformly sheared flow is unstable, ii) the possibility of stable particle transport in the near-wall shear flow over the entire range of particle inertia, and iii) the occurrence of the stable particle fluctuating velocity statistics for the equilibrium homogeneous flow with contraction and expansion when the averaged particle momentum equations posses steady-state solutions.
3. The explicit quadratic algebraic model developed gives a plausible prediction of the particle stress anisotropy.

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