VELOCITY DERIVATIVES IN AN ATMOSPHERIC SURFACE LAYER AT $\mathrm{Re}_\lambda = 10^4$. FURTHER RESULTS.

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ABSTRACT

We present a set of new results obtained from a field experiment at $\mathrm{Re}_\lambda = 10^4$. The emphasis is on the velocity derivatives, both the (tensor of) spatial derivatives $\partial u_i/\partial x_j$ and the temporal derivatives $\partial u_i/\partial t$. In the former we address new aspects of geometrical statistics, reduction of nonlinearity, and comparison of strain dominated regions and regions with concentrated vorticity. In the latter new results are presented on measurements without employing the Taylor hypothesis which allowed to evaluate the local, $\mathbf{a}_l \equiv \partial \mathbf{u}/\partial t$, and the convective, $\mathbf{a}_c \equiv (\mathbf{u}\cdot\nabla)u$, accelerations, and the relation between $\mathbf{a}_l$ and $\mathbf{a}_c$, as well as to assess the feasibility of direct measurements of the streamwise derivative $\partial/\partial x_1$ without invoking the Taylor hypothesis.

INTRODUCTION

A field experiment was performed in which all the three velocity components and all the nine components of the velocity gradients tensor at $\mathrm{Re}_\lambda = 10^4$ were measured. This was done by implementation in the field of techniques used by Tsinober et al. (1992, 1997) in laboratory experiments. Several essential technological innovations were introduced in the manufacturing process of the 20 hot-wire probe in view of specific requirements of a field experiment. A special high precision calibration unit was designed and manufactured for computer controlled three-dimensional calibration of the probe. More details on the experiments are given in Kholmyansky and Tsinober (2000) and Kholmyansky et al. (2000, 2001). These references along with conventional information contain results on direct coupling of large and small scales in terms of statistics of small scale quantities (e.g. enstrophy) conditioned on large scale quantities (e.g. energy of velocity fluctuations). This direct coupling is a manifestation of one of the aspects of nonlocality of turbulence in physical space (Tsinober, 2000). Another aspect of nonlocality in physical space is reflected in the nonlocal relation between vorticity and strain fields and other quantities associated with vorticity and strain, such as the relation between the third order quantities, $\omega_i\omega_k s_{ik}$ and $-s_{ij}s_{jk}s_{ki}$, where ω_i - is the vorticity vector, and s_{ij} - the rate of strain tensor, Tsinober (1998), Kholmyansky et al. (2000, 2001).

Another set of results concerns the issues of geometrical statistics, reduction of nonlinearity, acceleration and Taylor hypothesis. In this communication we present further results on these latter issues.

GEOMETRICAL STATISTICS

It is well known that there is a distinct and strong tendency of alignment between vorticity, ω, and the eigenvector, λ_2, corresponding to the intermediate eigenvalue, Λ_2, of the rate of strain tensor, s_{ij}, see references in Tsinober (1998) and Kholmyansky et al. (2001). There are no such strong tendencies of alignment between vorticity, ω, and other two eigenvectors, λ_1 and λ_3. The eigenvector λ_1 corresponds to the largest (positive) eigenvalue, Λ_1, of the rate of strain tensor, s_{ij}, and the eigenvector λ_3 corresponds to the smallest (negative) eigenvalue, Λ_3, of the rate of strain tensor, s_{ij} ($\Lambda_1 > \Lambda_2 > \Lambda_3$).

It has been shown also that there is a strong tendency of strict alignment between vorticity, ω, and the vortex stretching vector $\mathbf{W}$, $W_i = \omega_j s_{ij}$, Tsinober et al. (1992, 1997), Kholmyansky et al. (2001). This alignment is in conformity with the predominance of vortex stretching over vortex compressing, i.e. the fact that enstrophy production, $\omega_i \omega_j s_{ij}$, is a positively skewed quantity, $\langle \omega_i \omega_j s_{ij} \rangle > 0$. Indeed, since $\omega_i \omega_j s_{ij} \equiv \omega \cdot \mathbf{W}$, the tendency of strict alignment between ω and $\mathbf{W}$ is in conformity with the positiveness of $\langle \omega_i \omega_j s_{ij} \rangle$. However, it appears that the main positive contribution to $\langle \omega_i \omega_j s_{ij} \rangle$ comes not from the term $\omega^2 \Lambda_2 \cos^2(\omega, \lambda_2)$, associated with the second eigenvalue, Λ_2, of the rate of strain tensor, in spite of the preferential alignment between vorticity, ω, and the eigenvector, λ_2, corresponding to the intermediate eigenvalue, Λ_2, of the rate of strain tensor, s_{ij}, Tsinober et al. (1997), Kholmyansky et al. (2001). Namely, the main positive contribution to $\langle \omega_i \omega_j s_{ij} \rangle$ comes from the term $\omega^2 \Lambda_1 \cos^2(\omega, \lambda_1)$, associated with the positive eigenvalue, Λ_1, of the rate of strain tensor, s_{ij}, though there is no tendency for alignment between vorticity, ω, and the eigenvector, λ_1, corresponding to the largest (positive) eigenvalue, Λ_1, of the rate of strain tensor, s_{ij}. The reasons for such a behavior were explained in Kholmyansky et al. (2000, 2001). Briefly this is associated with cancellation of positive and negative contributions in the term $\omega^2 \Lambda_2 \cos^2(\omega, \lambda_2)$, since Λ_2 assumes both positive and negative values (and is positively skewed) and because Λ_1 is much larger than Λ_2.

In view of the above facts it is of interest to have an idea about the relation between the alignment of vorticity, ω, and the eigenframe, λ_i, of the rate of strain tensor, s_{ij} and of vorticity, ω, and the vortex stretching vector $\mathbf{W}$, $W_i = \omega_j s_{ij}$. This can be done by looking at the joint PDFs of $\cos(\omega, \mathbf{W})$ and $\cos(\omega, \lambda_i)$. These are shown in Figure 1.

One can see from the Figure 1 that strong alignment between ω and $\mathbf{W}$ is associated both with strong alignment of ω and λ_1 (Figure 1*a*) and of ω and λ_2 (Figure 1*b*).

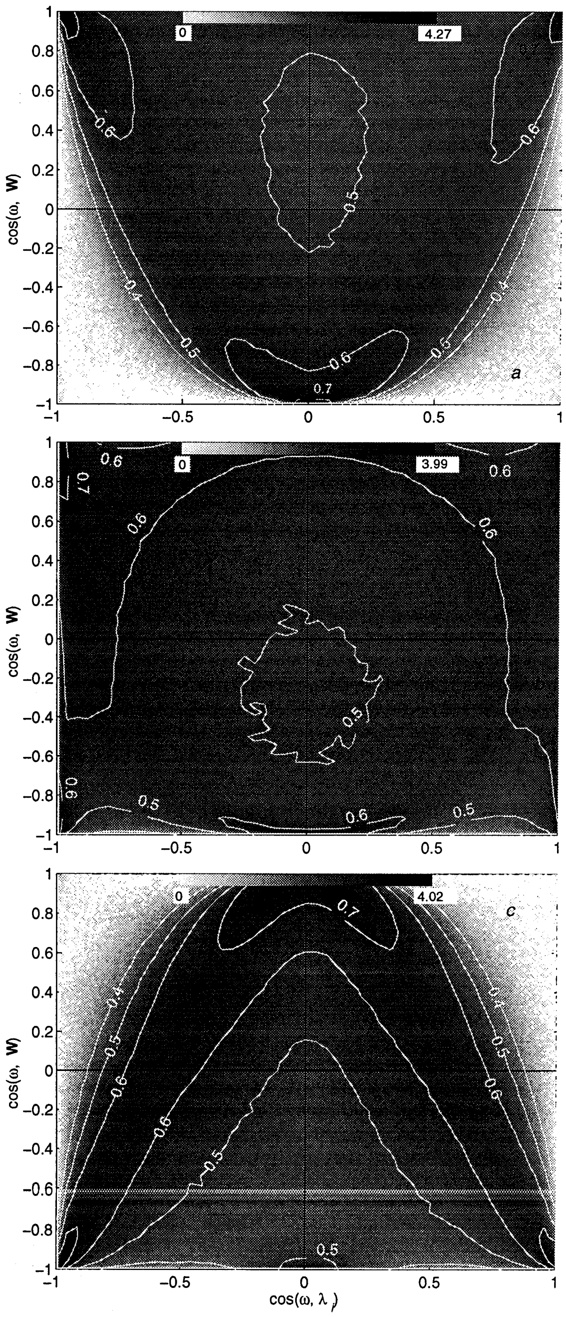

Figure 1: Joint PDFs of $\cos(\omega, \mathbf{W})$ and $\cos(\omega, \lambda_i)$. *a* - $i = 1$, *b* - $i = 2$, *c* - $i = 3$. The density of the gray scale shows log of the number of events.

This corresponds to the fact that the positive contribution to $\langle \omega_i \omega_j s_{ij} \rangle$ comes both from the term $\omega^2 \Lambda_1 \cos^2(\omega, \lambda_1)$ and from the term $\omega^2 \Lambda_2 \cos^2(\omega, \lambda_2)$, although the contribution from the former is two up to three times larger than that from the latter. The behavior of the joint PDFs shown in Figure 1 is in agreement with the simple relation $\cos(\omega, \mathbf{W}) = \Lambda_k \cos^2(\omega, \lambda_k) \left\{ \Lambda_k^2 \cos^2(\omega, \lambda_k) \right\}^{-1/2}$. It follows from this relation (under some assumptions) that $\cos(\omega, \mathbf{W}) \sim 1$, if either $\cos(\omega, \lambda_1) \sim 1$ or $\cos(\omega, \lambda_2) \sim 1$, whereas

$\cos(\omega, \mathbf{W}) \sim -1$ if $\cos(\omega, \lambda_3) \sim 1$, Tsinober et al. (1997).

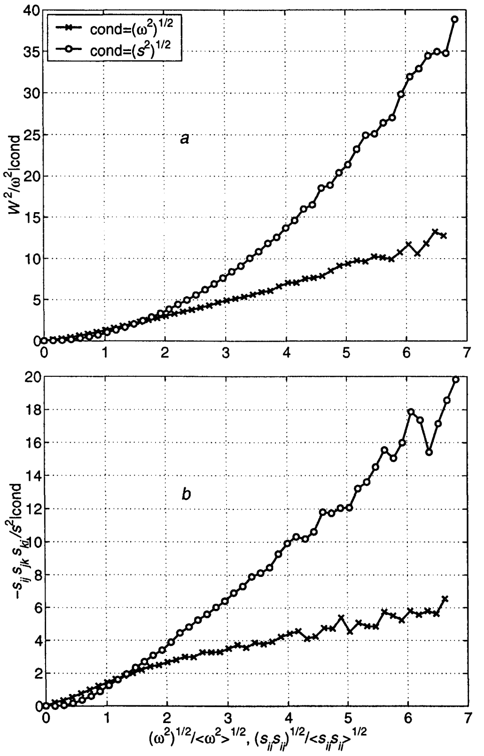

Figure 2: Conditional averages of *a)* W^2/ω^2, $(W_i \equiv \omega_i s_{ij})$ and *b)* $s_{ij}s_{jk}s_{ki}/(s^2)$, in slots of ω and s.

REDUCTION OF NONLINEARITY. STRAIN AND VORTICITY DOMINATED REGIONS

Reduction of nonlinearity is understood here as in Tsinober (1998), Tsinober et al. (1999) and Kholmyansky et al. (2001). Namely, all the physically meaningful nonlinearities appear to be much smaller in the regions with concentrated vorticity (large enstrophy) than in the regions dominated by strain. This is true of such quantities as $\omega_i\omega_j s_{ij}$, $\omega_i\omega_j s_{ij}/\omega^2$, $s_{ij}s_{jk}s_{ki}$, $s_{ij}s_{jk}s_{ki}/s^2$, W^2, $(W_i \equiv \omega_j s_{ij})$, W^2/ω^2, $s_{ij}s_{jk}s_{im}s_{jm}$, $s_{ij}s_{jk}s_{im}s_{jm}/s^2$ and $W^2/(\omega^2) - \{\omega_i\omega_j s_{ij}/(\omega^2)\}^2$. All these quantities appear in the equations for vorticity, ω_i, enstrophy, ω^2, total strain, $s^2 = s_{ij}s_{ij}$, etc.[1] The reduction of nonlinearities $\omega_i\omega_j s_{ij}/\omega^2$ and $W^2/(\omega^2) - \{\omega_i\omega_j s_{ij}/(\omega^2)\}^2$ was shown in Kholmyansky et al. (2000, 2001). Here we show two additional examples in Figure 2, clearly demonstrating the phenomenon of reduction of nonlinearity in the above sense.

[1]The quantity $W^2/(\omega^2) - \{\omega_i\omega_j s_{ij}/(\omega^2)\}^2$ is the inviscid rate of change of direction of the vorticity vector. It appears in the equation for the unit vector of vorticity, $\widetilde{\omega}_i = \omega_i/\omega$, i.e. it is responsible for tilting of vorticity.

	$\frac{\omega_i\omega_j s_{ij}}{\omega^2}$	$\frac{-s_{ij}s_{jk}s_{ki}}{s^2}$	$\frac{W^2}{\omega^2}$	$\frac{W^2}{(\omega^2)-\{\omega_i\omega_j s_{ij}/(\omega^2)\}^2}$
ω^2	0.14	0.11	0.37	0.33
s^2	0.24	0.28	0.79	0.71

Table 1: Correlation coefficients between nonlinearities versus enstrophy and strain.

Var($\mathbf{a}$)	Var($\mathbf{a}_l$)	Var($\mathbf{a}_c$)	Corr($\mathbf{a}_l, \mathbf{a}_c$)
$5.3 \cdot 10^3$	$35.7 \cdot 10^3$	$40.6 \cdot 10^3$	-0.932

Table 2: Variance of total, local and convective accelerations and the correlation coefficient between $\mathbf{a}_l$ and $\mathbf{a}_c$.

This behavior implies that all the above nonlinearities are expected to be more correlated with strain rather than with vorticity. This is seen from the examples shown in Table 1 and Figure 3.

ACCELERATIONS AND TAYLOR HYPOTHESIS

In order to tackle the issue of accelerations and the related problems of validity of Taylor hypothesis and more generally of random Taylor (or sweeping decorrelation) hypothesis it was necessary to build a special five array probe with the central array moved out along the probe axis, see photo in Figure 4. Such an arrangement allowed to evaluate directly the streamwise derivative $\partial/\partial x_1$ without invoking the Taylor hypothesis. This was done by forming a difference between the value obtained at the 'sticking' out array and the average of the values obtained at the remaining four arrays. We present here the first tentative results.

These issues take their origin from Taylor (1935). It was suggested by Tennekes (1975) that Taylor's 'frozen-turbulence' approximation should be valid for the analysis of the consequences of large-scale advection of the turbulent microstructure, i.e. $a/a_l < 1$ and $a/a_c < 1$, where $\mathbf{a}_l = \partial\mathbf{u}/\partial t$, $\mathbf{a}_c = (\mathbf{u}\cdot\nabla)\,\mathbf{u}$, $\mathbf{a} = \mathbf{a}_l + \mathbf{a}_c$, $a = |\mathbf{a}|$, $a_l = |\mathbf{a}_l|$, $a_c = |\mathbf{a}_c|$, $\mathbf{u}$ is the total instantaneous velocity vector. This in turn is possible if there is mutual (statistical) cancellation between the local acceleration, $\mathbf{a}_l$, and the convective acceleration, $\mathbf{a}_c$. Since these quantities are vectors, the degree of this mutual cancellation should be studied both in terms of their magnitude and the geometry of vector alignments.

In terms of magnitude it appears from our experiments that the variance of the total acceleration is more than six times smaller than that of the local and the convective components, the latter are of the same magnitude.

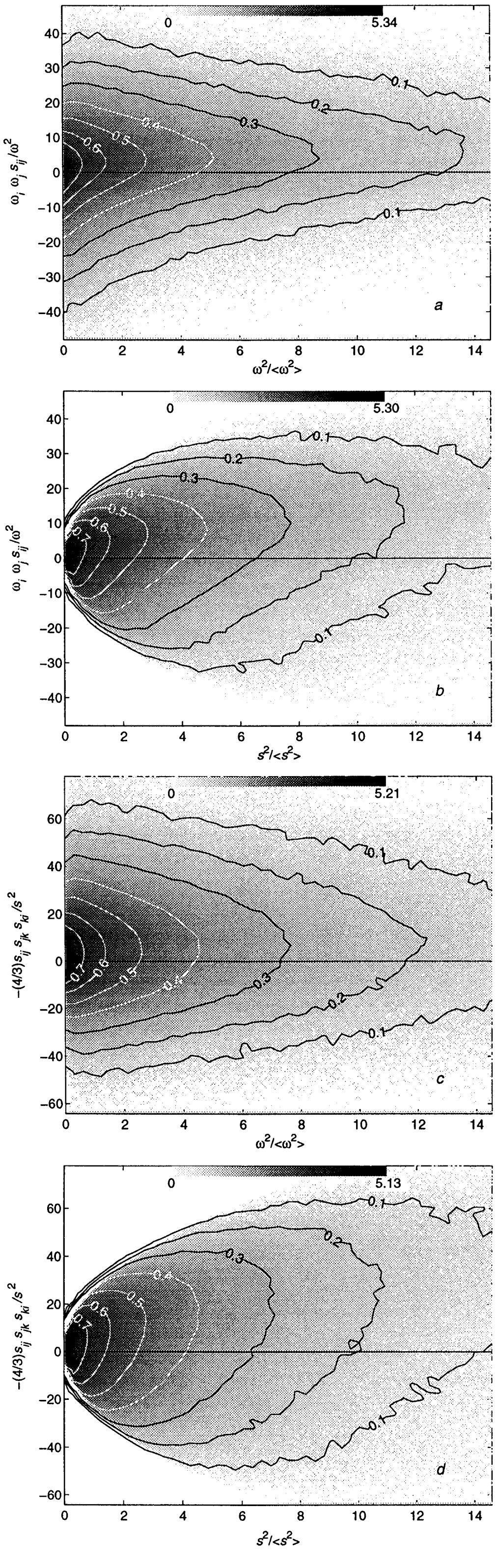

Figure 3: Joint PDFs of $\omega_i\omega_j s_{ij}/\omega^2$ (*a*, *b*) and $-(4/3)s_{ij}s_{jk}s_{ki}/s^2$ (*c*, *d*) with ω^2 (*a*, *c*) and s^2 (*b*, *d*). The density of the gray scale shows log of the number of events.

A remarkable result is that the local acceleration, $\mathbf{a}_l$, and the convective acceleration, $\mathbf{a}_c$ are strongly negatively correlated with the correlation coefficient -0.93, see Table 2.

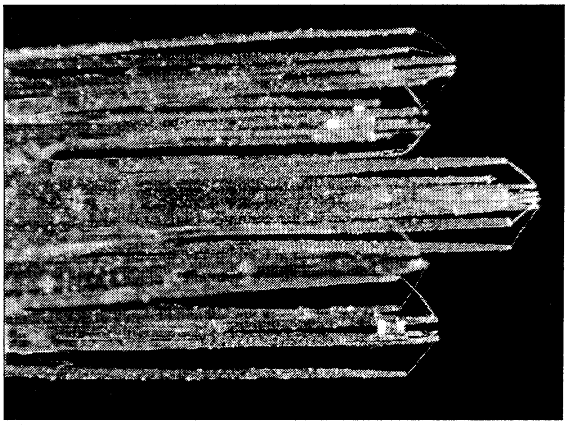

Figure 4: Microphoto of the tip of the probe with central array 1 mm ahead of the other ones.

$i=1$	$i=2$	$i=3$
0.853	0.870	0.875

Table 3: Correlation coefficients between the true velocity derivatives in the streamwise direction $\partial u_i/\partial x_1$ and their counterparts obtained via Taylor hypothesis.

In terms of geometrical relations the local acceleration, $\mathbf{a}_l$, and the convective acceleration, $\mathbf{a}_c$, have a strong tendency for antialignment. This is seen from Figure 5. The results shown in Figure 5 and Table 2 are in good agreement with those obtained in numerical simulations, Tsinober et al. (2001), and via 3-D particle tracking, Lüthi et al. (2001). It is noteworthy that the results by Tsinober et al. (2001) and those by Lüthi et al. (2001) were obtained for moderate $\mathrm{Re}_\lambda \leq 3\cdot 10^2$, whereas those given here are at $\mathrm{Re}_\lambda = 10^4$.

The inset in Figure 5 shows the joint PDF of the true velocity derivative $\partial u_1/\partial x_1$ and its value obtained via Taylor hypothesis. Such plots for the two other derivatives $\partial u_2/\partial x_1$, $\partial u_3/\partial x_1$ look practically the same. The correlation coefficient between the true velocity derivative $\partial u_1/\partial x_1$ and its value obtained via Taylor hypothesis is about 0.85. Similar values were obtained for the two other derivatives $\partial u_2/\partial x_1$, $\partial u_3/\partial x_1$, see Table 3.

It should be emphasized that the results presented in this section show that the true derivative $\partial/\partial x_1$ in the streamwise direction is evaluated quite reasonably. However, higher quality measurements are needed in order to make definitive conclusions regarding the validity of the Taylor hypothesis.

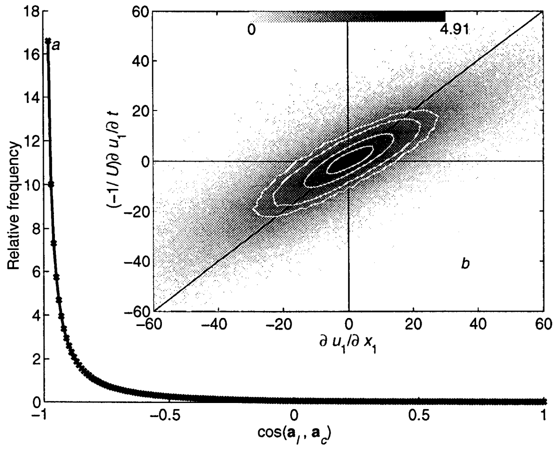

Figure 5: PDF of the cosine of the angle between $\mathbf{a}_l$ and $\mathbf{a}_c$ *(a)*. Inset *(b)* - joint PDF between the true $\partial u_1/\partial x_1$ and the one computed using the Taylor hypothesis. The density of the gray scale shows log of the number of events. Isolines correspond to the values 0.1, 0.2, 0.4 and 0.6 of the maximum value of the logarithm.

CONCLUDING REMARKS

The results given here conform with and confirm one of the main conclusions of Tsinober and Kholmyansky (2000) and Kholmyansky et al. (2001) that the basic physics of turbulent flow at high Reynolds number $Re_\lambda \sim 10^4$, at least qualitatively, is the same as at moderate Reynolds numbers, $Re_\lambda \sim 10^2$. This appears to be true not only of such basic processes as enstrophy and strain production, geometrical statistics, the role of concentrated vorticity and strain, and reduction of nonlinearity, but also in respect with the relation between the local, $\mathbf{a}_l \equiv \partial \mathbf{u}/\partial t$ and the convective, $\mathbf{a}_c \equiv (\mathbf{u} \cdot \nabla)u$, accelerations. However, in this case there exists a distinct Reynolds number dependence, at least in the range of Reynolds numbers $30 \leq Re_\lambda \leq 3 \cdot 10^2$, investigated in Tsinober et al. (2001). Further work is needed to find out such a dependence at higher Reynolds numbers.

Our results prove the feasibility of correct measurements of the streamwise derivatives without invoking the Taylor hypothesis, thus enabling to address a number of important issues associated with accelerations and related matters.

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