INTERACTIONS OF TWO AND THREE TURBULENT SIMPLE WAKES

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ABSTRACT

Turbulent complex wakes generated by two and three cylinders in a side-by-side arrangement were investigated experimentally. One cylinder was slightly heated; the temperature difference is about $1^{o}C$ so that the temperature could be treated as a passive scalar. A combination of an X-wire and a cold wire was used to measure the velocity and temperature fluctuations. The present objective is to examine the interactions between turbulent simple wakes and their effects on the momentum and heat transport phenomena. The superposition hypothesis, as proposed by Bradshaw and his co-workers, is examined for its validity and extent. Using the experimental data of a single-cylinder wake as base, the hypothesis is used to assess the turbulence field, up to the third order velocity products, of complex wakes. It is found that the complex interactions do not seem to have any effect on the fine-scale turbulence, at least up to the scales in the inertial sub-range. On the other hand, the temperature spectra in the inertial sub-range have been affected; its slope has been appreciably increased compared with the single-cylinder data. The gradient transport assumption is found to be valid for the turbulence field, but not for the temperature field. The heat flux and temperature gradient do not approach zero simultaneously near the centerlines of simple wakes, thus giving rise to a substantial variation in the heat transport. This leads to a significant drop in the turbulent Prandtl number.

INTRODUCTION

Bradshaw (1976) defined a complex turbulent flow as one with externally applied rates of strain or where interactions of two or more basic turbulent flows are involved. One example of a complex flow with externally applied rates of strain is an asymmetric flow studied experimentally by Hanjalic and Launder (1972); the flow was formed by a two-dimensional fully developed channel flow between walls of drastically different roughness. Basic flows refer to simple flows such as jets, wakes, fully developed channel flows and boundary layers. For example, Fabris and Fejer (1974) formed a complex flow consisting of 31 hexagonally arranged parallel jets. Since most flows of engineering interest are complex, interest in documenting and predicting such flows is growing. The present study is primarily concerned with complex flows formed by two-dimensional simple wakes generated by circular cylinders. In particular, the complex wake created by the interactions of two and three side-by-side circular cylinders is examined.

The complex wake generated by two and more cylinders has drawn considerable attention in the past due to its importance in many engineering applications. For example, complex wakes are found behind tube bundles in heat exchangers, fuel and control guide rods in nuclear reactors, piers and bridge pilings, oil and gas pipelines, cooling-tower arrays, suspension bridges and high rise buildings. These complex wakes are usually formed by the interactions of a number of simple wakes generated by the individual structure. Research into this kind of flows has been largely focused on the Strouhal map, the pressure, the mean and fluctuations of lift and drag coefficients, measured in the immediate vicinity of the cylinders (Bearman & Wadcock 1973; Zdravkovich 1977), while data in the downstream region is mostly limited to qualitative descriptions.

Zdravkovich (1968) conducted a smoke visualization of the laminar wake behind three cylinders in various triangular configurations. He observed that multiple Karman vortex streets could co-exist. However, the vortices decayed very quickly and an entirely new single vortex street was formed. Williamson (1985) visualized a laminar wake with a Reynolds number ($Re = U_{\infty}d/\nu$) of 100 ~ 200 behind a pair of side-by-side cylinders in the range of T/d = 1.85 ~ 4.0. Here, U_{∞} is the free-stream velocity, d is the diameter of the cylinders, ν is the fluid kinematic viscosity and T is the distance between the cylinder axes. He noted that, as a result of interaction, the wakes of the two cylinders could amalgamate to form a single wake. On the other hand, quantitative data on simple wake interactions is scarce. Cheng and Moretti (1988) reported the mean velocity and turbulent intensity profiles, measured by Pitot tubes, Kiel probes and hot-wires, up to $4.5d$ downstream of a single tube row with a T/d = 1.3. Palmer and Keffer (1972) measured an asymmetrical turbulent wake generated by two side-by-side cylinders of unequal diameter at different downstream distances from the cylinders. Their aim was to create and investigate the region of turbulent 'energy reversal' where the turbulent kinetic energy production turns negative. However, these studies did not focus on the interaction between the individual wakes. Fabris (1984) studied the interaction of two turbulent wakes generated by a pair of cylinders in a side-by-side arrangement. The T/d ratio was set at 8 and their measurements were carried out in the far field. From this brief review, it appears that the near field interactions of simple wakes have not been extensively investigated. Therefore, one of the objectives of the present work is to assess the interactions in the near field of simple wakes generated by individual circular cylinders and their effects on the turbulence field, including the velocity products up to the third order and the spectral characteristics.

In analyzing complex turbulent flows, Bradshaw *et al.* (1973) proposed a superposition scheme. According to their hypothesis, when two simple shear layers merged to form a complex shear layer, the characteristics of the complex shear layer could be deduced from the individual simple shear layer provided the interaction effects were weak on the turbulence structures. In other words, the turbulence fields of the two simple shear layers could be superimposed to form the turbulence field of the complex shear layer. It is well known that the non-linear Navier-Stokes equations forbid the superposition of two or more turbulence fields. However, this hypothesis seemed to be quite valid for those complex flows that were formed by interactions of simple flows. They demonstrated that, based on the hypothesis, a fully developed duct flow could be well predicted by a calculation method using empirical data obtained from isolated fully developed boundary layers. Weir *et al.* (1981) did similar calculation for a plane jet using the data of two mixing layers that originated at the two lips of the jet nozzle. They found that there was a fairly good agreement between the calculations and the measurements, except in the triple velocity products near the centerline. The validity of the superposition hypothesis was further substantiated by the calculation of a turbulent near-wake of a flat plate using the data of the two shear layers developed on opposite sides of the plate (Andreopoulos and Bradshaw 1980). In the present study, the validity of the hypothesis will be further assessed against the complex wakes formed by the interactions of two or more simple wakes. In particular, the simple wakes generated behind circular cylinders will be considered. Since the hypothesis has not been tested against this type of complex wakes before, the present study could extend the range of validity of the hypothesis.

It is of interest to simultaneously investigate the characteristics of heat transport in a complex wake. This can be achieved by slightly heating one cylinder. As such, the coupling between the velocity and the temperature field can be neglected so that the temperature can be treated as a passive scalar. Another advantage of a small temperature difference between the cylinder and the ambient fluid is that the temperature sensitivity of the hot wire can be ignored when transforming the hot-wire voltages into velocities. This is beneficial to the present investigation because hot-wires are used to measure the velocity field. Furthermore, the investigation allows the turbulent Prandtl number to be examined. This data is important for the thermal eddy diffusivity model because it is used in conjunction with the eddy diffusivity for momentum to evaluate the thermal eddy diffusivity. Note that the use of turbulent heat transfer models at a level higher than the thermal eddy diffusivity model is quite complicated (Kays 1992). However, the use of a diffusivity model requires the assumption of gradient transport. Consequently, the validity and extent of the gradient transport assumption for both heat and momentum transport are examined in some detail.

EXPERIMENTAL DETAILS

Experiments to investigate the behavior of complex wakes were carried out in an open-return, low turbulence wind tunnel with a square cross-section (0.3 m ×0.3 m) of 0.8 m long. The wakes were generated by one, two or three brass cylinders (d = 3.8mm) arranged side-by-side (Fig. 1). The cylinders were installed horizontally in the mid-plane and spanned the full width of the working section. They were located at 20 cm downstream of the exit plane of the contraction. This resulted in a maximum blockage of about 3.8% and an aspect ratio of 79. The transverse spacing between the cylinders was varied from T/d = 1.5 to 3. Only cylinder 1 (Fig. 1) was electrically heated in these experiments. The maximum temperature difference between the cylinder and the ambient fluid, Θ_1, was approximately 0.8 ~ 1.1°C. At this level of heating, the temperature can be safely treated as a passive scalar at the two measurement stations, x/d = 10 and 20, where x is the stream-wise coordinate and measured from the center of the cylinder. Measurements were made at a free-stream velocity U_∞ of 7m/s, or Re = 1800. In the free-stream, the longitudinal turbulence intensity was measured to be approximately 0.5%.

A three-wire probe (an X-wire plus a cold wire, the latter placed about 1 mm upstream of the X-wire crossing point and orthogonal to the X-wire plane) was used to measure the velocity fluctuations in the stream-wise and lateral directions, u and v, respectively, and the temperature fluctuation, θ. The hot wires were etched from a 5mm diameter Wollaston (Pt-10% Rh) wire to a length of about 1 mm. For the cold wire, a temperature coefficient of 1.69×10^{-3}°C^{-1} was used. Constant-temperature and constant-current circuits were used for the operation of the hot wires and the cold wire. An overheat ratio of 1.8 was adopted for the X-wires, while a current of 0.1 mA was used in the cold wire. The sensitivity of the cold wire to velocity fluctuations was negligible since the length-to-diameter ratio was sufficiently large to allow the neglect of any low-wave-number attenuation of the temperature variance. The frequency response of the wire, as indicated by -3dB frequency, was estimated to be 2.2kHz at the wind speed investigated. This was sufficient to avoid any high frequency attenuation of the main quantities of interest to the present study. Signals from the circuits were offset, amplified and then digitized using

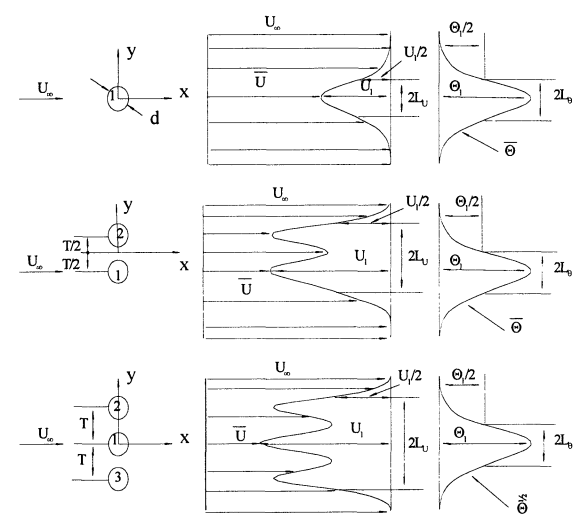

Figure 1: Definition sketch .

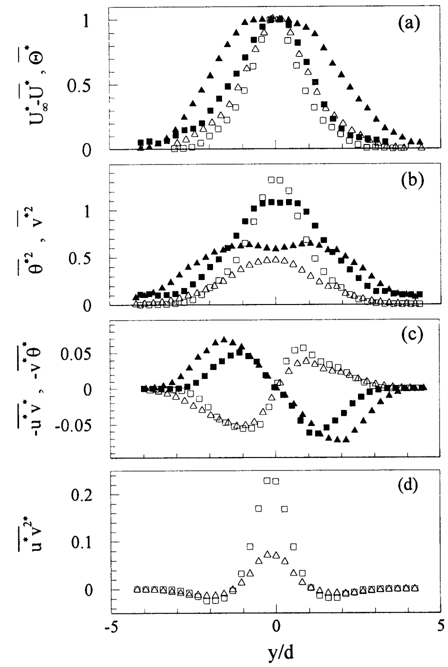

Figure 2: Lateral distributions of (a) mean velocity (open symbols) and temperature (solid symbols), (b) transverse velocity (open symbols) and temperature variance (solid symbols), and (c) shear stress (open symbols) and heat flux (solid symbols) in a single cylinder wake.
(■, □) x/d = 10; (▲,Δ) x/d =20.

a 16 channel (12bit) A/D board and a personal computer at a sampling frequency of 3.5kHz per channel. The duration of each record was about 15s.

A comparison (not shown) between the present measurements and those of Antonia *et al.* (1993) indicated good agreement in the Reynolds shear stress $\overline{uv}$, the heat flux $\overline{v\theta}$, and the gradients $\partial\bar{U}/\partial y$ and $\partial\bar{\Theta}/\partial y$ of the mean velocity $\bar{U}$ and the mean temperature $\bar{\Theta}$, thus providing a validation of the present measurements.

VALIDATION OF THE SUPERPOSITION HYPOTHESIS

In this study, the single-cylinder wake is a simple wake and can be used as the building block for the two- and three-cylinder wakes that were generated by the side-by-side cylinders. Therefore, the superposition hypothesis is used to calculate the turbulence field of the complex wakes using the data of the single-cylinder wake (Fig. 2). The calculations are compared with the measured data of the complex two- and three-cylinder wakes.

Mean Velocity

Fig. 3 presents the cross-stream distribution of the mean velocity in the two-cylinder wake. The measurements and the predictions from the superposition hypothesis are in agreement with each other, irrespective of the *x/d* locations and the cylinder separation distance *T/d*. The discrepancies are within experimental uncertainty, which is about ±2% (Zhou and Antonia 1992), and are mainly caused by the velocity calibration of the X-wires. Similar results are also observed for the three-cylinder wake; however, for the sake of brevity, they are not shown. It is evident from these results that the superposition hypothesis is quite valid for the two- and three-cylinder wakes, as far as the mean field of the complex wake is concerned.

Second Order Velocity Products

The measured shear stress $\overline{uv}$ and that calculated according to the superposition hypothesis agree reasonably well, both qualitatively and quantitatively (Fig. 4). However, substantial discrepancies are noticed in the lateral normal stress $\overline{v^2}$ (Fig. 5). At *T/d* = 3.0, the calculated $\overline{v^2}$ is in qualitative agreement with the measurement, but is substantially smaller in magnitude. When *T/d* is reduced to 1.5, the calculation based on the superposition hypothesis fails to yield results that are consistent with the experimental data, qualitatively as well as quantitatively. It has been observed that, when *T/d* is between 1.5 and 2.0, the gap flow between the individual simple wakes could stably deflect, thus giving rise to an asymmetric distribution of the measured $\overline{v^2}$ at *T/d* = 1.5. This phenomenon could become much worse when *T/d* < 1.5 (Williamson 1985; Moretti 1993). Since the superposition hypothesis does not take the gap flow deflection into account, it is not surprising that the calculations are not in good agreement with the measurements at *T/d* = 1.5. The dramatic difference in behavior in the calculations of $\overline{uv}$ and $\overline{v^2}$ based on the superposition hypothesis seems to suggest that $\overline{uv}$ and $\overline{v^2}$ develop differently.

Third Order Velocity Products

Fig. 6 presents the results of the lateral transport of the shear stress, $\overline{uv^2}$, for the three-cylinder wake. The calculations of $\overline{uv^2}$ based on the superposition hypothesis appears to be in qualitative agreement with experiment, except near *y/d* = 0 for the *T/d* = 1.5 case. However, the results show considerable discrepancies for the *T/d* = 3.0 case.

Based on the superposition hypothesis, Bradshaw *et al.* (1973) successfully predicted a fully developed plane duct flow using the data of two isolated fully developed boundary layers. They concluded that the interactions between the boundary layers did not significantly change the turbulence structure. The relatively weak effects of interactions could be attributed to the 'time-sharing' process of large eddies. In other words, the large eddies from either shear layer could occur at the same location but not at the same time. Weir *et al.* (1980) formed a two-dimensional jet by superposing two mixing layers, and found that the triple velocity products near the centerline were significantly affected by the interactions. They suggested that the relatively high turbulent intensities in the mixing layers gave rise to an intense interaction and significant nonlinear effects. Andreopoulos and Bradshaw (1980) investigated the interaction between turbulent shear layers in the near-wake of a flat plate and proposed that interactions took place by fine-grained mixing, instead of large eddy 'time sharing'. In the complex wakes investigated in this study, it is most likely that the interactions occur through opposite-signed large-scale eddies from the different simple wakes. Therefore, fine-grained mixing could not play a dominant role. In addition, intense interactions and nonlinear effects could be expected because of the relatively high turbulent intensities in the wake. The poor agreement in $\overline{v^2}$ and $\overline{uv^2}$ is therefore not unexpected. Evidently, the interactions have altered the turbulence structures and cannot be completely represented by the 'time sharing' process of large eddies. Williamson (1985) observed

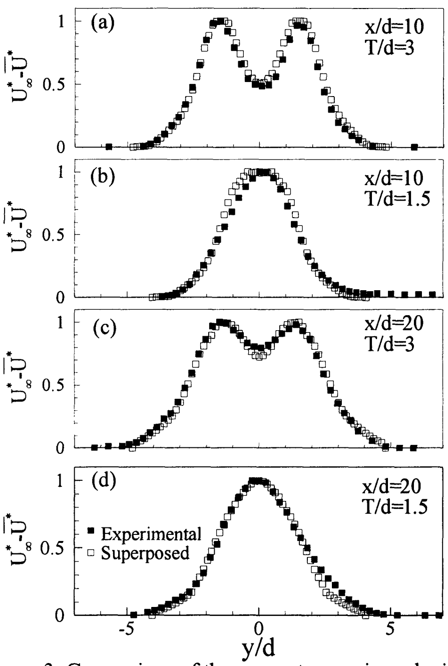

Figure 3: Comparison of the mean streamwise velocity between measurement (■) and superposition hypothesis (□) in a two-cylinder wake.

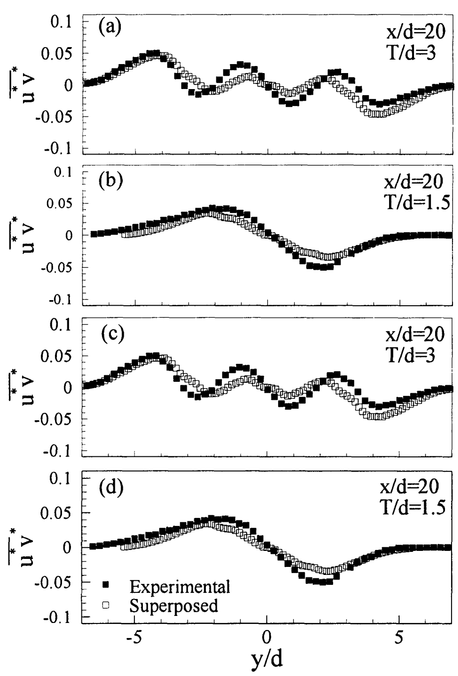

Figure 4: Comparison of the shear stress between measurement (■) and superposition hypothesis (□) in a two-cylinder wake (a, b) and a three-cylinder wake (c, d).

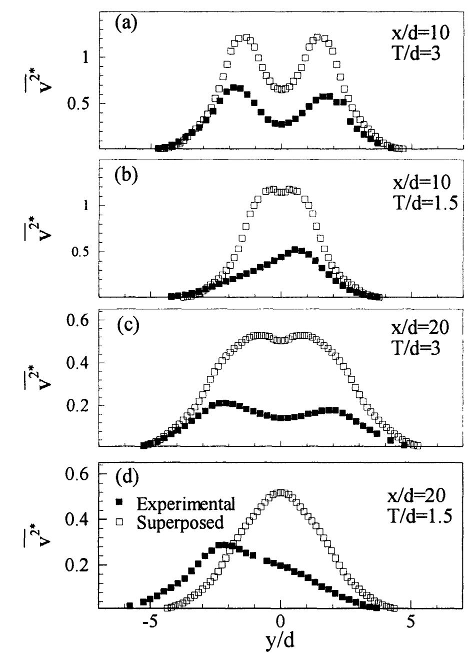

Figure 5: Comparison of $\overline{v^2}$ between measurement (■) and calculation (□) in a two-cylinder wake.

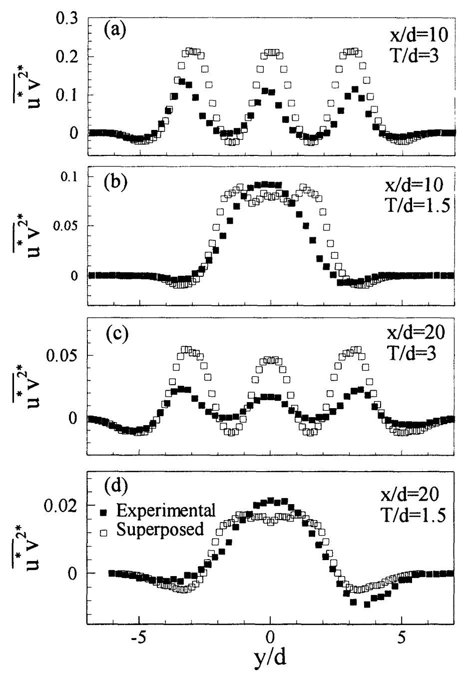

Figure 6: Comparison of $\overline{uv^2}$ between measurement (■) and superposition hypothesis (□) in a three-cylinder wake.

that, in the range of Re = 100 ~ 200, the vortex streets behind two cylinders might be anti-phase as well as in-phase. However, the anti-phase vortex streets appeared to be dominating. The two configurations of vortex streets have also been noted at higher Re (see for example Kamemoto 1976 at Re = 662). It seems plausible that the in-phase vortex streets are consistent with the time-sharing concept of large eddies. Therefore, it is likely that the strong interactions occur between the anti-phase vortex streets. Note that the lateral velocities associated with the interacting vortices in two anti-phase streets have opposite signs. As a consequence, they tend to cancel out each other, thus leading to a smaller $\overline{v^2}$ or $\overline{uv^2}$. This could provide an explanation as to why the calculations based on the superposition hypothesis are substantially larger than the measurements in Figs. 5 and 6.

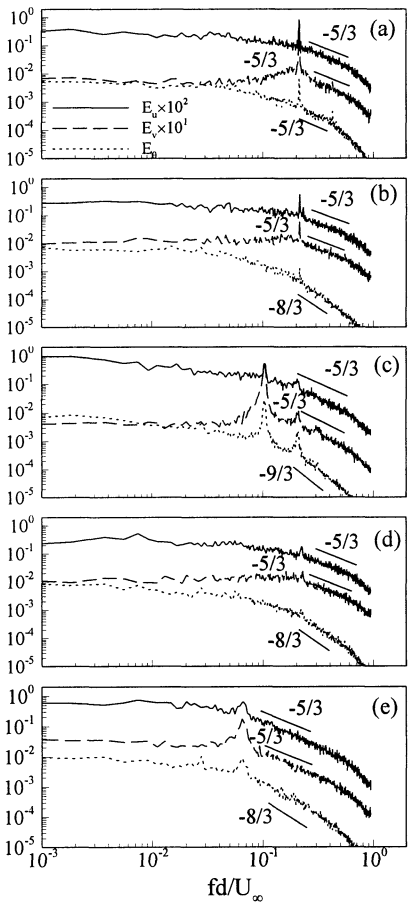

Figure 7: Power spectra E_θ, E_u and E_v of the fluctuating temperature (θ), streamwise and lateral velocities (u, v), x/d =20. (a) Single cylinder, y/d = 0.5. (b) Two cylinders, T/d = 3.0, y/d = -1.0; (c) two cylinders, T/d = 1.5, y/d = -0.25; (d) three cylinders, T/d = 3.0, y/d = 0.5; (e) three cylinders, T/d = 1.5, y/d = 0.5.

SPECTRAL CHARACTERISTICS

Fig. 7 presents some typical spectra E_u, E_v and E_θ of u, v and θ at x/d = 20 and 0.5d above the centerline of the heated simple wake generated by cylinder 1 (Fig. 1). All spectra exhibit a peak at the average vortex shedding frequency f_s. When T/d = 3.0, the Strouhal number, $f_s d/U_\infty$, of the two-and three-cylinder wake is identical to that of the single-cylinder wake (~ 0.21). However, as T/d is reduced to 1.5, $f_s d/U_\infty$ drops to 0.104 in the two-cylinder case and to 0.065 in the three-cylinder case. It is well documented that when T/d is between 1.5 and 2.0, two vortex-shedding frequencies occur in the wake of a cylinder row. This result is confirmed by the present experiment. It is further noted that the two frequencies ($f_s d/U_\infty$ = 0.104 and 0.102) in the two-cylinder wake are not the same as those (0.067 and 0.065) in the three-cylinder wake. It seems that the shedding frequencies are also dependent on the number of cylinders.

In the single-cylinder wake, the slope of the inertial sub-range of E_θ is – 5/3, which is identical to that of E_u or E_v. This slope is the same as that documented in the literature, e.g. Tennekes and Lumley (1972). In the complex wakes, the inertial sub-range of E_u and E_v does not appear to be affected by the interactions of the simple wakes. As a result, the slope remains the same as in the single-cylinder case. This, however, is not the case for E_θ. There is an appreciable increase, up to – 9/3, in the magnitude of the slope of the inertial sub-range in all the complex wakes. It is evident that the interactions between simple wakes affect not only large-scale temperature fluctuations but also the scales in the inertial sub-range.

GRADIENT TRANSPORT ASSUMPTION

The single- and three-cylinder wake data are reasonably symmetrical about the centerline (y/d=0) of the heated cylinder. In the case of the two-cylinder wake, only the lower cylinder was heated. This arrangement allows the interactions between two turbulent simple wakes, one heated and the other not, to be investigated. The asymmetric nature of this arrangement is quite different from that of the single-cylinder and three-cylinder case. The spread of the thermal boundary was comparable with that of the turbulent flow in the lower side (y/d < 0), but was entirely contained within a fully turbulent fluid in the upper side (y/d > 0).

Figs. 8 and 9 present the gradients, $\partial\overline{U}/\partial y$ and $\partial\overline{\Theta}/\partial y$, and the ratios $-\overline{uv}/\partial\overline{U}/\partial y$ and $-\overline{v\theta}/\partial\overline{\Theta}/\partial y$. The gradients were estimated from a least squares fit to $\overline{U}$ and $\overline{\Theta}$. In the single-cylinder wake, the gradient transport assumption is quite valid, that is, the shear stress approaches zero at the same time as the mean velocity gradient. The heat flux and the temperature gradient also become zero simultaneously. The ratio $-\overline{uv}/\partial\overline{U}/\partial y$ is virtually a constant, so is $-\overline{v\theta}/\partial\overline{\Theta}/\partial y$. In the complex wakes, because of weak interactions, the gradient transport assumption is still valid at T/d = 3.0. When the separation distance is reduced to T/d =1.5, the gradient transport assumption is still valid for the momentum transport. However, $\overline{v\theta}$ (not shown) does not necessarily approach zero near the centerlines of the individual simple wakes when $\partial\overline{\Theta}/\partial y$ goes to zero (Figs. 9c and 9d). Consequently, $-\overline{v\theta}/\partial\overline{\Theta}/\partial y$, which otherwise is approximately constant, changes significantly at y/d = 0 and ± 1.5 (Fig. 9b).

CONCLUSIONS

The effects of interactions between simple wakes on the velocity field and the temperature field have been investigated using a three-wire probe. This investigation leads to the using

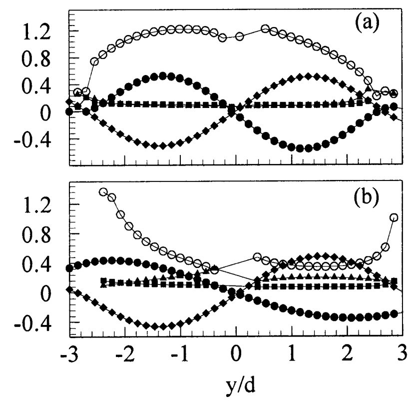

Figure8: Lateral $-\overline{uv}/(\partial\overline{U}/\partial y)$ (■), $-\overline{v\theta}/(\partial\overline{\Theta}/\partial y)$ (▲), $\partial\overline{U}/\partial y$ (◆), $\partial\overline{\Theta}/\partial y$ (●), and the turbulent Prandtl number (O) in a single cylinder wake. (a) x/d =10, (b) x/d =20.

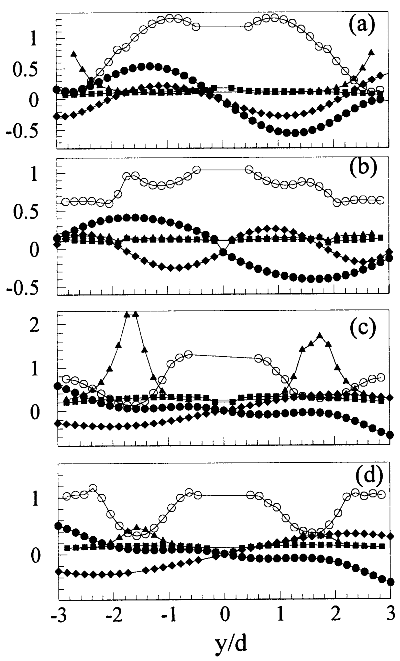

Figure 9: Lateral distributions of $-\overline{uv}/(\partial\overline{U}/\partial y)$ (■), $-\overline{v\theta}/(\partial\overline{\Theta}/\partial y)$ (▲), $\partial\overline{U}/\partial y$ (◆), $\partial\overline{\Theta}/\partial y$ (●), and the turbulent Prandtl number (O) in a three-cylinder wake. (a) x/d =10, T/d = 3.0; (b) 20, 3.0; (c) 10, 1.5; (d) 20, 1.5.

a three-wire probe. This investigation leads to the following conclusions.

1. The superposition hypothesis can be used to predict the mean velocity fields of complex cylinder wakes using the experimental data of a single-cylinder wake. The hypothesis is also qualitatively supported by the second-order velocity products. However, discrepancy between measurements and calculations occurs qualitatively and quantitatively for the third-order velocity products. Evidently, the turbulence structures have been altered by the interactions between simple wakes. It is conjectured that the vortices in the in-phase vortex streets might not interact strongly following the 'time-sharing' process. The vortices in the anti-phase vortex streets could have interacted intensely. This interaction could be responsible for the alteration of the turbulence structures.

2. The interactions between simple wakes do not seem to have any affect on the fine-scale turbulence, at least up to the inertial sub-range of the velocity spectra. However, the inertial sub-range of the temperature spectra has been affected, the slope being appreciably increased in magnitude.

3. The gradient transport assumption has been validated in terms of momentum transport in both simple and complex wakes. The assumption, however, does not seem to be always valid in terms of passive scalar transport in complex wakes. It is found that $\overline{v\theta}$ and $\partial\overline{\Theta}/\partial y$ do not approach zero simultaneously near the centrelines of the simple wakes that make up the complex wake, thus leading to a drastic variation in $-\overline{v\theta}/\partial\overline{\Theta}/\partial y$.

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