SCALING LAWS OF TEMPERATURE AND VELOCITY FLUCTUATIONS IN TURBULENT THERMAL CONVECTION

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ABSTRACT

Experiments by Castaing et al. (1989) showed that the Nusselt number versus Rayleigh number power law scaling exponent in Rayleigh-Benard convection is 2/7 rather than the classical 1/3 over a large range of Rayleigh number (10^7-10^{12}). They derived two scaling theories (λ-I and λ-II) that result in the 2/7 power law scaling. Adrian (1996) derived corresponding scaling laws for the vertical profiles of the r.m.s. temperature and velocity fluctuations, and provided experimental evidence in support of the λ-layer scalings. However, due to the scatter in the experimental data for the r.m.s. temperature profiles in unsteady non-penetrative convection, the data was not able to select between the two λ-layer scalings. The present set of experiments in Rayleigh-Benard convection were conducted to provide a set of well-converged data that might support of the λ-layer scalings. However, the r.m.s. data over the outer layer do not conclusively select between the λ-I and λ-II scalings. The data are fit by a power-law with exponent –0.4, not with the –1/2 exponent required by the λ-I theory. And, the log-law required by the λ-II theory was found not to be a good fit to the data. Thus, neither of the theories adequately describes the temperature fluctuation data.

INTRODUCTION

λ-layer Scaling

Measurements of heat transfer in Rayleigh-Benard convection in helium gas at low temperature by Castaing et al. (1989) showed that the Nusselt (Nu) number versus Rayleigh (Ra) number power law scaling exponent is very close to 2/7 over a large range of Ra (10^7-10^{12}). This contradicts the classical 1/3 power law result of Priestley (1959). Castaing et al. (1989) proposed a new scaling theory, called λ-layer scaling, and derived a 2/7 scaling law under two different circumstances (λ-I and λ-II). The λ-layer theories postulate a model of the flow consisting of three layers: a very thin λ-layer adjacent to the wall, a thicker mixing layer above the λ-layer, and an even thicker core layer above the mixing layer.

In the core layer, the λ-layer theory assumes that the inviscid length, velocity, and temperature scales of Deardorff (1970) apply. The length, velocity and temperature scales in the λ-layer for the λ-I and λ-II theory are summarized in Table 1. All symbols are defined in the nomenclature section.

TABLE 1: LENGTH, VELOCITY, AND TEMPERATURE SCALES IN THE λ-LAYER THEORY

	Length	Velocity	Temperature
core	$z^*=L/2$	$w^*=(\beta g Q_0 z^*)^{1/3}$	$\theta^*=Q_0/w^*$
λ-I	$\lambda=z^*/Nu$	$w_h=\beta g \Delta T \lambda^2/\nu$	ΔT
λ-II	$\lambda=z^*/Nu$	$w_h=\beta g \Delta T \lambda^2/\nu$	$\Delta_m=\kappa\nu/\beta g \lambda^3$

In the λ-I theory, the thermals ejected from the λ-layer move through the core, which is not in vigorous motion, such that they stay intact. Castaing et al. (1989) then assert that velocity scales in the core and the λ-layer are the same. In the λ-II theory, the core flow consists of vigorous circulations, such that the thermals get ripped apart and are thoroughly mixed. Thus, they assert that the temperature scale in the λ-layer and core is the same. In both cases, matching of the velocity and temperature scales respectively in the mixing layer yields the 2/7 scaling law. Castaing et al. (1989) argue that the second model is the more physically plausible. However, it is not known which of the two models of the core flow occurs. And, since both models lead to a 2/7 scaling, measurements of Nu vs. Ra cannot select between them.

It should be noted that even though the 2/7 scaling is observed over a wide range of Rayleigh numbers, it is not asymptotic. Siggia (1994) argues if there is an asymptotic scaling then it must be the 1/3 scaling. However, recent experiments in low-temperature helium gas by Chavanne et al. (1996) indicate that above Ra of 3×10^{10} there is a departure from the 2/7 power law in which the exponent grows without reaching an asymptotic value. In particular, no 1/3 power law region was observed. Thus, the issue of the Nu vs. Re scaling is not resolved.

λ-layer Scaling for Velocity and Temperature

The λ-layer scaling arguments of Castaing et al. (1989) were used by Adrian (1996) to derive corresponding scaling laws for the vertical (z-direction) profiles of r.m.s. temperature fluctuations (σ_θ) and r.m.s. vertical velocity fluctuations (σ_w) by asymptotic matching of the core and λ-layer scales in the mixing layer. Both λ-layer theories lead to log-laws for σ_w (as opposed to a 1/3-power law for the classical theory). In the case of σ_θ, the λ-I theory leads to a power law with exponent $-1/2$, while the λ-II theory leads to a log law (in contrast to the $-1/3$ power law for the classical theory). These results are summarized in Table 2.

TABLE 2: COMPARISON OF CLASSICAL, λ-I, AND λ-II SCALING LAWS

	Classical	λ-I	λ-II
Temperature	$\sigma_\theta \sim z^{-1/3}$	$\sigma_\theta \sim z^{-1/2}$	$\sigma_\theta \sim \ln z$
Velocity	$\sigma_w \sim z^{1/3}$	$\sigma_w \sim \ln z$	$\sigma_w \sim \ln z$

Adrian (1996) used LES (Schmidt and Schumann, 1989) and experimental (Adrian et al., 1986) data of non-penetrative convection and DNS (Kerr, 1996), LES (Moeng and Rotunno, 1990) and experimental (Deardorff and Willis, 1967) of Rayleigh-Benard convection to examine the velocity scaling. In all cases, the r.m.s. velocity data was fit by a logarithmic curve, supporting the λ-layer scalings for velocity. However, since both λ-layer scaling laws result in logarithms, the velocity measurements are unable to select between the λ-I and λ-II theories.

Adrian (1996) used LES (Schmidt and Schumann, 1989) and experimental (Adrian et al., 1986) data of non-penetrative convection to examine the temperature scaling. However, due to the scatter in the experimental data, the curves were fit equally well by a logarithmic curve and a -1/2 power law. The scatter of the experimental data in non-penetrative convection occurs as a result of the unsteady nature of the flow, which does not allow for long averaging times.

Thus, well-converged temperature statistics are required to select between the two λ-layer theories. Long averaging times are possible in Rayleigh-Benard convection due to the statistically steady nature of the flow. The objective of this study is to use Rayleigh-Benard convection to experimentally obtain well converged temperature statistics in order to select between the scaling theories.

RAYLEIGH-BENARD APPARATUS AND INSTRUMENTATION

Planform visualizations of turbulent thermal convection have shown that the convection cells that develop have a maximum horizontal extent less than approximately 5 layer depths. Thus, in order to approximate infinitely wide horizontal layers in the lab, a high aspect ratio (6:1 – 8:1) Rayleigh-Benard experiment has been constructed.

The planform dimension is 91 cm × 91 cm, while the layer depth can be varied from 11 cm –18 cm. The working fluid is de-ionized water. A layer depth of 12 cm, corresponding to an aspect ratio of 7.5:1 was used for the experiments described here. The experimental rig is shown schematically in Figure 1.

The layer is heated from below by four etched-foil resistance heating mats which are bonded to the bottom of a 2.5 cm thick aluminum plate using a pressure sensitive adhesive. The spatial uniformity of the temperature at the upper surface of this aluminum plate was examined under non-convective conditions using thermo-chromic liquid crystals. The temperature distribution was found to be uniform to within the resolution of the crystals, approximately 0.1 K (less than 1.5% of the temperature drop across the layer, ΔT). The electrical power to the resistance mats was provided by four DC power supplies. The mean voltage and current output of the power supplies was stable to within 0.1% over the course of an experiment, with a ripple of less than 0.2% of the output voltage.

The layer is cooled from above by a 7.5 cm thick aluminum plate that has rectangular aluminum channels attached to its upper surface. The channels had cooling water pumped through them at a rate of 680 liters/minute by a pair of centrifugal pumps. At this flow rate the temperature rise in the cooling water was less than 0.5% of ΔT at the highest heat flux. An array of 10 thermocouples located 12 mm from the lower surface of the aluminum plate was used to measure the temperature distribution under experimental conditions. The mean temperature distribution was found to be uniform to within 2% of ΔT at all Rayleigh numbers. The lower plate was leveled to within 0.0004 radians of horizontal by means of leveling screws and a depth micrometer. The distance between the two plates was maintained to within 12.0 ± 0.1 cm by means of machined Plexiglas spacers. The variations in the spacing between the two plates is due to the fact that the plates are not perfectly flat.

The side walls of the rig are made from Plexiglas with glass inserts for optical access. In order to minimize cooling due to conduction through the side walls, they were insulated using 5 cm of Styrofoam insulation. This resulted in less than 1% of the total heat flux being lost through the side walls at all Rayleigh numbers used. Shadowgraph flow visualization indicated that there were no large-scale circulations due to side-wall cooling or non-uniformity of the upper or lower plate temperature distribution.

The temperature measurements in the fluid layer were made using 0.3 mm diameter thermocouple probes, which are smaller than the Kolmogorov ($\eta \sim 0.6$ mm) scale for the highest Rayleigh number used. Thus, all the relevant scales in the flow are captured. The data were taken by a PC using a 16-bit A/D board and 8^{th} order low-pass elliptical filters, such that the system has noise rejection sufficient for a temperature resolution of 0.01 K.

In low aspect ratio cells (~1:1) the horizontal position of the thermocouple is important. That is, the temperature and velocity statistics change with the horizontal location of the probe (Castaing et al., 1989). However, in the high aspect ratio cell used here, the statistics were found to be independent of horizontal position (as long as the probe is not near a side wall). For all the measurements made here, the probes were horizontally displaced by approximately 10 cm from the center of the cell.

EXERIMENTAL PARAMETERS USED

The r.m.s. temperature profiles were measured over a decade of Rayleigh numbers between 1.9×10^8 and 2.2×10^9. This is well within the regime in which the 2/7 power law was found to apply. And, the Nu vs. Re power law scaling exponent obtained in the present experiments, 0.295, is in reasonable agreement with the 2/7 (0.286) power law.

The profiles consisted of 18 z-positions in the lower half of the layer between z/z*=0.02 and z/z*=1.00. The experimental parameters are summarized Table 3. All parameters are evaluated at the bulk (core) temperature, T_b.

TABLE 3: EXPERIMENTAL PARAMETERS AND SCALES

Ra	1.9×10^8	4.5×10^8	1.1×10^9	2.2×10^9
Nu	36.3	40.9	61.9	70.7
Pr	6.3	5.8	4.9	4.0
ΔT	6.1 K	12.1 K	21.9 K	30.1 K
T_b	24.1 °C	27.0 °C	34.3 °C	43.2 °C
H	0.12 m	0.12 m	0.12 m	0.12 m
z*	60.0 mm	60.0 mm	60.0 mm	60.0 mm
w*	3.4 mm/s	4.6 mm/s	7.0 mm/s	8.7 mm/s
θ*	0.08 K	0.13 K	0.24 K	0.31 K
λ	1.65 mm	1.47 mm	0.97 mm	0.85 mm
w_h	44 mm/s	82 mm/s	93 mm/s	140mm/s
Δ_m	0.012 K	0.014 K	0.036 K	0.039 K

Due to the statistically steady nature of Rayleigh-Benard convection, long averaging times are able to be used in order to obtain well converged statistics. The estimate of the r.m.s. of the temperature fluctuation at each point was made using 20 000 samples, each taken approximately one integral time scale apart. That is, approximately 10 000 independent samples were used for each data point.

Rayleigh-Benard convection is a relatively slow process. The measured integral time scales ranged from 2 seconds (Ra = 2×10^9) to 4 seconds (Ra = 2×10^8). Thus, it took approximately 4 to 8 days of continuous operation to obtain each of the r.m.s. temperature profiles.

RESULTS AND DISCUSSION

The data have been normalized using the core layer scales (Deardorff scales). These scales were found to collapse the data better than the λ-layer scales.

Figure 2 is a log-log plot of the data across the entire half-layer. The solid line is the best power-law fit to the data between 0.02 ≤ z/z* ≤ 0.60. Beyond approximately z/z* = 0.60 in Rayleigh-Benard convection, the curve is not expected to follow a power-law (or log-law) since it must reach a maximum at z/z* = 1.0 due to symmetry. A scaling exponent of –0.4 was found to best fit the data. Even though this curve is a good fit to the data across almost the entire layer, this exponent lies between the –1/3 exponent for the classical scaling theory and the –1/2 exponent for the λ-I scaling theory. Thus, the data do not support the λ-I scalings.

A slight decrease of the magnitude of the scaling exponent with increasing Rayleigh number (from -0.42 at 2×10^8 to -0.38 at 2×10^9) was observed. However, due to the relatively small range of Rayleigh number achieved, no conclusions are made about this trend.

Figure 3 is a semi-log plot of the data with the solid line being the best log law fit to the data between 0.02 ≤ z/z* ≤ 0.60. It can be seen that the log-law is not a good fit to the data beyond z/z* = 0.3 and below z/z* = 0.06. So, although the data do not conclusively support the λ-II scaling, they do not invalidate it either.

Figure 4 is a plot of the probability density function of the temperature fluctuations at the vertical center of the cell. Castaing et al. (1989) showed that when normalized by the local r.m.s. value, the data collapse onto a single curve, as is the case here. The probability density function of centerline temperature fluctuations can be fit by back-to-back exponential functions, which are the solid lines on the plot:

$$P\left(\frac{T'}{\sigma_\theta}\right) \propto \exp\left(-c\left|\frac{T'}{\sigma_\theta}\right|\right) \qquad (1)$$

Here we obtain c = 1.3 on the cold side and c = 1.4 on the hot side. The slight asymmetry may be due to fact that the convection is not perfectly symmetrical about the layer half-depth plane due to Prandtl number variations between the cold and hot surfaces. Exponential tails (c = 1.2) were obtained by Castaing et al. (1989) in their unit aspect ratio cell only for the probe at the center of the cell. When the probe was horizontally offset the tails weren't exponential and the pdfs were not symmetrical. Exponential tails were only observed in the same group's 6.7:1 aspect ratio helium gas cell when the temperature signal was high-pass filtered using a filter the length of the cell turnover time (Wu and Libchaber, 1992). It is not known why exponential tails were not observed in the unfiltered signal, as is the case here. In DNS of Rayleigh-Benard convection, Sirovich et al. (1989) obtained exponential tails (c = 1.25) as well. Christie and Domaradzki (1994) have shown that small scales produce exponential tails while large scales produce

Gaussian tails. Thus, the exponential tails observed here suggest that the flow may be dominated by small scales.

CONCLUDING REMARKS

The r.m.s. data obtained from this set of Rayleigh-Benard experiments do not conclusively support either the λ-I or λ-II scaling. The data are fit by a power-law but not with the –1/2 exponent required by the λ-I theory. And, the log-law was found not to be a convincing fit to the data. Some aspects of the measurements are not ideal. In particular the spatial resolution of the temperature probe may result in erroneous data close to the wall. Also, the high heat fluxes lead to significant variation of the thermo-physical properties. Thus, further work is needed before a conclusive assessment of the scaling can be made.

FUTURE WORK

Work has commenced on obtaining the velocity fields in unsteady non-penetrative and Rayleigh-Benard convection using stereo digital-PIV, such that all three velocity vectors are measured on a planar domain spanning the layer depth (or half-depth). Using the assumption of axi-symmetry with respect to the horizontal directions, the full three-dimensional velocity correlation tensor can be obtained. With this data, the nature of the core flow can be examined directly.

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NOMENCLATURE

c_p	specific heat [$Jkg^{-1}K^{-1}$]
g	gravitational acceleration [ms^{-2}]
H_0	heat flux at lower surface [Wm^{-1}]
L	layer depth [m]
Nu	Nusselt number $Nu = \frac{Q_0 z_*}{\kappa \Delta T}$
Pr	Prandtl number $Pr = \frac{\nu}{\kappa}$
Q_0	kinematic heat flux at lower surface [Kms^{-1}] $Q_0 = \frac{H_0}{\rho c_p}$
Ra	Rayleigh number $Ra = \frac{\beta g L^3 \Delta T}{\kappa \nu}$
T_b	bulk (core) temperature [°C]
w_h	lambda layer velocity scale [ms^{-1}]
w*	core layer velocity scale [ms^{-1}]
z	vertical distance from lower surface [m]
z*	layer half-depth (L/2) [m]

Greek Symbols

β	coefficient of thermal expansion [K^{-1}]
ΔT	temperature drop across layer [K]
Δ_m	lambda layer temperature scale [K]
η	Kolmogorov length scale [m]
κ	thermal diffusivity [m^2s^{-1}]
λ	lambda layer length scale [m]
ν	kinematic viscosity [m^2s^{-1}]
σ_θ	r.m.s. of temperature fluctuations [K]
σ_w	r.m.s. of vertical velocity fluctuations [ms^{-1}]
θ*	core layer temperature scale [K]
ρ	fluid density [$kg\ m^{-3}$]

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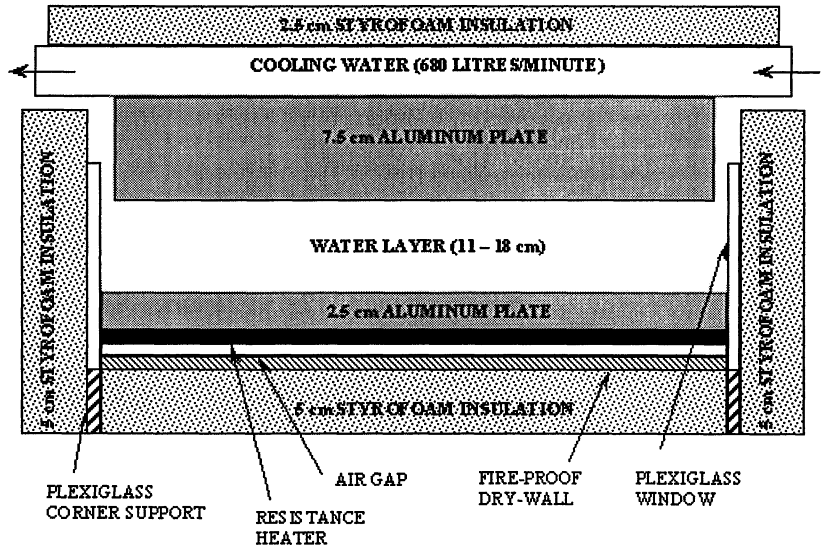

Figure 1: Schematic of the experimental rig.

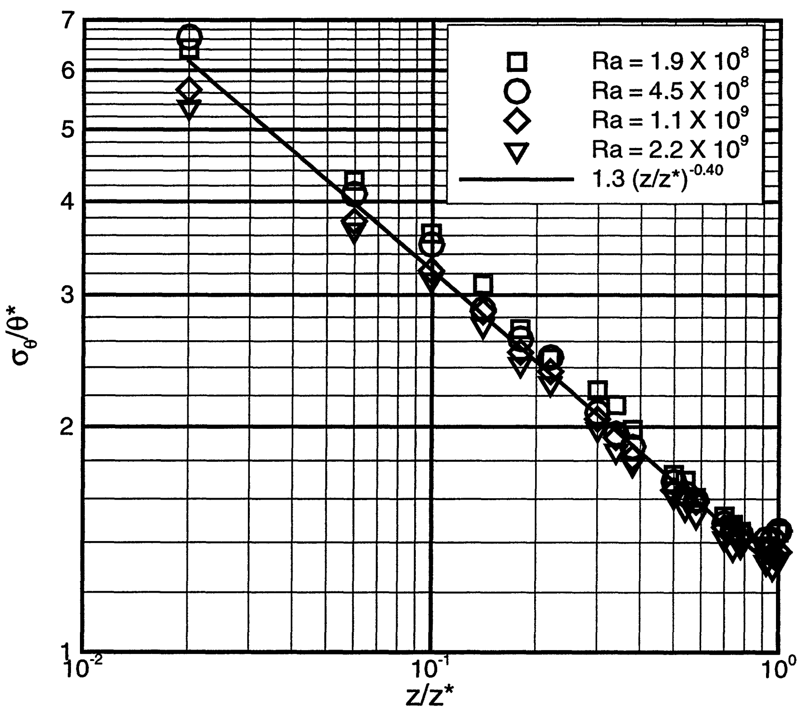

Figure 2: R.M.S. of the temperature fluctuations. Solid line is the best power-law fit to the data from 0.02<z/z*<0.60.

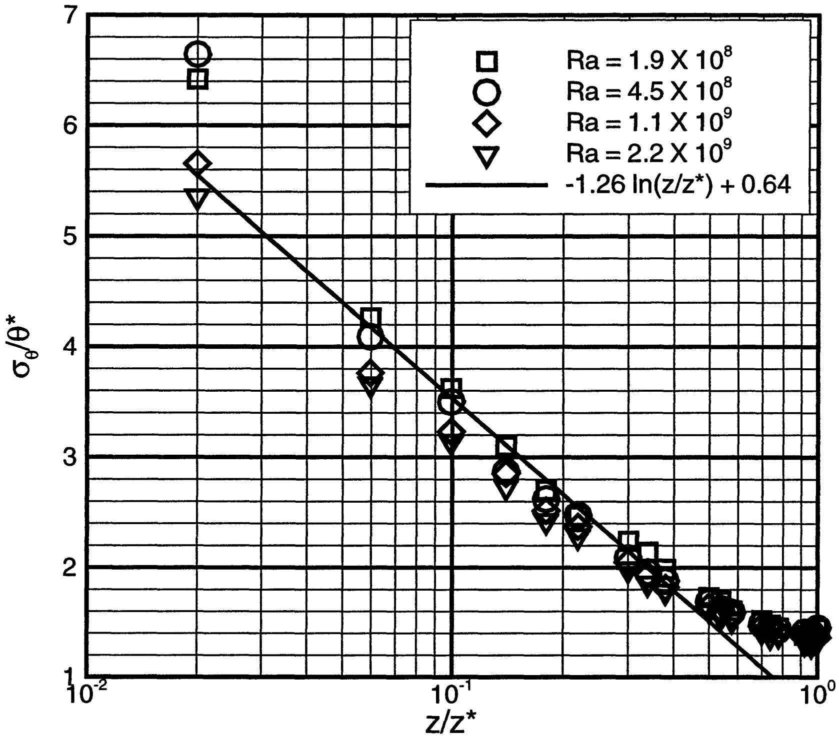

Figure 3: R.M.S. of the temperature fluctuations. Solid line is the best log-law fit to the data from 0.02 < z/z* < 0.60.

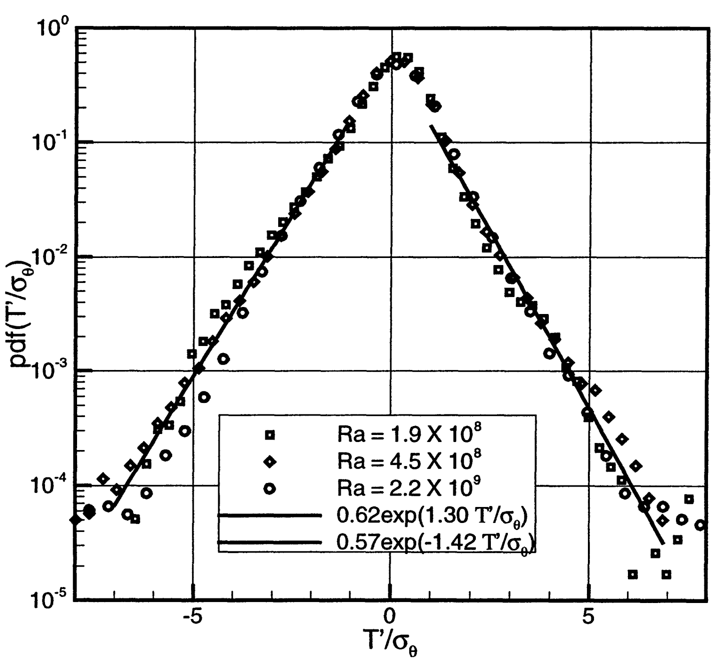

Figure 4: Probability density function of the centerline temperature fluctuations.