A NONLINEAR LOW REYNOLDS TURBULENCE CLOSURE BASED ON $k-\epsilon$ AND $k-\omega$

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ABSTRACT

A quadratic low Reynolds turbulence closure ensuring non-zero anisotropies on the wall is described.

The influence of anisotropy and of the length-scale determining parameter is assessed by combination of the nonlinear constitutive relation with both a k-ϵ and a k-ω based (SST) turbulence model.

The performance of these models is compared to a classical linear low Reynolds k-ϵ model and a non-linear high-Reynolds k-ω model for two testcases: channel flow and Backward facing step (BFS) flow.

INTRODUCTION

Most turbulence models used nowadays are based on the Boussinesq hypothesis linking the stress and strain tensors within a linear constitutive relation. Although many popular turbulence models originate from this approach, it has the inherent weakness not to reproduce the strong anisotropies, which significantly influence the mean flow in e.g. curved channels, stagnating flows,.... For fully developed channel flow this approach even erroneously reduces to an isotropic model. To model anisotropy, two approaches are possible: 1) the use of Reynolds stress models (RSM), 2) the use of nonlinear constitutive relations. The second method is appealing as the computational cost remains low. On this basis, Gatski and Speziale (1993) developed a high Reynolds version of an anisotropic model. In this paper we propose a low Reynolds anisotropic constitutive relation which allows to model the anisotropy up to the wall. This new model is compared with a classical linear model and the nonlinear model of Gatski.

A second, but less known, weakness of presently used turbulence models is the non-universal prediction of turbulence time or length scales, especially in more complex flows, e.g. separated flows. These scales appear in the constitutive relation and have an important influence on the prediction of both the turbulent normal and shear stresses. The turbulent time or length scale can change drastically in non-equilibrium flows, leading to incorrect representations of the Reynolds-stresses, even when an anisotropic constitutive relation is perfectly tuned. In most turbulence models, the turbulence dissipation ϵ or the turbulence frequency ω are used to define these scales. In this paper, the developed nonlinear model is combined with either a k-ϵ turbulence model or the SST model of Menter (1994), which is based on a k-ω model. The present nonlinear relation necessitates a low Reynolds extension of the SST model.

PROPOSED TURBULENCE MODEL

A quadratic constitutive relation based on the ideas of Speziale and Abid (1995) and Shih *et al.* (1993) is used. A general and co-ordinate invariant relationship between stresses, strains and vorticities, up to quadratic terms, can be written as

$$\begin{aligned} 2b_{ij} = & \frac{\overline{u_i'u_j'}}{k} - \frac{2}{3}\delta_{ij} \\ = & -2c_\mu(\tilde{S}_{ij} - \frac{1}{3}\delta_{ij}\tilde{S}_{ll}) \end{aligned}$$

$$+ c_1(\tilde{S}_{ik}\tilde{S}_{kj} - \frac{1}{3}\delta_{ij}\tilde{S}_{lk}\tilde{S}_{kl})$$
$$+ c_2(\tilde{\Omega}_{ik}\tilde{S}_{kj} - \tilde{S}_{ik}\tilde{\Omega}_{kj})$$
$$+ c_3(\tilde{\Omega}_{ik}\tilde{\Omega}_{kj} - \frac{1}{3}\delta_{ij}\tilde{\Omega}_{lk}\tilde{\Omega}_{kl}) \qquad (1)$$

where $\tilde{S}_{ij}$ and $\tilde{\Omega}_{ij}$ are the components of the non-dimensional shear and rotation tensors:

$$\tilde{S}_{ij} = \tau\frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) , \quad \tilde{\Omega}_{ij} = \tau\frac{1}{2}(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i})$$

and where b_{ij} are the components of the anisotropy tensor, τ is a turbulent time scale, and c_μ, c_1, c_2, c_3 are apparent viscosities.

First order term

Realizability, defined as the non-negativity of the turbulent normal stresses, together with the Schwarz-inequality between any fluctuations, is a physical and mathematical principle that should be ensured by any turbulence model. As the standard k-ϵ model is not realizable, the first objective is to introduce a realizable first order model.

Consider as an example an accelerating flow where $S_{11} = -S_{22} > 0$ and all other shear components are zero. If only the first order term in equation (1) is used, the normal stress $\overline{u'_1u'_1}$ can be written as

$$\frac{\overline{u'_1u'_1}}{2k} = \frac{1}{3} - c_\mu\frac{k}{\epsilon}S_{11} = \frac{1}{3} - c_\mu\tilde{S}_{11} \qquad (2)$$

For standard k-ϵ, with $c_\mu = 0.09$, equation (2) produces a negative value when $\tilde{S}_{11} > 3.7$. Physically $\overline{u'_1u'_1}$ decreases when $\tilde{S}_{11}$ increases. Using the dimensionless shear $\tilde{S} = \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}$, the realizability conditions are:

$$\frac{\overline{u'_1u'_1}}{2k} > 0 \qquad \text{for } 0 < \tilde{S} < \infty$$
$$\frac{\overline{u'_1u'_1}}{2k} \to 0 \text{ and } (\frac{\overline{u'_1u'_1}}{2k})_{\tilde{S}} \to 0 \qquad \text{for } \tilde{S} \to \infty$$

These conditions can be satisfied by using

$$c_\mu = \frac{1}{A + 1.5\tilde{S}} \qquad (3)$$

In order to determine the constant A, the value of $\tilde{S}$ in the equilibrium region of a channel flow is considered: $\tilde{S} \approx 3.3$. As the standard k-ϵ model has been tuned for this flow, we choose $A = 6$ which reproduces $c_\mu = 0.09$ for this region.

To extend the model for validity in the wall region, low-Reynolds number modifications are necessary. In line with the accompanying low-Reynolds turbulence model of Yang and Shih (1993), the turbulent time scale is taken as

$$\tau = \frac{k}{\epsilon} + \sqrt{\frac{\nu}{\epsilon}}$$

Due to the replacement of $c_\mu = 0.09$ by equation (3), a new damping function is needed in order to adjust the turbulent viscosity correctly near the wall. As in the Yang-Shih model, $R_y = \frac{\sqrt{k}y}{\nu}$ was used as a wall distance parameter. The DNS-data of Kim *et al.* (1987) for channel flow at $Re_\tau = \frac{u_\tau h/2}{\nu} = 395$ were used to determine the damping function f_μ.
The shear components for channel flow are: $S_{11} = S_{22} = 0$ and $S_{12} = S_{21} = \frac{1}{2}u_y$. As a consequence, in this flow, the quadratic terms do not influence $\overline{u'_1u'_2}$ and the normal stresses do not influence the turbulent production. This allows the development of a first order model without knowing the exact form of the quadratic terms. The function f_μ was constructed by comparing $\overline{u'_1u'_2}$ from the DNS-data with $\overline{u'_1u'_2} = -2kc_\mu\tilde{S}_{12}$ as predicted by the high Reynolds model: $f_{\mu,theor} = \frac{-\overline{u'_1u'_2}}{2k\tilde{S}c_\mu}$. The following function was obtained:

$$f_\mu = 1 - e^{(-.04R_y{}^{0.5} - 1E-4R_y{}^2 - 8E-11R_y{}^5)} \quad (4)$$

This function f_μ respects the theoretical limit to the wall: $\overline{u'_1u'_2} \sim \vartheta(y^3)$ implies $f_\mu \sim \vartheta(y)$ because $k \sim \vartheta(y^2)$, which is satisfied as $R_y \sim \vartheta(y^2)$.

Second order terms

One can easily verify that in a simple shear flow, the quadratic terms only affect the normal stresses whereas the first order term only influences the shear stresses. As the present study intends to reproduce the near-wall anisotropy, the modelling should focus on the second order terms. The goal is to reproduce the asymptotic behaviour of the fluctuating component perpendicular to the wall. As this component vanishes faster than the others when approaching the wall, one can derive the wall-value of b_{22}: $b_{22} = \frac{\overline{u'_2u'_2}}{2k} - \frac{1}{3} \to -\frac{1}{3}$ when $y \to 0$ as $\overline{u'_2u'_2} \sim \vartheta(y^4)$ and $k \sim \vartheta(y^2)$.

To ease the modelling, a regrouping of the quadratic terms is done, such that b_{22} is solely affected by one term.

In equation (1) the c_3-term can be omitted as Mansour *et al.* (1991) showed that there is no effect of pure rotation on initially isotropic turbulence. The two remaining quadratic terms are then regrouped into two terms T'_1 and T'_2, which can be cast in a common form:

$$c'_\mu[\quad \alpha_1(\tilde{S}_{ik}\tilde{S}_{kj} - \frac{1}{3}\delta_{ij}\tilde{S}_{lk}\tilde{S}_{kl}) + \alpha_2(\tilde{\Omega}_{ik}\tilde{S}_{kj} - \tilde{S}_{ik}\tilde{\Omega}_{kj})]$$

where α_1 and α_2 are constants and c'_μ is an apparent viscosity to be determined for both T'_1 and T'_2. For the case of simple shear (only $\tilde{S}_{12} = \tilde{S}_{21} = \tilde{\Omega}_{12} = -\tilde{\Omega}_{21} = a \neq 0$) such a term has the following contribution to the normal anisotropies:

$$b_{11} = c'_\mu[\alpha_1 \frac{1}{3}a^2 + \alpha_2(2a^2)]$$
$$b_{22} = c'_\mu[\alpha_1 \frac{1}{3}a^2 - \alpha_2(2a^2)]$$

For $\alpha_1 = 6$ and $\alpha_2 = 1$, a first term T'_1 is obtained which only influences b_{11}:

$$T'_1 = c'_{\mu 1}[\quad 6(\tilde{S}_{ik}\tilde{S}_{kj} - \frac{1}{3}\delta_{ij}\tilde{S}_{lk}\tilde{S}_{kl}) + (\tilde{\Omega}_{ik}\tilde{S}_{kj} - \tilde{S}_{ik}\tilde{\Omega}_{kj})]$$

For $\alpha_1 = 6$ and $\alpha_2 = -1$, a second term T'_2 is obtained which only influences b_{22}:

$$T'_2 = c'_{\mu 2}[\quad 6(\tilde{S}_{ik}\tilde{S}_{kj} - \frac{1}{3}\delta_{ij}\tilde{S}_{lk}\tilde{S}_{kl}) - (\tilde{\Omega}_{ik}\tilde{S}_{kj} - \tilde{S}_{ik}\tilde{\Omega}_{kj})]$$

This formulation does not allow a sufficient wall-anisotropy. If modelled using T'_1 and T'_2, the b_{22}-component is determined as $b_{22} = c'_{\mu 2}(2a^2 + 2a^2) = c'_{\mu 2}(4a^2) = c'_{\mu 2}u_y^2\tau^2$. When using $\tau = \frac{k}{\epsilon}$, this results in a wall value $b_{22}^w = 0$. Even when using $\tau = \frac{k}{\epsilon} + \sqrt{\frac{\nu}{\epsilon}}$ as a time-scale, the corresponding wall value $\sqrt{\frac{\nu}{\epsilon}}$ is too small to provide $b_{22}^w = -\frac{1}{3}$, unless allowing a very large $c'_{\mu 2}$ when $\tilde{S}$ is small. This is in contrast with a homogeneous shear, where a small shear should imply a small anisotropy.

In order to allow non-zero wall anisotropies without the creation of unphysical behaviour, the second order terms are modified into:

$$T_1 = c_{\mu 1}(6\widehat{S_{ik}S_{kj}} - 2\delta_{ij}\widehat{S_{lk}S_{kl}} + (\widehat{\Omega_{ik}S_{kj}} - \widehat{S_{ik}\Omega_{kj}}))$$
$$T_2 = c_{\mu 2}(6\widehat{\widehat{S_{ik}S_{kj}}} - 2\delta_{ij}\widehat{\widehat{S_{lk}S_{kl}}} - (\widehat{\widehat{\Omega_{ik}S_{kj}}} - \widehat{\widehat{S_{ik}\Omega_{kj}}}))$$

with

$$\widehat{S_{ik}\Omega_{kj}} = S_{ik}\Omega_{kj}(\frac{k}{\epsilon})^2 f_\mu + (1-f_\mu)max[min(\beta_1^2 S_{ik}^w\sqrt{\frac{\nu}{\epsilon^w}}\Omega_{kj}^w\sqrt{\frac{\nu}{\epsilon^w}}, \beta_2^2), -\beta_2^2] \qquad (5)$$

$$\widehat{\widehat{S_{ik}\Omega_{kj}}} = S_{ik}\Omega_{kj}(\frac{k}{\epsilon})^2 f_\mu + (1-f_\mu)(2\beta_2)^2\frac{S_{ik}^w}{S^w}\frac{\Omega_{kj}^w}{\Omega^w} \qquad (6)$$

where the superscript w means the value taken on the wall. The other tensor products are obtained by replacing Ω_{kj} by S_{kj} and/or S_{ik} by Ω_{ik} in eq. (5) and (6).

Just as in the previous formulation, the effect of these terms is to redistribute the turbulent kinetic energy among the normal Reynolds stresses $\overline{u'_i u'_i}$. For simple shear flow, T_1 determines b_{11} and T_2 determines b_{22}.

The last terms in (5) and (6) are near-wall corrections. To ensure $b_{22} = -\frac{1}{3}$ on the wall in different flows, a (constant) non-zero value of the T_2 term is provided by using the wall pattern of shear and rotation in eq. (6).

In order to determine the constant β_2 in the last term of (6), DNS-data for channel flow were used. The anisotropies for positions going from the equilibrium region up to the wall were plotted as a function of the parameter $\widehat{\widehat{S}} = \sqrt{2\widehat{\widehat{S_{ij}S_{ij}}}}$ in the same graph as some experimental and DNS-values for a few simple shear flows. Using this graph, two restrictions led to the choice of the constant β_2:

1. a one-one relation between anisotropies and $\widehat{\widehat{S}}$ should exist in order to use $\widehat{\widehat{S}}$ for the modelling
2. the anisotropies for simple shear flows should still be properly predicted.

The present choice of the constant $\beta_2 = 14$ satisfies these conditions.

The near-wall corrections which are given by equation (6) are not used in the term T_1 which determines b_{11}, as the physical wall-value of b_{11}, although being non-zero, is not constant, and differs depending on the flow. Therefore, instead of using a wall-pattern, in equation (5) the wall-shear values are made dimensionless by using the wall-value of the Kolmogorov time-scale.

The constant β_1 in (5) was chosen so that for the channel flow $\widehat{S_{ik}S_{kj}} = \widehat{\widehat{S_{ik}S_{kj}}}$, resulting in $\beta_1 = 13.76$ when using DNS-data. When using the numerically obtained value of ϵ^w, the constant should be set at $\beta_1 = 16.24$.

In equation (5), the limiting of the wall-correction to a value between $-\beta_2^2$ and β_2^2 ensures the wall-value of $\widehat{S}$ does not exceed the wall value of $\widehat{\widehat{S}}$, thus prohibiting too small values of b_{11} (which would occur if $\widehat{S} > 2\beta_2$).

To obtain a realizable model, the quadratic terms should vanish when shear and/or rotation go to infinity. Again, the available DNS-data for channel flow were used to find the unknown coefficients $c_{\mu 1}$ and $c_{\mu 2}$ using curve fitting. An additional constraint for $c_{\mu 2}$ is the value on the wall in order to obtain $b_{22} = -\frac{1}{3}$. As a result, following coefficients were obtained:

$$
\begin{aligned}
c_{\mu 1} &= 1/max(p(x_1), 5)\ , \quad c_{\mu 2} = 1/p(x_2) \\
p(x_1) = &\quad -4.679 + 2.349x_1 + 0.7054x_1^2 \\
&\quad -0.01768x_1^3 + (-2.55E-5 \\
&\quad +9.6116E-7x_1)e^{0.65062x_1} \\
p(x_2) = &\quad 3(-17.995 + 8.7904x_2 - 5.8264x_2^2 \\
&\quad +0.2419x_2^3 - 3.4922E - 3x_2^4) \\
x_1 = &\quad .5(\widehat{S} + \widehat{\Omega}),\ x_2 = .5(\widehat{\widehat{S}} + \widehat{\widehat{\Omega}}) \\
\widehat{\widehat{S}} = &\quad \sqrt{2\widehat{\widehat{S_{ij}S_{ij}}}}\ , \widehat{\widehat{\Omega}} = \sqrt{2\widehat{\widehat{\Omega_{ij}\Omega_{ij}}}}
\end{aligned}
$$

Summary of the derived nonlinear model

In summary, the developed constitutive relation is:

$$
\begin{aligned}
&2b_{ij} = -2c_\mu f_\mu(\tilde{S}_{ij} - \frac{1}{3}\delta_{ij}\tilde{S}_{ll}) \\
&+c_{\mu 1}(6\widehat{S_{ik}S_{kj}} - 2\delta_{ij}\widehat{S_{lk}S_{kl}} - (\widehat{S_{ik}\Omega_{kj}} - \widehat{\Omega_{ik}S_{kj}})) \\
&+c_{\mu 2}(6\widehat{\widehat{S_{ik}S_{kj}}} - 2\delta_{ij}\widehat{\widehat{S_{lk}S_{kl}}} + (\widehat{\widehat{S_{ik}\Omega_{kj}}} - \widehat{\widehat{\Omega_{ik}S_{kj}}}))
\end{aligned}
$$

Proposed models

The present nonlinear model is combined with two different turbulence models to evaluate the influence of both anisotropy and time scale.

The **NLYS** model uses the nonlinear model along with the transport equations of the k-ϵ model taken according to Yang and Shih (1993):

$$
\begin{aligned}
\frac{Dk}{Dt} &= \frac{\partial}{\partial x_m}((\nu + \frac{\nu_t}{\sigma_k})\frac{\partial k}{\partial x_m}) + P_k - \epsilon \\
\frac{D\epsilon}{Dt} &= \frac{\partial}{\partial x_m}((\nu + \frac{\nu_t}{\sigma_\epsilon})\frac{\partial \epsilon}{\partial x_m}) + \frac{c_{\epsilon 1}f_1P_k - c_{\epsilon 2}f_2\epsilon}{\tau} + E
\end{aligned}
$$

where E is a low-Reynolds-term.

As in the standard k-ϵ model, in the Yang-Shih model, $\sigma_k = 1.$, $\sigma_\epsilon = 1.3$, $c_{\epsilon 1} = 1.44$ and $c_{\epsilon 2} = 1.92$, while the low-Reynolds modifications are:

$$
\begin{aligned}
&f_1 = 1\ , \quad f_2 = 1 - 0.22e^{(-(\frac{k\tau}{6\nu})^2)}\ , \quad \nu_t = c_\mu f_\mu k\tau\ , \\
&\tau = \frac{k}{\epsilon} + \sqrt{\frac{\nu}{\epsilon}}\ , \quad E = \nu\nu_t(\frac{\partial^2 u_i}{\partial x_k \partial x_j})(\frac{\partial^2 u_i}{\partial x_k \partial x_j})
\end{aligned}
$$

For this model, when using the numerically obtained value of ϵ^w, the constant β_1 should be set at $\beta_1 = 16.24$.

The model **NLSSTL** basically uses an SST framework as suggested by Menter (1994), but extended for low Reynolds effects (SSTL). As the k-ω model has the disadvantage of being quite dependent on the imposed freestream values for ω, and the k-ϵ model does not have this deficiency, Menter suggested using the k-ω model of Wilcox (1993a) in the near wall region and blending towards the standard k-ϵ model in the outer part of the boundary layer. In this paper the SST model is extended for low-Reynolds effects (**SSTL**) by using the low-Reynolds k-ω model of Wilcox (1993) in the near wall region.

The transport equations are then given by:

$$
\begin{aligned}
\frac{Dk}{Dt} &= -\overline{u_i'u_j'}\frac{\partial u_i}{\partial x_j} - \beta^*\omega k + \frac{\partial}{\partial x_m}((\nu + \sigma_k\nu_t)\frac{\partial k}{\partial x_m}) \\
\frac{D\omega}{Dt} &= -\frac{\gamma}{\nu_t}\overline{u_i'u_j'}\frac{\partial u_i}{\partial x_j} - \beta\omega^2 + \frac{\partial}{\partial x_m}((\nu + \sigma_\omega\nu_t)\frac{\partial \omega}{\partial x_m}) \\
&\quad + 2(1 - F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}
\end{aligned} \tag{7}
$$

In the SST model, the turbulent viscosity is given by

$$
\nu_t = min\left(\alpha^*\frac{k}{\omega}; \frac{a_1}{\Omega F_2}\right)
$$

where $a_1 = 0.31$ and Ω is the absolute value of the vorticity and F_2 is given by: $F_2 = tanh(arg_2^2)$, $arg_2 = max(2\frac{\sqrt{k}}{0.09\omega y}; \frac{500\nu}{y^2\omega})$.

The parameters in the model (α^*, γ, σ_k, σ_ω,β, β^*) are interpolated between values in the original model, commonly denoted here by ϕ_1, and parameters in the transformed standard k-ϵ model, commonly denoted here by ϕ_2, using the function F_1:

$$
\phi = F_1\phi_1 + (1 - F_1)\phi_2
$$

where, as in Menter's model, the function F_1 is given by:

$$
\begin{aligned}
&F_1 = tanh(arg_1^4) \\
&arg_1 = min\left[max(\frac{\sqrt{k}}{0.09\omega y}; \frac{500\nu}{y^2\omega}); \frac{4\sigma_{\omega 2}k}{CD_{k\omega}y^2}\right] \\
&CD_{k\omega} = max\left(2\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}, 10^{-20}\right)
\end{aligned}
$$

The parameters of set 1 (ϕ_1: low-Reynolds k-ω) are:

$$
\alpha_1^* = \frac{\alpha_0^* + Re_T/R_k}{1 + Re_T/R_k}\ , \ \gamma_1 = \frac{5}{9}\frac{\alpha_0 + Re_T/R_\omega}{1 + Re_T/R_\omega}\ ,
$$

$$\beta_1^* = 0.09\frac{5/18 + (Re_T/R_\beta)^4}{1 + (Re_T/R_\beta)^4} \ , \ \beta_1 = 3/40 \ ,$$
$$\sigma_{k1} = 0.5 \ , \ \sigma_{\omega 1} = 0.5 \ , \text{ with } \alpha_0^* = \beta_1/3 \ ,$$
$$\alpha_0 = 1/10 \ , \ R_\beta = 8 \ , \ R_k = 6 \ , \ R_\omega = 27/10$$

where Re_T is the turbulence Reynolds number defined by $Re_T = \frac{k}{\omega\nu}$.

The parameters of set 2 (ϕ_2: standard k-ϵ) are: $\alpha_2^* = 1.$, $\sigma_{k2} = 1.0$, $\sigma_{\omega 2} = 0.856$, $\beta_2 = 0.0828$, $\beta_2^* = 0.09$, $\kappa = 0.41$, $\gamma_2 = \beta_2/\beta_2^* - \sigma_{\omega 2}\kappa^2/\sqrt{\beta_2^*}$.

In the **NLSSTL** model, the turbulent viscosity and first order term in the constitutive relation are determined by the SSTL formulation, but the proposed quadratic terms have been added. The value of ϵ, needed in the constitutive relations, is determined as $\epsilon = \beta^*\omega k$. For this case, in eq. (5), the value $\beta_1 = 13.76$ can be maintained.

RESULTS

Figures 1 to 3 show results for a channel flow. The linear k-ϵ model of Yang-Shih (1993) is denoted as **LYS**. The nonlinear model of Gatski and Speziale (1993), combined with the high Reynolds k-ω model of Wilcox (1993a) is denoted as **KOGS**. The velocity and shear stress profiles are well reproduced by all models. Only **KOGS** underpredicts the turbulent kinetic energy due to the high Reynolds k-ω version used. The difference between the models is most obvious in the profiles of the normal Reynolds-stresses. The present model agrees very well with the DNS-data both when using the k-ϵ and the low-Reynolds SST formulation. Indirectly, this means that the time scale is well defined by both turbulence models for this flow configuration. As expected, the linear YS-model produces identical profiles for uu, vv and ww corresponding to an unphysical isotropic turbulence. The too low k-peak prediction in the **KOGS** model is entirely due to the use of the high Reynolds k-ω version. The constitutive relation fails to reproduce the normal stresses, especially near the wall. The vv and ww profiles have nearly identical evolutions with $v \sim O(y)$ near the wall instead of the correct $v \sim O(y^2)$. The present model predicts this near-wall behaviour correctly.

To see the influence of the turbulence scale, the more complex flow pattern of a Backward Facing Step (BFS) is chosen. The considered BFS ($Re_h = 5100$) was calculated by Le and Moin (1992) using DNS. This flow is already quite complex because of the occuring recirculation region. In our calculations, the grid, extending from 3 stepheights before to 19 behind the step, had 185 x 129 nodes. Figure 4 shows results for the friction coefficient. With the linear Yang-Shih k-ϵ model, the recirculation length is seriously underpredicted. This is however also the case for the nonlinear version of the Yang-Shih model. Surprisingly, the better predicted C_f-evolutions are not primarily related to the used anisotropy model, but rather to the used turbulence model. The k-ω based models, i.e. **KOGS** and **NLSSTL**, predict the recirculation region much better than the k-ϵ based models. The used anisotropy model has only a secondary effect when comparing **KOGS** and **NLSSTL**. This stresses the importance of the turbulent time scale which is apparently better represented with a k-ω than with a k-ϵ based turbulence model. In figure 5, the k-profile inside the recirculation zone is best predicted with the present anisotropy model together with the low Reynolds version of the SST turbulence model.

CONCLUSIONS

The use of a nonlinear model which predicts anisotropy up to the wall allows accurate anisotropy predictions, but does not significantly improve flow predictions in separated flows, when used with a k-ϵ based model.

The use of a k-ω based model like the SST model, which has been adapted for low-Reynolds effects, significantly improves BFS-predictions. This suggests the better representation of the turbulent time and length scales by such a model.

The combination of a low Reynolds modification of the SST model with nonlinear constitutive relations provides the best overall performance.

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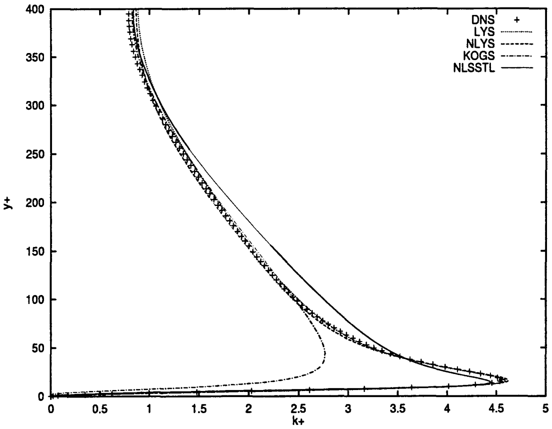

Figure 1. Turbulent kinetic energy profile for channel flow, $Re_\tau = 395$.

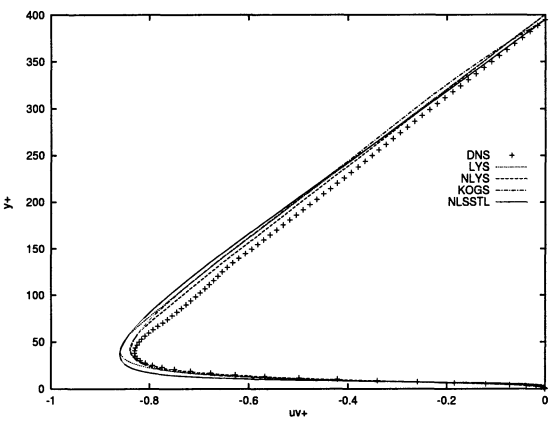

Figure 2. Turbulent shear stress profile for channel flow, $Re_\tau = 395$.

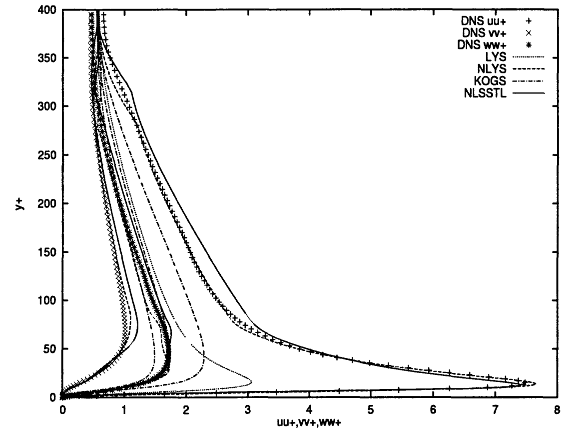

Figure 3. Normal Reynolds-stresses profile for channel flow, $Re_\tau = 395$.

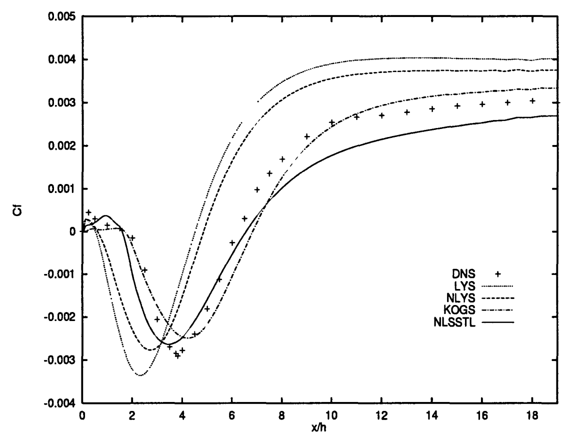

Figure 4. Step-wall skin friction coefficient for BFS, $Re_h = 5100$.

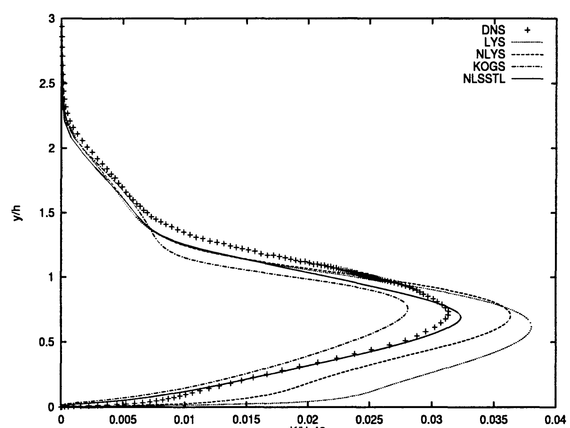

Figure 5. Turbulent kinetic energy profile for BFS, $Re_h = 5100$, at streamwise location X/H=4.

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