REYNOLDS NUMBER DEPENDENCE AND INVARIANT ASSUMPTION IN TURBULENT BOUNDARY LAYER

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ABSTRUCT

The logarithmic velocity region is considered in zero-pressure-gradient turbulent boundary layers. We propose a new definition of log-law region with using the probability profiles of streamwise velocity fluctuation. The measure called Kullback Leibler divergence is applied to distinguishing the probability profiles. The ratio of boundary layer thickness,δ, and the upper end of logarithmic region,δ_L, is investigated, and also the Reynolds number dependence of pdf profile is mentioned.

INTRODUCTION

The logarithmic velocity region is considered in zero-pressure-gradient turbulent boundary layer. One of the recent interesting topics is the mean velocity distribution in the overlap region in wall-bounded shear flows.(1) We have believed so far the log-law velocity profile as firmly established result in turbulence research, however, some researches cast doubts about its existence. At present, we have no answer to the universal scaling form in the wall bounded shear flows, but even if the log law; $U^+ = A \cdot \log y^+ + B$, is a good representation of the experimental data, we still have several questions. Is the slope A universal constant? Additive constant B is independent of the Reynolds number, isn't it? The researchers use the different values to fit their experimental data. We assume, these disagreements come from the indistinct definition of the log-law region. It is not clear how far the log-law region extends from the wall. The outer edge of the logarithmic region, δ_L, is scaled by the boundary layer thickness δ, and the ratio δ_L/δ is reported to be constant; $\delta_L/\delta \simeq 0.2$.(2) On the other hand, Purtel et al. suggested this value is a decreasing function of the Reynolds number.(3) So the purpose of this paper is to give the definition of the log-law region and then investigate the Reynolds number dependence of A, B, and δ_L/δ. And also the pdf profile in the log-law region is discussed.

EXPERIMENTAL CONDITION

In a wind tunnel with a test section $0.32 \times 1.06m$ in area and $2.6m$ in length, a typical two-dimensional turbulent boundary layer is generated. The data are measured at $1900mm$ downstream from the leading edge with using I-type probe, of which the sensitive region is made of tungsten wire whose diameter is 2.5, 3.1, $5.0\mu m$ and $0.5mm$ in length. The probe is operated by a constant temperature anemometer, and the velocity is sampled during $30sec$ by 12-bit A/D converter at 10kHZ. The shape parameter and the wake factor in this experimental condition are plotted as a function of the Reynolds number in Fig. 1.

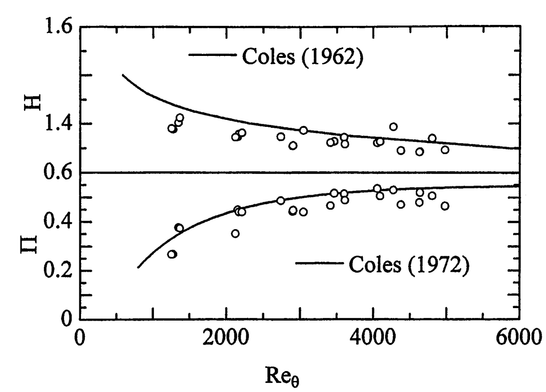

Fig. 1 The shape parameter and the wake factor in this experimental condition are plotted as a function of the Reynolds number.

THE PROCEDURE TO EXTRACT THE LOG-LAW REGION

The extent of the logarithmic region is classically considered as an equilibrium region where the total shear stress is constant.[4] But the "equilibrium" is interpreted here as the condition that the pdf profile of the normalized streamwise velocity component does not change. That is, the pdf profile remains unique in the log-law region. We call this idea the invariant assumption of pdf profile.

When the instantaneous velocity in streamwise component is decomposed into mean and fluctuation as $\tilde{u} = U + u'$, we think about the pdf of normalized velocity; $u = u'/u_r$, where u_r is r.m.s. value of u'. If the invariant region of the pdf profile exist, we regard this as the log-law region.[5] In confirming this assumption, we used the pdf-equation approximated near the wall region.

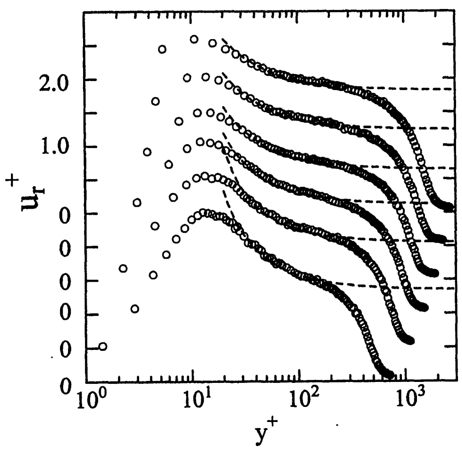

Fig. 2 The turbulent intensity normalized by the inner variable is interpolated by Eq. (4) for several Reynolds numbers, R_θ = 1270, 2205, 2890, 3590, 4290, 4700.

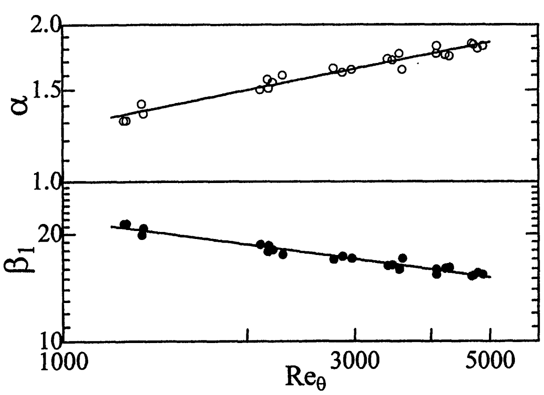

Fig. 3 The coefficients α and β_1 are plotted as a function of Reynolds number. The solid lines are $\alpha = 0.324 \cdot R_\theta^{0.21}$ and $\beta_1 = 86.32 \cdot R_\theta^{-0.21}$.

The pdf is the expected value of the Dirac's delta function in this formula, and it is taken the starting point;

$$P_f(u_p) \equiv \delta(u - u_p) \quad , \quad u = u'/u_r \, , \tag{1}$$

is defined as the fine grained pdf, where u is a normalized random variable and u_p is a sample variable.[6] And then the pdf is defined as

$$f(u_p) \equiv \langle P_f(u_p) \rangle \, , \tag{2}$$

where $\langle \quad \rangle$ denotes an ensemble average. So the invariant assumption is,

$$\partial f(u_p)/\partial x = 0 \quad , \quad \partial f(u_p)/\partial y = 0 \, , \tag{3}$$

in log-law region. We have derived the log-law profile from the pdf equation at close to the wall subject to Eq. (3).[5] The detailed explanation is omitted here, but we adopted the following rational expansion to interpolate the turbulence intensity distribution in the log-law region.

$$u_r^+ = \alpha + \beta_1 (y^+ - \gamma)^{-1} + \beta_2 (y^+ - \gamma)^{-2} + \cdots \, . \tag{4}$$

The coefficient α means $\alpha = \lim_{y^+ \to \infty} u_r^+$ and γ is the outer edge of the buffer layer. The slope of the logarithmic profile is derived as

$$A = C \alpha \beta_1 \, , \tag{5}$$

where C is constant; $C \simeq 0.2$. We make sure whether Eq. (4) can predict the experimental data, and the result is plotted in Fig. 2. In the logarithmic region Eq. (4) can well interpolate the turbulence intensity in several Reynolds numbers. The coefficients, α, β_1, are plotted in Fig. 3 in which they are a function of the Reynolds number. The coefficient α is scaled like $\alpha = 0.324 \cdot R_\theta^{0.21}$ and $\beta_1 = 86.32 \cdot R_\theta^{-0.21}$. Therefore, within the experimental accuracy, the product of α and β_1 is constant independent of the Reynolds number, that is, $\alpha\beta_1 \simeq 28.0$. Then from the Eq. (5), A is constant independent of Reynolds number. About the additive constant B, we are sure from the theoretical procedure that it depends on the Reynolds number.

Naturally in experimental data analysis, the invariant assumption of pdfs must be a little relaxed. We extract the region where the pdf has a "similar" profile but not the "same " one. The Kullback Leibler divergence (KLD) [7] is used to distinguish the pdf's profile, which is defined as,

$$D(P \| Q) \equiv \sum_{\{\mathbf{s}\}} P(s_i) \log_e \left(P(s_i)/Q(s_i) \right) \, , \tag{6}$$

where $P(\mathbf{s})$ and $Q(\mathbf{s})$, $\{\mathbf{s}\} = \{s_1, s_2, \cdots\}$, are discrete probability distributions. KLD has a non-negative value for any $P(\mathbf{s})$ and $Q(\mathbf{s})$, and it is zero only when $P(\mathbf{s})$ is the same with $Q(\mathbf{s})$. As KLD has a smaller value, then $P(\mathbf{s})$ and $Q(\mathbf{s})$ are more similar. That is, it is a indicator to evaluate how much $P(\mathbf{s})$ resembles $Q(\mathbf{s})$.

RESULTS AND DISCUSSION

We used KLD and extract the log-law region subject to the invariant assumption of pdfs. The result is shown in Fig.4. The solid symbols indicate the log-law region defined by the invariant assumption of pdfs. The outer edge of the logarithmic region, δ_L, is a function of Reynolds number, then it is plotted in Fig. 5. The ratio δ_L/δ is a decreasing function of R_θ and it approaches to 0.2. And also the additive constant B is shown in the graph. This value also shows slight but evident dependence on Reynolds number.

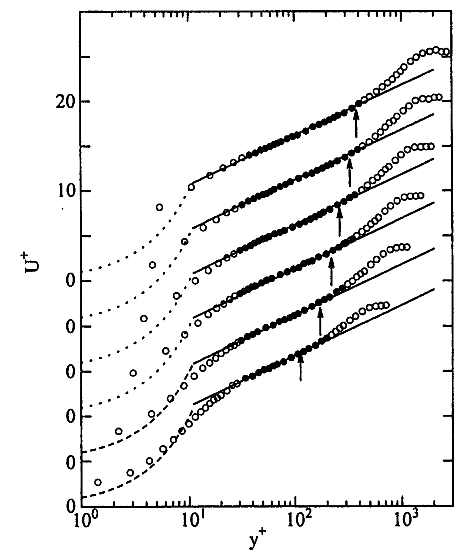

Fig. 4 Mean velocity distribution for several Reynolds number flows, R_θ = 1270, 2205, 2890, 3590, 4290, 4700, in which the log-law region is indicated by solid symbols. The arrows indicate the position where $\delta_L/\delta = 0.2$.

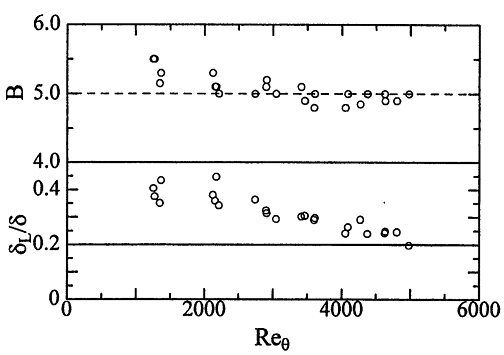

Fig. 5 Reynolds number dependence of δ_L and B.

The pdf profile in the log-law region is investigated more carefully. A simple way is to compare pdfs with Gaussian distribution. Figure 6 shows the KLD computed at each position from the wall adopting the velocity fluctuation probability and the Gaussian profile. The KLD has a minimum value within the log-law region, which is smaller for the higher Reynolds number, and also the peak position sensitively depends on the Reynolds number. That is, the pdf profile is not independent of the Reynolds number but suggests approaching the Gaussian distribution in the inviscid limit. Also this figure shows the invariant assumption is appropriate nature, but in this limit, the invariant assumption will be satisfied exactly.

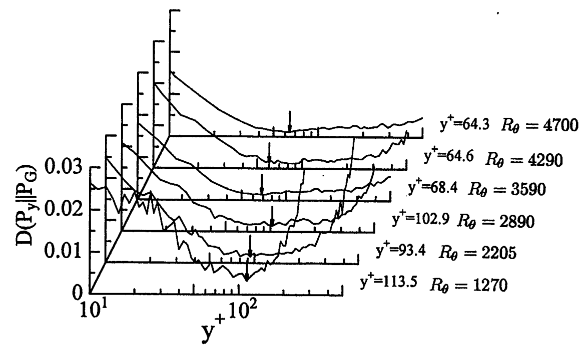

Fig. 6 Kullback Leibler divergence distribution computed by the experimental data and the Gaussian distribution.

CONCLUSIONS

In zero-pressure gradient boundary layers, we present the definition of the log-law region as the extent where the pdf profile of normalized u-component velocity fluctuation remains unique. Although the pdf profile depends delicately on the Reynolds number, the log-law region is well extracted by the invariant assumption of pdfs (exactly speaking, this is the relaxed invariant assumption). If the log-law is a good representation of the experimental data ($R_\theta < 5000$), our results are that A is a universal constant, but B depends on the Reynolds number. The ratio of the outer edge of logarithmic region and the boundary layer thickness, δ_L/δ, is a decreasing function of R_θ.

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