COMPARISON BETWEEN EULERIAN AND LAGRANGIAN STRATEGIES FOR THE DISPERSED PHASE IN NONUNIFORM TURBULENT PARTICLE-LADEN FLOWS

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ABSTRACT

For gas - solid two-phase flows as well as for gas - liquid systems the correct prediction of fluid and particle properties is of major relevance. Form the experimental point of view the measurement techniques to quantify the relevant parameters of the underlying fluid flow (single phase) can be regarded as highly reliable. The task is more difficult when trying to get reasonable data for the characterization of the relevant parameters of the dispersed phase within a turbulent two-phase flow. Especially the determination of the turbulence quantities of the dispersed phase (normal and shear stresses) has been studied within only a few projects in the past. In this study the experimental investigations of Mostafa et al. (1989) were taken as a basis to compare the Euler/Euler- and the Euler/Lagrange approach with respect to the predictions of the turbulence properties of the dispersed phase. In these experiments the configuration of turbulent axisymmetric particle-laden gas jets is considered, which provides the required measured data concerning the turbulent properties of the solid phase.

INTRODUCTION

Nowadays, two approaches are mainly used to describe the dispersed phase in a two-phase flow (solid, droplet or bubble suspensions). In the so called *Lagrangian* method the discrete elements are tracked through a random fluid field by solving their equations of motion. In the second methodology, both phases are handled as two interpenetrating continuums and are governed by a set of differential equations representing conservation laws; this approach is named as *Eulerian.* In this last context, for establishing the dispersed elements equations, two possibilities come out. First, the second phase is considered as a fluid for all effects. This corresponds to the well-known two-fluid model. Second, the non-continuous phase is thought of as a cloud of material elements, whose behaviour is driven by a probability density function (PDF), depending on each element variable, that responds to a kinetic transport equation similar to the Maxwell-Boltzmann one. The continuum equation for the second phase is obtained as the statistical moments of such PDF-evolution equation.

In spite of the lack of a complete agreement about the final form of the equations and the constitutive relations used for the dispersed phase, the Eulerian strategies continue to be attractive from an engineering point of view because of their simplicity and computational efficiency.

However, the traditional closures, even giving approximated values for the mean fields, fail in the predictions of particle turbulent quantities specially in nonuniform flows. To overcome this fact considerable effort has been devoted during the last years to develop turbulence closures at the level of second moments of the particulate phase (Reeks (1993), Wang et al. (1997), Hyland et al. (1998), Février & Simonin (1998)), but those are still in the research stage. As a matter of fact, the traditional Lagrangian strategies for modelling the particle phase, grossly underpredict the streamwise velocity fluctuation of the particles in nonuniform flows (i.e. jets). On the other hand, the classical Eulerian methods make no attempt to predict the particle Reynolds stress tensor.

In previous works (Laín (1997), Laín & Aliod (1999)), it was suggested that in axisymmetric jet flows laden with high inertia particles, the streamwise and transversal velocity fluctuations of the solid phase could keep the approximate relation, $u'^2/v'^2 \approx 10$ as soon as a fully developed flow is reached (here u'^2 corresponds to the axial normal stress and v'^2 to the radial one). Such a ratio is much higher than the homologous one for the continuous phase, of order one. The purpose of this paper is to propose a simple, computationally efficient, disperse phase second order model, that able to approximately predict the enhacing of the streamwise normal stress with respect to the tangential and radial components.

The computational results are compared with the measurements of Mostafa et al. (1989) giving a reasonable agreement in all availables quantities.

LAGRANGIAN TRACKING ALGORITHM

The dispersed phase has been regarded within the Lagrangian framework on the one side. There the single particle is observed on its way through the flow field by solving the equations of the particle motion and the particle position. Performing an order of magnitude analysis of the relevant time scales of the flow system results in a reduced form of the particle equation of motion neglecting the virtual mass and the basset force. Finally, the following system of equations has to be solved:

$$x_{i,t} = v_i \quad (1)$$

$$v_{i,t} = \frac{3\rho C_D\,(u_i - v_i)\mid u_i - v_i \mid}{4\rho^d d_p} + g_i \cdot \left(1 - \frac{\rho}{\rho^d}\right) \quad (2)$$

The drag coefficient is given as follows:

$$C_D = \frac{24.0}{Re_p}[1.0 + \frac{1}{6}Re_p^{0.66}] \quad Re_p < 1000 \quad (3)$$
$$C_D = 0.44 \quad Re_p \geq 1000$$

In the above equations (Eq. 1 - 3) the superscript d is used to identify a single particle. The instantaneous fluid velocity along the particle trajectory was obtained by applying the Langevin equation model. C_D is the drag coefficient (for details, see Sommerfeld et al. (1993)).

In order to obtain the relevant information about the underlying flow field the continuous phase has to be solved as well. Therefore, the time-averaged Navier-Stokes equations were solved in connection with an appropriate turbulence model. These equations include particle source terms in order to take into account the effect of two-way coupling. These source terms were calculated using a modified version of the Particle Source in Cell - approximation of Crowe et al. (1977). A detailed description of this procedure is given in Kohnen et al. (1994).

DISPERSED PHASE TRANSPORT EQUATIONS

Using the Dispersed Elements PDF-Indicator Function ensemble conditioned average (Aliod & Dopazo (1990), Prosperetti & Zhang (1994)), the following equations for the dispersed phase, in the context of isothermal dilute flows (Laín (1997)), are initially considered:

Mass conservation equation:

$$\left[\rho^d \alpha^d\right]_{,t} + \left[\rho^d \alpha^d V_i\right]_{,i} = 0 \quad (4)$$

Momentum conservation equation:

$$\left[\rho^d \alpha^d V_j\right]_{,t} + \left[\rho^d \alpha^d V_i V_j\right]_{,i} = \left[-\alpha^d \rho^d \overline{v'_j v'_i}^d\right]_{,i} + I_j^D + f_j^{dV} \quad (5)$$

Fluctuating kinetic energy equation $k^d = \overline{k'^d}^d = \frac{1}{2}\overline{v'_i v'_i}^d$:

$$\left[\rho^d \alpha^d k^d\right]_{,t} + \left[\rho^d \alpha^d V_j k^d\right]_{,j} = \left[-\alpha^d \rho^d \overline{k'^d v'_j}^d\right]_{,j} + \alpha^d \mathcal{P}^d + I^W \quad (6)$$

Here, V, α^d, Æ are the ensemble averaged dispersed elements velocity, volume fraction and fluctuating velocity with respect to V. ρ^d is the density of the discrete elements, which is supposed to be constant. I_j^D is the standard interaction term due to the aerodynamic drag, the volumetric forces f_j^{dV} take into account the weight and the buoyancy. $\mathcal{P}^d$ is the standard production term also found in single phase flows and I^W is the fluctuating work exchanged with the fluid.

Now it is worth to recall the closure used for I^W:

$$I^W = C_D \alpha^d \rho^d \left(k\theta - k^d\right); \qquad \theta = \frac{\tau_L}{\tau_L + C_D^{-1}} \qquad \tau_L = 0.4\tfrac{k}{\epsilon} \quad (7)$$

where C_D is again the coefficient of the standard drag law for monodispersed spherical particles of diameter d_p, as it is already defined in eq. 3. It should be stressed, that the particle Reynolds number Re_p, appearing in this equation, is based on the relative velocity between both phases and μ is the fluid viscosity. The inverse of C_D is just the response time of the particle, τ_p.

The performance of the closure (7) in axisymmetric jets laden with high inertia particles has been assessed in Laín (1997), where a pretty accurate prediction of k^d

is obtained for a set of experiments in such a configuration.

The transport equation governing the particle velocity correlations can be found in different works, for example Simonin (1991). For dilute flows, neglecting collisions between the discrete elements, it can be written:

$$\frac{D\left(\rho^d\alpha^d\overline{v_i'v_j'}^d\right)}{Dt} - \mathcal{D}_{v'v',ij} = \alpha^d\mathcal{P}_{ij}^d + I_{ij}^W \qquad (8)$$

where $\mathcal{D}_{v'v',ij}$ represent the transport by particle velocity fluctuations of the stresses, $\mathcal{P}_{ij}^d$ is the production contribution (which does not need to be positive defined):

$$\mathcal{P}_{ij}^d = -\rho^d\left(\overline{v_i'v_m'}^d V_{j,m} + \overline{v_j'v_m'}^d V_{i,m}\right) \qquad (9)$$

and I_{ij}^W is the exchanged work rate between the dispersed phase and the fluid which is expressed as:

$$I_{ij}^W = \alpha^d\rho^d C_D\left(-2\overline{v_i'v_j'}^d + \overline{u_i'v_j'}^d + \overline{u_j'v_i'}^d\right) \qquad (10)$$

u' states for the fluid fluctuating velocity with respect to the ensemble averaged value $\mathrm{U} = \overline{\mathrm{u}}$.

Following the theoretical work of Reeks (1993), in the limit of large inertial particles in simple shear flows, the Boussinesq-Prandtl hypothesis is feasible for modelling the particle shear stresses. They are split up in an homogeneous component, whose structure is the same as if the local carrier flow would be homogeneous, and a deviatoric component involving terms proportional to the mean shear of both, the dispersed and carrier flows. However, for long particle response times, the deviatoric component dominates over the homogeneous contribution reaching a finite value of $-\frac{1}{2}\epsilon_\infty S^d$, where ϵ_∞ is the long-time particle diffusion coefficient in the *transverse direction* and S^d the shear gradient of the dispersed phase. In addition, in this limit the diffusivity momentum coefficient, μ^d, is said to be proportional to ϵ_∞. Therefore, despite the fact, that the diffusivity momentum should be a tensor, μ^d can be written as an scalar quantity in the limit. In this context, the following expresion for the particle shear stresses can be written:

$$-\rho^d\overline{v_x'v_r'}^d = \mu^d\left[V_{x,r} + V_{r,z}\right] \qquad (11)$$

with $\mu^d \propto \rho^d\overline{v_r'v_r'}^d C_D^{-1}$ and (x,r) denote the axial and transversal coordinate, respectively. The closure outlined above, eq. 11, will be taken as a basis for the following cases considered in the present work.

ALGEBRAIC PARTICLE STRESS MODEL (APSM)

As the simplest approximation, an algebraic model formulation for the particle normal stresses can be proposed extending the ideas of the Algebraic Stress Model (ASM) developed by Rodi (1972) for single phase flow. It is assumed, that the sum of convection and diffusion terms of the Reynolds stresses, $\overline{v_i'v_i'}^d$ (the sum is not understood on the repeated index i), is proportional to the sum of the convection and diffusion terms of the turbulent kinetic energy k^d.

$$\begin{aligned}
&\frac{D\left(\rho^d\alpha^d\overline{v_i'v_i'}^d\right)}{Dt} - \mathcal{D}_{v'v',ii} \\
&\approx \frac{\overline{v_i'v_i'}^d}{k^d}\left(\frac{D\left(\rho^d\alpha^d k^d\right)}{Dt} - \mathcal{D}_{k^d}\right) \\
&= \frac{\overline{v_i'v_i'}^d}{k^d}\left(\alpha^d\mathcal{P}^d + I^W\right) \qquad (12)
\end{aligned}$$

where $\mathcal{D}_{k^d}$ represent the transport by particle velocity fluctuations of k^d. Introducing the approximate balance for the normal shear stresses (into the equations 6 and 8) leads to:

$$\overline{v_i'v_i'}^d \approx \frac{k^d}{\alpha^d\mathcal{P}^d + I^W}\left(\alpha^d\mathcal{P}_{ii}^d + I_{ii}^W\right) \qquad (13)$$

Here, the only non-closed terms appear in the fluid-particle correlation included in I_{ii}^W (eq.10). Février & Simonin (1998) have worked out several methods for handling this fluid-particle correlations, deriving algebraic as well as differential equations for them. Unfortunately, the required CPU time increases fast as the number of equations increases.
The approach proposed in this work is simpler. A relationship between the fluid-particle correlation, $\overline{u_i'v_i'}^d$, and the fluid and particle stresses is assumed in the following way:

$$\overline{u_i'v_i'}^d = \frac{1}{2}\left(\overline{v_i'v_i'}^d + \overline{u_i'u_i'}\,\theta_{ii}\right) \qquad (14)$$

where the tensor θ_{ij} is written as:

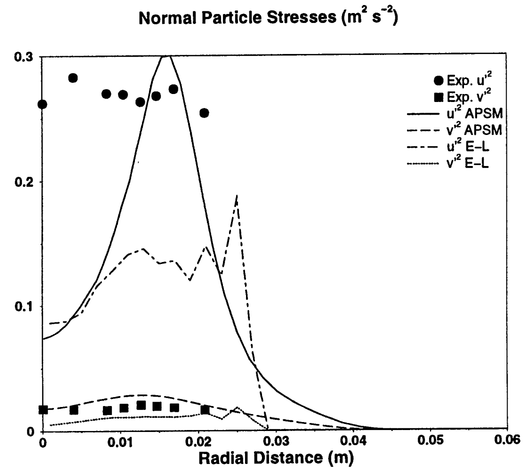
$$\theta_{ij} = \frac{C_D\tau_{L_{ij}}}{1 + C_D\tau_{L_{ij}}}; \qquad \tau_{L_{ij}} = C_L\frac{\overline{u_i'u_j'}}{\varepsilon} \qquad (15)$$

with $C_L = 0.4$. Eq. 15 can be seen as a natural extension of eq. 7. Inserting eq. 15 into eq. 10 yields the following expression for I_{ii}^W:

$$I_{ii}^W = \alpha^d\rho^d C_D\left(\overline{u_i'u_i'}\,\theta_{ii} - \overline{v_i'v_i'}^d\right) \qquad (16)$$

Equations 11, 13, and 16 constitute a system of three equations for the particle normal stresses, that can be solved, if expressions for the fluid stresses and k^d are provided.

The results are shown in the Figure 1. There, the particle normal stresses, the axial (u'^2) and radial components (v'^2), are shown in comparison with the experiments of Mostafa et al. (1989) and the output of



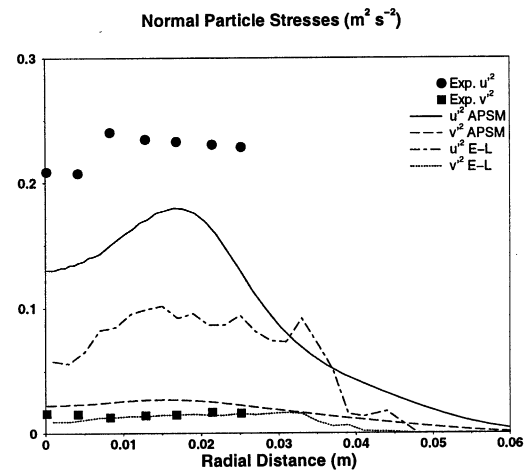


Figure 1. Normal particle Reynolds stresses for the experiments of Mostafa et al. (1989) in two transversal sections: $X/D = 6.2$ (left) and $X/D = 12.45$ (right) versus the output of the APSM and Euler-Lagrange approach (E-L). u'^2 corresponds to the axial streamwise direction and v'^2 to the transversal direction.

the classical Euler-Lagrange approach for two sections, $X/D = 6.2$ and $X/D = 12.45$. X denotes the distance downstream the nozzle and D its diameter. It is necessary to point out that the Euler-Lagrange calculations provide good enough values for all the fluid variables, including the Reynolds stresses and the mean velocities of the particles, but they underpredict considerably their axial fluctuating component. This is a typical situation, that also appears in the classical two-fluid model (cf. Issa & Oliveira (1998)).

While the results for the transversal direction are similar in both strategies of calculus (even the Euler-Lagrange method seems to work a little bit better), the situation is different for the streamwise component. In both cases, it is underpredicted, especially in the symmetry axis, but the APSM version is noticeably closer to the experiments.

Despite the fact, that the performance of APSM is not good enough, the improvements achieved are encouraging to think, that a simplified Reynolds Stress Particle Model can enhance even more the quality of the predictions. This task will be carried out in the next section.

REYNOLDS STRESS PARTICLE MODEL (RSPM)

The model proposed is based on the set of equations 8 including the definition of eq. 9 and the closure of eq. 16. Moreover, as already mentioned, the closure defined by eq. 11 for the shear stresses will be assumed in the context of non-uniform, strongly anisotropic flows laden with high inertia particles.

The term representing the transport by particle velocity fluctuations in eq. 8 is closed by, for practical purposes, using a Boussinesq approximation:

$$\mathcal{D}_{v'v',ij} = \left[\alpha^d \frac{\mu^d}{\sigma^d_{ij}} \left[\overline{v'_i v'_j}^d \right]_{,k} \right]_{,k} \tag{17}$$

This includes an implicit summation in k. σ^d_{ij} are the turbulent Schmidt numbers. In this case we only need to consider $i = j$. The values chosen here have been $\sigma^d_{xx} = 0.3$ and $\sigma^d_{rr} = \sigma^d_{ww} = 1.0$ (w is the azimuthal direction). The selection of σ^d_{xx} was suggested for the value used in the k^d equation (Laín (1997)), while for the others the simplest value is assigned since the performance of the RSPM does not depend appreciably on them.

In summary, the proposed RSPM consists of a system of three equations (Eq. 8 with $i = j$), one definition for the production contribution, eq. 9 and the closure approximations, eqn. 11, 16 and 17.

The comparisons with the experiments of Mostafa et al. (1989) are shown in Fig. 2 for the two transversal sections $X/D = 6.2, 12.45$. There, the calculated profiles for the three particle normal stresses and the kinetic energy calculated from them are plotted versus the experimental data. It is remarkable, that the anisotropy of the stresses is reasonably well captured, in spite of the simplicity of the model, which only presents three extra equations with respect to the standard models.

Figure 3 shows the different contributions of eq. 8 in the section $X/D = 12.45$, for $u'^2 = \overline{v'_x v'_x}^d$, $v'^2 = \overline{v'_r v'_r}^d$ and $w'^2 = \overline{v'_w v'_w}^d$: convection, diffusion, production and interaction. Their arrangement is:

$$Convection = Diffusion + Production + Interaction \tag{18}$$

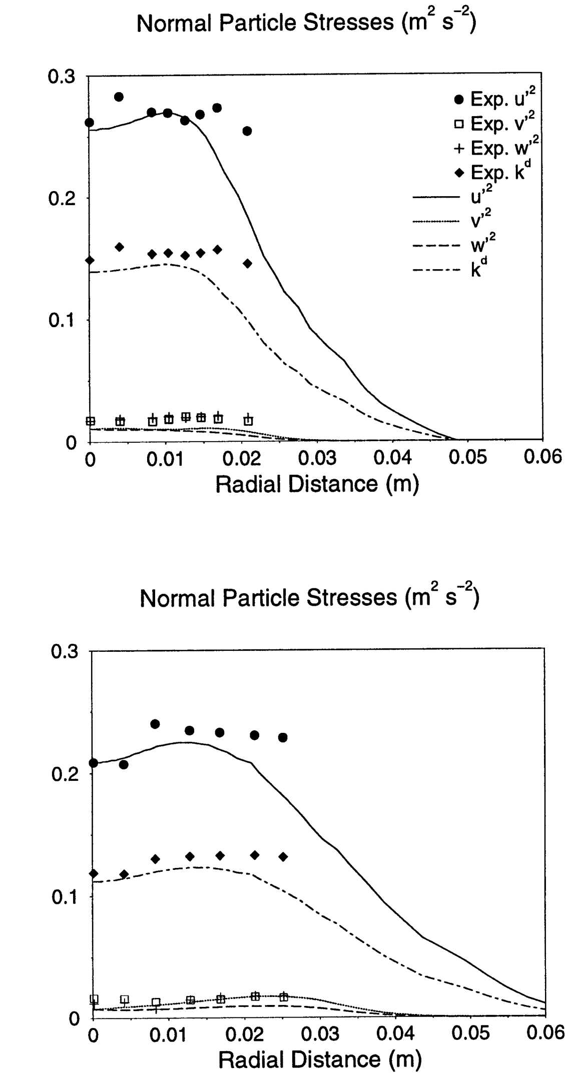


Figure 2. Normal particle Reynolds stresses for the experiments of Mostafa et al. (1989) in two transversal sections: $X/D = 6.2$ (left) and $X/D = 12.45$ (right) versus the output of the RSPM. In addition to Figure 1 w'^2 corresponds to the azimuthal direction and $k^d = 0.5/(u'^2 + v'^2 + w'^2)$.

Besides, the residue of adding all this contributions is shown in these Figures. The axial stress reveals a principal equilibrium between the convection and interaction terms, modulated by the diffusion and production. This fact is very similar to the result obtained for the k^d equation (Laín (1997), Laín & Aliod (1999)), which is not surprising, because in such an anisotropic configuration u'^2 is mainly responsible for the development of k^d. It is worth to note, that the shape of the interaction term is very similar to that obtained by Wang et al. (1997) in a channel flow using a more

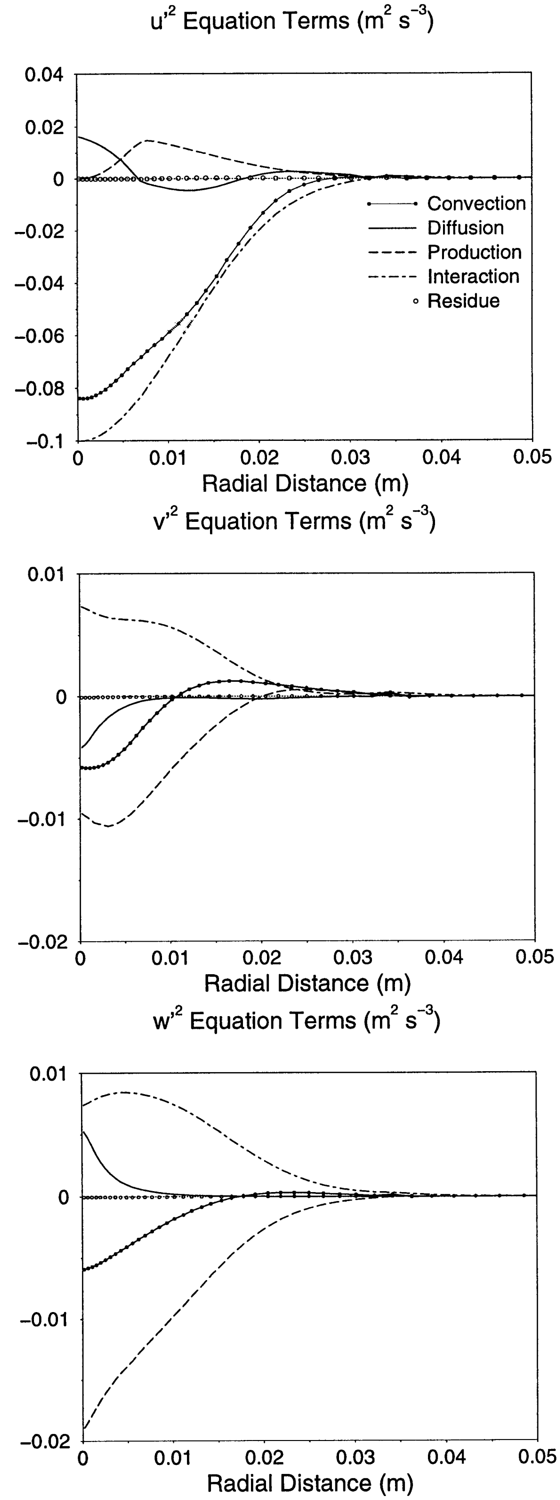


Figure 3. Snapshots of the terms present in the equations for the normal stresses of the dispersed phase in the axial station $X/D = 12.45$ of the experiments of Mostafa et al. (1989).

comprehensive formulation in connection with LES.

In the case of v'^2 and w'^2 the four contributions are of the same order. It should be noted, however, that the production terms are negative and the interaction terms positive. This can be interpreted in the case of v'^2 as follows: The interaction of particles with the fluid, I^W_{rr}, provides the necessary energy for the expansion of the dispersed phase in the jet along the radial direction, represented by the interaction between the own radial stresses and $V_{r,r}$, which is consistent with the results of Laín (1997).

CONCLUSIONS

In the present study we compared different Eulerian-Eulerian strategies with an Eulerian-Lagrangian strategy with respect to their applicability to predict the particle normal and shear stresses. Concerning the Eulerian-Eulerian method a simple model was developed in order to describe the anisotropy of the aforementioned turbulent quantities and applied to a jet flow. It was demonstrated, that the APSM was not able to predict this anisotropy in a satisfactory way. Only the more advanced RSPM shows a reasonably good agreement with the underlying experimental data. Even the Eulerian - Lagrangian method reveals some difficulties in the prediction of these second order terms for the particles. Some work has to be done in order to solve this problem. In addition, resolving the individual terms in the transport equation for the particle velocity correlations indicates, that the production term of the axial component is positive and the other two components are negative. This could be explained by an energy transfer among those components. In the future, a closure for the fluid particle correlation obtained by PDF will be included applicable for inhomogeneous flows as well. Moreover, the model has to be validated against additional experimental configurations.

REFERENCES

Aliod, R. and Dopazo, C., 1990, "A statistically conditioned averaging formalism for deriving two-phase flow equations", *Part. Part. Syst. Charact.*, Vol. 7, pp. 481-494.

Février, P. and Simonin, O.,1998, "Constitutive Relations for Fluid-Particle Velocity Correlations in Gas-Solid Turbulent Flows", *Proc. 3rd Int. Conf. Multiphase Flow*, Paper 538, Lyon (France).

Hyland, K. E., Simonin, O., and Reeks, M.W., 1998, "On the Continuum Equations for Two Phase Flows", *Proc. 3rd Int. Conf. Multiphase Flow*, Paper 567, Lyon (France).

Issa, R.I. and Oliveira, P.J., 1998, "Accounting for non-equilibrium turbulent fluctuations in the Eulerian two-fluid model by means of the notion of induction period". *Proc. 3rd Int. Conf. Multiphase Flow*, Paper 307, Lyon (France).

Kohnen, G., Rüger, M. and Sommerfeld, M., 1994, "Convergence behaviour for numerical calculations by the Euler/Lagrange method for strongly coupled phases", *Num. Meth. for Multiphase Flows*, FED Vol. 185, Editors: (Crowe et al.), pp. 191-202.

Laín, S., 1997, "Nuevo Modelo Euleriano-Euleriano para Flujos Bifásicos Turbulentos Dispersos", *Ph.D. thesis*, University of Zaragoza, Spain.

Laín, S. and Aliod, R., 1999, "Analysis and discussion on the Eulerian dispersed particle equations in non-uniform turbulent gas-solid two - phase flows", to appear in *Proc. 4th Int. Symp. on Engineering Turbulence Modelling and Measurements*, Corsica (France).

Mostafa, A.A., Mongia, H.C., McDonell, V.G. and Samuelsen, G.S., 1989, "Evolution of particle laden jet flows: A theoretical and experimental study", *AIAA J.*, Vol. 27, pp. 167-183.

Prosperetti, A. and Zhang, D.Z., 1994, "Averaged equations for inviscid disperse two-phase flow", *J. Fluid Mech.*, Vol. 267, pp. 185-219.

Reeks, M.W., 1993, "On the constitutive relations for dispersed particles in nonuniform flows I: Dispersion in a simple shear flow", *Phys. Fluids*, A 5 (3), pp. 750-761.

Rodi, W., 1972, "The prediction of free turbulent layers by use of a two-equation model of turbulence". *Ph.D. thesis*, Imperial College, UK.

Simonin, O., 1991, "Prediction of the dispersed phase turbulence in particulate laden jet". *Proc. Fourth Int. Symp. on Gas-Solid Flows*, ASME Fed, Vol. 121.

Sommerfeld, M., Kohnen, G., and Rüger, M., 1993, "Some open questions and inconsistencies of Lagrangian particle dispersion models", *Proc. Ninth Symp. on Turbulent Shear Flows*, Kyoto, Japan, Paper 5.1.

Wang, Q., Squires, K.D. and Simonin, O., 1997, "Large eddy simulation of turbulent gas-solid flows in a vertical channel and evaluation of second-order models", *Proc. 11th Symp. TSF*, pp. 27-18 – 27-23, Grenoble (France).