A KINETIC MODEL FOR THE TRANSPORT OF ARBITRARY-DENSITY PARTICLES IN TURBULENT SHEAR FLOWS

Leonid I. Zaichik
Laboratory of Mathematical Modelling
Institute for High Temperatures of the Russian Academy of Sciences
Moscow 111250, Russia

Vladimir M. Alipchenkov
Department of Mechanical Engineering
Electrogorsk Research and Engineering Center of Nuclear Plants Safety
Electrogorsk, MR 142530, Russia

ABSTRACT
The purpose of this paper is to present a statistical model based on a kinetic PDF equation for predicting transport of arbitrary-density particles in two-phase turbulent shear flow. The model developed is used to simulate the motion of particles and bubbles in both homogeneous sheared and pipe flows.

INTRODUCTION
Much progress in the theory of two-phase turbulent flows which has been achieved up to now relates mainly to gas-solid flows. Such flows are characterized by very large values of the ratio between the densities of the dispersed particulate and continuous fluid phases. The theory of two-phase turbulent flows at small ratio of the particle density to the fluid one is developed to a less extent. This fact is connected with two major causes. The first is of necessity take into consideration the instantaneous forces acting on a particle by the surrounding fluid flow field to predict the particle-turbulence interaction, because turbulence controls particle motion in this case not only by means of the fluctuating drag force, whereas those are negligible in two-phase gas-solid flows. The second cause can be attributed to effects induced by interface deformation and bubble compression in gas-liquid flows. In this paper, the influence of instantaneous forces on interactions between particles and turbulent fluid eddies is only considered. The particles (bubbles) are assumed to retain their form, and the 'back-effect' of particles on fluid turbulence is not allowed for.

As an efficient approach to describe the transport of particles in the Eulerian continuum modelling manner, introducing the probability density function (PDF) of the particle velocity distribution can be regarded. The statistical method based on the PDF is a more consecutive and accurate way to generate the continuum conservation equations for the dispersed phase in comparison with the traditional deterministic method since it makes possible to avoid the usage of any additional heuristic assumptions to close the governing transport equations. Kinetic equations for the PDF of particle velocity were deduced by Derevich and Zaichik (1988), Reeks (1991, 1992), and Zaichik (1997). In those papers, the only drag force of all interfacial forces acting on a particle was allowed for, and, consequently, the kinetic equations obtained there are valid mainly in the case of motion of heavy particles in the gas. In the present paper, equations for the PDF and its moments, which describe particle-turbulence interactions with accounting for instantaneous forces and therefore retain validity within full range of values of the ratio between the particle and fluid densities (from heavy particles in the gas to bubbles in the liquid), are proposed.

KINETIC EQUATION FOR THE PDF
The motion of a single spherical particle in a fluid turbulent flow field is given by the following equations

$$\frac{d\mathbf{R}_p}{d\tau} = \mathbf{v}_p, \quad \frac{d\mathbf{v}_p}{d\tau} = \frac{\mathbf{u}-\mathbf{v}_p}{\tau_{u*}} + \mathbf{f}_A + \mathbf{f}_B + \mathbf{f}_L + \mathbf{F}_g \qquad (1)$$

$$\mathbf{f}_A = A\frac{D\mathbf{u}}{D\tau}, \quad \mathbf{f}_B = k_B\int_0^{\tau}\frac{d(\mathbf{u}-\mathbf{v}_p)}{d\tau_1}\frac{d\tau_1}{\sqrt{\tau-\tau_1}}, \quad \mathbf{f}_L = L(\mathbf{u}-\mathbf{v}_p)\times \mathrm{rot}\,\mathbf{u}$$

$$\mathbf{F}_g = \frac{(1-\rho_f/\rho_p)}{1+C_A\rho_f/\rho_p}\mathbf{g}, \quad \tau_{u*} = \tau_u\left(1+C_A\frac{\rho_f}{\rho_p}\right), \quad \tau_u = \frac{4\rho_p d_p}{3\rho_f C_D|\mathbf{u}-\mathbf{v}_p|}$$

$$A = \frac{(1+C_A)\rho_f/\rho_f}{1+C_A\rho_f/\rho_f}, \quad k_B = \frac{9\rho_f}{\rho_p d_p}\sqrt{\frac{\nu}{\pi}}\left(1+C_A\frac{\rho_f}{\rho_p}\right)^{-1}, \quad L = \frac{C_L\rho_f/\rho_p}{1+C_A\rho_f/\rho_p}$$

where $\mathbf{R}_p$ and $\mathbf{v}_p$ are the Lagrangian position and the velocity of the particle, $\mathbf{u}$ is the fluid velocity, ρ_f and ρ_p are the fluid and particle densities, d_p is the particle diameter, ν is the fluid viscosity, and τ_u is the particle relaxation time. The second equation in (1) describes the balance of forces acting on the moving particle. The terms on the right-hand side are respectively the interfacial drag, the forces due to fluid pressure gradient and added virtual mass, the Basset history effect, the shear-induced lift force, and the gravity-buoyancy force.

Let $P(\mathbf{v},\mathbf{x},\tau)$ be the PDF of particle velocity. From the particle evolution given in (1), this PDF evolves by

$$\frac{\partial P}{\partial \tau}+v_i\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial v_i}\left[\left(\frac{U_k-v_k}{\tau_{u*}}+F_{Ai}+F_{Bi}+F_{Li}+F_{gi}\right)P\right]$$

$$=-\frac{\partial}{\partial v_i}\left(\frac{\langle u_i'p\rangle}{\tau_{u*}}+\langle f_{Ai}'p\rangle+\langle f_{Bi}'p\rangle+\langle f_{Li}'p\rangle\right) \quad (2)$$

$$F_{Ai}=\frac{\overline{D}U_i}{D\tau},\quad \frac{\overline{D}}{D\tau}=\frac{\partial}{\partial\tau}+U_k\frac{\partial}{\partial x_k}$$

$$F_{Bi}=k_B\int_0^\tau\frac{d(U_i-V_i)}{d\tau_1}\frac{d\tau_1}{\sqrt{\tau-\tau_1}},\quad F_{Li}=L(U_j-V_j)\left(\frac{\partial U_j}{\partial x_i}-\frac{\partial U_i}{\partial x_j}\right)$$

The terms on the right-hand side of (2) describe the interaction between particles and turbulent fluid eddies according to the interfacial forces in (1).

With a view to determine the correlation between the fluctuating fluid velocity and the particle probability density $\langle u_i'p\rangle$ in (2), the fluid velocity field is modeled by a Gassian process with a known autocorrelation function. Then, with the help of the Furutsu-Novikov formula for Gassian random functions we obtain

$$\langle u_i'p\rangle=\iint\langle u_i'(\mathbf{x},\tau)u_k'(\mathbf{x}_1,\tau_1)\rangle\langle\frac{\delta p(\mathbf{x},\tau)}{\delta u_k(\mathbf{x}_1,\tau_1)}\rangle d\mathbf{x}_1 d\tau_1 \quad (3)$$

$$\langle\frac{\delta p(\mathbf{x},\tau)}{\delta u_k(\mathbf{x}_1,\tau_1)}\rangle=-\frac{\partial}{\partial x_j}\langle p(\mathbf{x},\tau)\frac{\delta R_{pj}(\tau)}{\delta u_k(\mathbf{x}_1,\tau_1)}\rangle-\frac{\partial}{\partial v_j}\langle p(\mathbf{x},\tau)\frac{\delta v_{pj}(\tau)}{\delta u_k(\mathbf{x}_1,\tau_1)}\rangle$$

To find the functional derivatives in (3) we use a solution to (1) by neglecting the shear-induced lift force. Taking the Laplace transform yields from (1)

$$v_{pi}^L(s)=\frac{1+A\tau_{u*}s+k_B\tau_{u*}\sqrt{\pi s}}{1+\tau_{u*}s+k_B\tau_{u*}\sqrt{\pi s}}u_i^L(s)=\phi(s)u_i^L(s) \quad (4)$$

where s denotes the variable in the Laplace transform.

Operation of convolution gains from (4)

$$v_{pi}(\tau)=L^{-1}[\phi(s)u_i^L(s)]=\int_0^\tau\varphi(\tau-\tau_1)u_i(\mathbf{R}_p(\tau_1),\tau_1)d\tau_1 \quad (5)$$

where $\varphi(\tau)=L^{-1}[\phi(s)]$ is the Greene function.

Applying a functional-differentiation operator to (5) yields a set of integral equations for the functional derivatives

$$\frac{\delta v_{pi}(\tau)}{\delta u_j(\mathbf{x}_1,\tau_1)}=\delta_{ij}\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))\varphi(\tau-\tau_1)H(\tau-\tau_1)$$
$$+\int_{\tau_1}^{\tau}\varphi(\tau-\tau_2)\frac{\partial u_i(\mathbf{R}_p(\tau_2),\tau_2)}{\partial x_n}\frac{\delta R_{pn}(\tau_2)}{\delta u_j(\mathbf{x}_1,\tau_1)}d\tau_2 \quad (6)$$

$$\frac{\delta R_{pi}(\tau)}{\delta u_j(\mathbf{x}_1,\tau_1)}=\delta_{ij}\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))\int_{\tau_1}^{\tau}\varphi(\tau-\tau_2)d\tau_2$$
$$+\int_{\tau_1}^{\tau}\int_{\tau_2}^{\tau}\varphi(\tau-\tau_3)d\tau_3\frac{\partial u_i(\mathbf{R}_p(\tau_2),\tau_2)}{\partial x_n}\frac{\delta R_{pn}(\tau_2)}{\delta u_j(\mathbf{x}_1,\tau_1)}d\tau_2 \quad (7)$$

where H(x) is the Heaviside function: H(x<0)=0, H(x>0)=1.

To solve integral equations (6) and (7), we apply an iteration procedure with respect to fluid velocity nonuniformity. In this way, one can derive

$$\langle u_i'p\rangle=-\langle u_i'u_k'\rangle(f_{u*}\frac{\partial P}{\partial v_k}+\tau_{u*}g_{u*}\frac{\partial P}{\partial x_k}+\tau_{u*}\ell_{u*}\frac{\partial U_n}{\partial x_k}\frac{\partial P}{\partial v_n}$$
$$+\tau_{u*}^2h_{u*}\frac{\partial U_n}{\partial x_k}\frac{\partial P}{\partial x_n}+\tau_{u*}^2h_{u*}\frac{\partial U_n}{\partial x_k}\frac{\partial V_j}{\partial x_n}\frac{\partial P}{\partial v_j}) \quad (8)$$

$$f_{u*}=\int_0^\infty\Psi_{Lp}(\xi)\varphi(\xi)d\xi,\quad g_{u*}=\frac{1}{\tau_{u*}}\int_0^\infty\Psi_{Lp}(\xi)\int_0^\xi\varphi(\xi_1)d\xi_1 d\xi$$

$$\ell_{u*}=\frac{1}{\tau_{u*}}\int_0^\infty\Psi_{Lp}(\xi)\int_0^\xi\varphi(\xi-\xi_1)\int_0^{\xi_1}\varphi(\xi_1-\xi_2)d\xi_2 d\xi_1 d\xi$$

$$h_{u*}=\frac{1}{\tau_{u*}^2}\int_0^\infty\Psi_{Lp}(\xi)\int_0^\xi\int_{\xi_1}^\xi\varphi(\xi-\xi_2)d\xi_2\int_0^{\xi_1}\varphi(\xi_1-\xi_2)d\xi_2 d\xi_1 d\xi$$

The coefficients f_{u*}, g_{u*}, l_{u*}, h_{u*} in (8) characterize an entrainment of particles into the fluctuating motion of the carrier fluid, namely, they indicate whether the particle responds to turbulent velocity fluctuations and determine the degree of coupling between the fluid and particulate phases. To calculate these coefficients one has to determine the velocity correlation of fluid motion along a particle trajectory. If the velocity autocorrelation function is described by the frequently used exponential dependence $\Psi_{Lp}(\xi)=\exp(-\xi/T_{Lp})$, the entrainment coefficients in (8) are given by

$$f_{u*}=\phi(s=1/T_{Lp})=\frac{1+A\Omega_{u*}+B\Omega_{u*}}{1+\Omega_{u*}+B\Omega_{u*}},\quad g_{u*}=\frac{\phi(s=1/T_{Lp})}{\Omega_{u*}}$$

$$\ell_{u*}=\frac{\phi^2(s=1/T_{Lp})}{\Omega_{u*}},\ h_{u*}=\frac{\phi^2(s=1/T_{Lp})}{\Omega_{u*}^2},\ \Omega_{u*}=\frac{\tau_{u*}}{T_{Lp}},\ B=\frac{k_B}{\sqrt{\pi T_{Lp}}}$$

The two first terms on the right-hand side of (8) describe the eddy-particle interaction in a homogeneous unsheared flow, and the three last terms characterize the effect of velocity gradients. Correlation (8) has the same form as $\langle u_i'p\rangle$ obtained by Zaichik (1997), however, the entrainment coefficients in (8) take into consideration the influence of instantaneous forces on the mechanism of eddy-particle interactions.

From (8), the fluid-particle velocity correlation moments are found

$$\langle u_i' v_j' \rangle = f_{u*}\langle u_i' u_j' \rangle + \tau_{u*}\langle u_i' u_k' \rangle \left(\ell_{u*} \frac{\partial U_j}{\partial x_k} - g_{u*} \frac{\partial V_j}{\partial x_k} \right) \quad (9)$$

Further, using a Liouville equation for a particle ensemble in phase space and neglecting by some terms of the higher first order derivatives of the PDF, we present the correlation $\langle f_{Ai} p \rangle$ in (2) as

$$\langle f_{Ai}' p \rangle = A[-\frac{\overline{D}}{D\tau}(f_{u*}\langle u_i' u_k' \rangle \frac{\partial P}{\partial v_k}) + \frac{\langle u_i' u_k' \rangle - \langle u_i' v_k' \rangle}{\tau_{u*}} \frac{\partial P}{\partial v_k}$$
$$+ A(\langle u_i' \frac{\partial u_k'}{\partial \tau} \rangle + U_n \langle u_i' \frac{\partial u_k'}{\partial x_n} \rangle + \langle u_i' u_n' \rangle \frac{\partial U_k}{\partial x_n} + \langle u_i' \frac{\partial u_k'}{\partial x_n} \rangle) \frac{\partial P}{\partial v_k}$$
$$+ \langle u_i' v_k' \rangle \frac{\partial P}{\partial x_k} - f_{u*}\langle u_k' u_n' \rangle \frac{\partial U_i}{\partial x_k} \frac{\partial P}{\partial v_n} + \frac{\partial \langle u_i' u_k' \rangle}{\partial x_k} P] \quad (10)$$

For simplicity, $\langle f_{Bi} p \rangle$ is disregarded in (2) because, as a rule, the Basset force does not contribute significantly to the balance of forces exerting on a particle and, in contrast to those due to pressure gradient and added mass, not cause any essential effect, but only slightly mitigates the influence of instantaneous forces. For high Reynolds number, the correlation between the lift force and the particle probability density is approximately given by

$$\langle f_{Li}' p \rangle = -L f_{u*} \langle u_j' u_k' \rangle \left(\frac{\partial U_j}{\partial x_i} - \frac{\partial U_i}{\partial x_j} \right) \frac{\partial P}{\partial v_k} \quad (11)$$

Substituting (8), (10), and (11) into (2) yields the following closed kinetic equation of the particle velocity distribution in turbulent shear flow

$$\frac{\partial P}{\partial \tau} + v_i \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial v_i}\left[\left(\frac{U_i - v_i}{\tau_{u*}} + A\frac{\partial \langle u_i' u_k' \rangle}{\partial x_k} + F_{Ai} + F_{Bi} + F_{Li} + F_{gi} \right) P\right]$$
$$= -\frac{\partial}{\partial v_i}\{\langle u_i' u_k' \rangle (f_{u*} \frac{\partial P}{\partial v_k} + \tau_{u*} g_{u*} \frac{\partial P}{\partial x_k} + \tau_{u*}\ell_{u*} \frac{\partial U_n}{\partial x_k}\frac{\partial P}{\partial v_n} + \tau_{u*}^2 h_{u*}^2 \frac{\partial U_n}{\partial x_k}\frac{\partial P}{\partial x_n}$$
$$+ \tau_{u*}^2 h_{u*} \frac{\partial U_n}{\partial x_k}\frac{\partial V_j}{\partial x_n}\frac{\partial P}{\partial v_j}) + A[\frac{\overline{D}}{D\tau} f_{u*}\langle u_i' u_k' \rangle \frac{\partial P}{\partial v_k} + \frac{\langle u_i' v_k' \rangle - \langle u_i' u_k' \rangle}{\tau_{u*}} \frac{\partial P}{\partial v_k}$$
$$- A(\langle u_i' \frac{\partial u_k'}{\partial \tau} \rangle + U_n \langle u_i' \frac{\partial u_k'}{\partial x_n} \rangle + \langle u_i' u_n' \rangle \frac{\partial U_k}{\partial x_n} + \langle u_i' \frac{\partial u_k' u_n'}{\partial x_n} \rangle) \frac{\partial P}{\partial v_k} - \langle u_i' v_k' \rangle \frac{\partial P}{\partial x_k}$$
$$+ f_{u*}\langle u_k' u_n' \rangle \frac{\partial U_i}{\partial x_k}\frac{\partial P}{\partial v_n}] + L f_{u*}\langle u_j' u_k' \rangle (\frac{\partial U_j}{\partial x_i} - \frac{\partial U_i}{\partial x_j})\frac{\partial P}{\partial v_k}\} \quad (12)$$

Equation (12) is accurate up to the second-rank derivatives. The terms on the right side describe the diffusion transfer in phase space caused by particle-turbulence interactions. In the limit of very small fluid density compared to the particle one ($\rho_f / \rho_p \to 0$), (12) reduces to the kinetic equations derived before by Derevich and Zaichik (1988), Reeks (1991), and Zaichik (1997).

GOVERNING EQUATIONS OF THE DISPERSED PHASE

From (12), we can gain a set of governing equations that represent the conservation of mass, momentum, and particulate turbulent stresses as the appropriate statistical moments of the PDF

$$\frac{\partial \Phi}{\partial \tau} + \frac{\partial \Phi V_k}{\partial x_k} = 0 \quad (13)$$

$$\frac{\partial V_i}{\partial \tau} + V_k \frac{\partial V_i}{\partial x_k} = -\frac{\partial}{\partial x_k}\left(\langle v_i' v_k' \rangle - A\langle u_i' u_k' \rangle\right) + \frac{U_i - V_i}{\tau_{u*}}$$
$$+ F_{Ai} + F_{Bi} + F_{Li} + F_{gi} - \frac{D_{p\,ik}}{\tau_{u*}} \frac{\partial \ln \Phi}{\partial x_k} \quad (14)$$

$$D_{p\,ij} = \tau_{u*}\left(\langle v_i' v_j' \rangle + g_{u*}\langle u_i' u_j' \rangle + \tau_{u*} h_{u*} \langle u_i' u_k' \rangle \frac{\partial U_j}{\partial x_k} - A\langle u_i' v_j' \rangle \right)$$

$$\frac{\partial \langle v_i' v_j' \rangle}{\partial \tau} + V_k \frac{\partial \langle v_i' v_j' \rangle}{\partial x_k} + \frac{1}{\Phi}\frac{\partial \Phi \langle v_i' v_j' v_k' \rangle}{\partial x_k} = -\langle v_i' v_k' \rangle \frac{\partial V_j}{\partial x_k} - \langle v_{ji}' v_k' \rangle \frac{\partial V_{ij}}{\partial x_k}$$
$$+ (1+A)[\langle u_i' u_k' \rangle (\ell_{u*} \frac{\partial U_j}{\partial x_k} - g_{u*}\frac{\partial V_j}{\partial x_k}) + \langle u_j' u_k' \rangle (\ell_{u*}\frac{\partial U_{ij}}{\partial x_k} - g_{u*}\frac{\partial V_i}{\partial x_k})]$$
$$+ L[(f_{u*}\langle u_i' u_k' \rangle - \langle v_i' v_k' \rangle)(\frac{\partial U_k}{\partial x_j} - \frac{\partial U_j}{\partial x_k}) + (f_{u*}\langle u_j' u_k' \rangle - \langle v_j' v_k' \rangle)(\frac{\partial U_k}{\partial x_i} - \frac{\partial U_i}{\partial x_k})]$$
$$+ \frac{2}{\tau_{u*}}\left(f_{u*}\langle u_i' u_j' \rangle - \langle v_i' v_j' \rangle \right) + A\{\frac{2(f_{u*} - 1)}{\tau_{u*}}\langle u_i' u_j' \rangle$$
$$+ f_{u*}[\langle u_i' u_k' \rangle (\frac{\partial U_j}{\partial x_k} + \frac{\partial V_j}{\partial x_k}) + \langle u_j' u_k' \rangle (\frac{\partial U_i}{\partial x_k} + \frac{\partial V_i}{\partial x_k})] + \frac{2}{\Phi}\frac{\overline{D} f_{u*}\Phi \langle u_i' u_j' \rangle}{D\tau}$$
$$- A(\frac{\overline{D}\langle u_i' u_j' \rangle}{D\tau} + \langle u_i' u_k' \rangle \frac{\partial U_j}{\partial x_k} + \langle u_j' u_k' \rangle \frac{\partial U_i}{\partial x_k} + \frac{\partial \langle u_i' u_j' u_k' \rangle}{\partial x_k})\} \quad (15)$$

Here Φ and V_i denote the particle averaged volume fraction and velocity. Equations (14) and (15) are written with an accuracy of the first-order velocity gradients. The first term on the right-hand side of (14) characterizes the turbulent migration ('turbophoresis') of particles due to gradients of both particulate and fluid turbulent stresses. The last term in (14) represents the particle turbulent diffusion.

Equation (15) describes the time evolution of the velocity second-moments, the convection and diffusion transfer, the generation from the average motion owing to velocity gradients, and the exchange of fluctuations between the particulate and fluid phases due to interfacial forces. For small particles or homogeneous unsheared flows, all the differential terms describing transport processes can be neglected, and (15) constricts to the following simple relation between the particulate and fluid turbulent stresses

$$\langle v_i' v_j' \rangle = \left[f_{u*}(1+A) - A \right] \langle u_i' u_j' \rangle = \frac{1 + A^2 \Omega_{u*}}{1 + \Omega_{u*}} \langle u_i' u_j' \rangle \quad (16)$$

Relationship (16) coincides with the familiar formula for the particle kinetic energy in isotropic homogeneous turbulence (Hinze, 1959).

If we use (16) to determine the particle velocity variance, the migration acceleration resulting from the particle-turbulence interaction will be given by

$$F_{Mi} = -\frac{\partial}{\partial x_k}\left(M\langle u_i' u_k' \rangle \right), \quad M = \frac{(1-A)(1-A\Omega_{u*})}{1+\Omega_{u*}} \quad (17)$$

As is apparent from (17), very heavy particles ($A \to 0$, $M>0$) migrate from high to low regions of the fluid turbulence energy. However, the migration coefficient M alters its sign, and hence the migration force changes its direction at A=1 and $A = 1/\Omega_{u*}$. Therefore, fairly inertial particles ($A<1$, $\Omega_{u*} > 1/A$) and rather small bubbles ($A>1$, $\Omega_{u*} < 1/A$) migrate from low to high regions of the turbulence energy. Of particular interest is that sufficiently large bubbles ($A>1$, $\Omega_{u*} > 1/A$) displaces due to the migration force similarly to heavy particles from a high-level turbulence energy region into a low-level turbulence zone. When $\rho_p/\rho_f = 1$ (A=1), as it should be expected, the migration force vanishes.

EDDY-PARTICLE INTERACTION TIME

A response of particles to the fluctuating motion of the carrier fluid is characterized by the ratio of the particle relaxation time to the fluid turbulence time scale defined along a particle trajectory (the so-called eddy-particle interaction time). For very small (non-inertial) particles, the eddy-particle interaction time, T_{Lp}, coincides with the integral Lagrangian turbulence scale for a fluid point, T_L, however, for sufficiently inertial particles, T_{Lp} can differ essentially from T_L. In this paper, T_{Lp} is determined on the basis of the familiar Corrsin approximation for predicting relation between the Lagrangian time autocorrelation function along a particle path and the Eulerian space-time velocity correlation function in a stationary, homogeneous, isotropic turbulent field

$$\Psi^{\ell}_{Lp}(\tau) = \int \Psi^{\ell}_{E}(\mathbf{r},\tau)\phi_p(\mathbf{r},\tau)d\mathbf{r}\,,\; \Psi^{n}_{Lp}(\tau) = \int \Psi^{n}_{E}(\mathbf{r},\tau)\phi_p(\mathbf{r},\tau)d\mathbf{r}$$

where ℓ and n mark the directions to be parallel and orthogonal to the mean relative velocity between particle and fluid $\mathbf{W} = \mathbf{V} - \mathbf{U}$. For the purpose of deriving simple explicit relations for the Lagrangian velocity correlations, the probability density of the particle displacement $\mathbf{r}$ after time τ is taken in the form of the Dirac delta-function

$$\phi_p(\mathbf{r},\tau) = \delta\left(\mathbf{r} - \mathbf{W}\tau - \frac{u_0\psi(\tau)}{\sqrt{3}}\mathbf{s}\right)$$

where u_0 is the turbulence intensity, and $\mathbf{s}$ denotes the unit vector in the direction $\mathbf{r}$. The function $\psi(\tau)$ that describes an effective run path of the particle in its fluctuating motion can be determined from a solution to equation (1). Then, accounting for only the two first forces in (1) as the most substantial ones, we obtain the following approximation

$$\psi(\tau) = \tau + (1-A)\tau_{u*}\left[\exp(-\tau/\tau_{u*}) - 1\right]$$

Representing the Eulerian space-time velocity correlation functions in the usual fashion for isotropic turbulence and using exponential approximations, we find the Lagrangian autocorrelation functions and corresponding turbulence time scales. Figure 1 shows the influence of the Stokes number, $St = \tau_{u*}/T_E$ where T_E is the Eulerian time macroscale, and the factor A, which includes the densities ratio and the added mass coefficient (hereafter C_A is taken as 0.5), on the particle-eddy interaction time when the 'crossing trajectories effect' is absent ($W = |\mathbf{W}| = 0$). It is seen a monotonous increase in T_{Lp} with increasing St for heavy particles (A<1). Naturally, $T_{Lp}=T_L$ for A=1 when $\rho_p = \rho_f$. For light particles or bubbles (A>1), a decrease in T_{Lp} occurs when St increases. In Fig. 1, the results of our model are added for comparison by the approximation obtained by Wang and Stock (1993) for heavy particles moving in the gas (A=0).

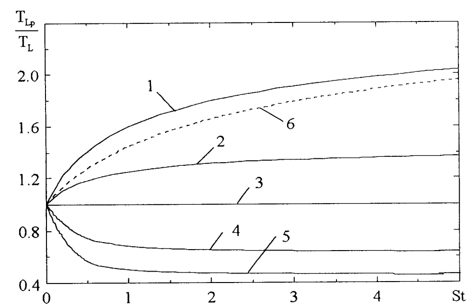

Figure 1. Influence of the Stokes number and the factor A on the eddy-particle interaction time: 1 - A=0, 2 - 0.5, 3 - 1, 4 - 2, 5 - 3; 6 - Wang and Stock (1993).

The 'crossing trajectories effect' causes a decrease in T_{Lp} with increasing the drift velocity, and, in the limit case of $W/u_0 \to \infty$, the following simple relations result for arbitrary-density particles (Csanady, 1963)

$$T^{\ell}_{Lp} = L/W\,,\quad T^{n}_{Lp} = L/2W \qquad (18)$$

HOMOGENEOUS SHEAR FLOW

The flow is assumed to be realized in the streamwise direction, x, and is characterized by a constant gradient of the fluid velocity ($G_u = dU_x/dy$) in the normal direction, y. Fluid velocity characteristics are given in accord with experiments by Tavoularis and Corrsin (1981) for an nearly homogeneous shear flow. Figure 2 demonstrates the behavior of the stremwise velocity fluid-particle covariance and particle variance, predicted respectively according to (9) and (15) at B=0, in dependence on the particle inertia parameter Ω_{u*}, the density factor A, and the shear parameter $S_u = T_{Lp}dU_x/dy$.

As shows Fig. 2(a), the streamwise velocity fluid-particle covariance of heavy particles reduces with increasing particle inertia in both the absence and the presence of mean velocity gradient. For bubbles (A=3), the effect of mean shear is of opposite tendency and results in a decrease in $\langle u'_x v'_x \rangle$. Velocity gradient effects particularly profound for bubbles in respect to the shear fluid-particle correlation, and $\langle u'_y v'_x \rangle$ can be even opposite in sign to $\langle u'_x u'_y \rangle$.

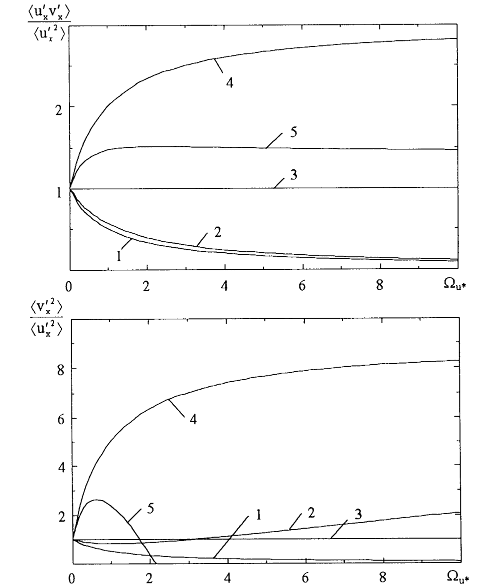

Figure 2. Velocity fluid-particle (a) and particle variance (b) in the homogeneous shear flow: 1,2 - A=0; 3 - A=1; 4,5 - A=3; 1,4 - S_u=0; 2,5 - S_u=1.

The influence of velocity shear on the particle velocity variance for heavy particles and bubbles is also found to be opposite. Figure 2(b) shows that the streamwise velocity fluctuations of heavy particles in sheared flow can elevate over the fluid ones, whereas the transverse particle velocity fluctuations are smaller than those in the fluid. The opposite tendency is observed for bubbles, and so the mean velocity shear decreases the streamwise bubble velocity fluctuations, while the transverse bubble fluctuating velocities can exceed the corresponding fluid ones.

BUBBLE DISTRIBUTION IN A VERTICAL PIPE

The model proposed is also employed for predicting behavior of bubbles in a vertical, fully developed, round pipe flow. Because of practical importance, numerous experimental investigations of bubble flows in pipes have been performed. As a result of these studies, a number of interesting phenomena has been revealed. One of the most remarkable phenomena is of considerable nonuniformity of void fraction distribution across the pipe section. So, in downward flow, the maximum of void fraction profile is located, as a rule, in the pipe center, whereas the void peak shifts towards the wall in upward flow.

Characteristics of fully developed pipe flow are only a function of the radial coordinate, r, and do not depend on the longitudinal coordinate, x.. The turbulent stresses of the bubble phase are taken in the locally-homogeneous approach by means of (16). In this case, the bubble distribution across the pipe section is governed by equation (14) for the radial momentum component

$$\frac{T_{Lp}}{\tau_{u*}} < u_r'^2 > \frac{d\ln\Phi}{dr} = -\frac{dM < u_r'^2 >}{dr} + \frac{C_L}{C_A}(U_x - V_x)\frac{dU_x}{dr} \quad (19)$$

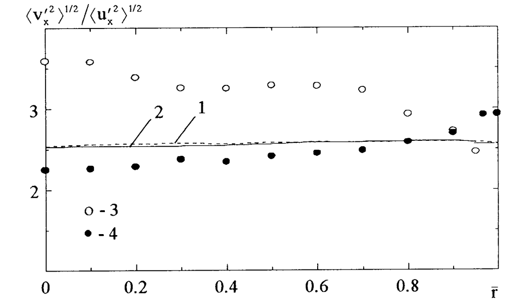

Figure. 4. Comparison between predicted (1,2) and measured (3,4) velocity fluctuations: 1,3 - $J_f = 0.376$ m/s, $J_g = 0.112$ m/s; 2,4 - $J_f = 1.391$ m/s, $J_g = 0.347$ m/s.

As can be observed from (19), the void fraction distribution in a vertical pipe flow is formed under the action of the turbulent migration and lift forces.

With the proviso $W >> u_*$ where u_* is the friction velocity, the eddy-bubble interaction time is prescribed according to (18), and the turbulence integral space scale is assumed to be of $L = 0.1D$. Thus, the bubble inertia parameter that controls the degree of coupling between the fluid and bubble phases is determined as

$$\Omega_{u*} = \frac{80C_A}{3C_D}\bar{d}_p, \quad \bar{d}_p = \frac{d_p}{D} \quad (20)$$

and, hence, if the drag coefficient may be taken as a constant, Ω_{u*} is only a function of the bubble size compared to the pipe diameter, D.

Figure 4 depicts both experimental distributions of bubble/fluid streamwise velocity fluctuations across the pipe section (Liu and Bankoff, 1993) and predictions according to (16) along with (20) at $C_D = 0.5$. The bubble fluctuating velocities are seen to be considerably in excess of the fluid ones, and measured and predicted values are in reasonable agreement. Thus, we can draw the conclusion that relation (20) can be used for making plausible evaluation of the bubble inertia parameter.

Integration of (19) yields

$$\bar{\Phi} = \Phi/\Phi(r=0) = \bar{\Phi}_L \bar{\Phi}_M \quad (21)$$

$$\bar{\Phi}_L = \exp\left[\mp C_0 Ga^{1/2} \int_0^{\bar{r}} \frac{\bar{d}_p^{1/2}\Omega_{u*}\bar{r}}{<\bar{u}_r'^2>(1+\bar{v}_t)} d\bar{r}\right]$$

$$\bar{\Phi}_M = \left[\frac{(M\langle \bar{u}_r'^2 \rangle)_{r=0}}{M\langle \bar{u}_r'^2 \rangle}\right]^{\Omega_{u*}M} \exp\left[\int_0^{\bar{r}} \ln(M\langle \bar{u}_r'^2 \rangle)\frac{d\Omega_{u*}M}{d\bar{r}} d\bar{r}\right]$$

$$\bar{r} = \frac{2r}{D}, \langle \bar{u}_r'^2 \rangle = \frac{\langle u_r'^2 \rangle}{u_*^2}, \bar{v}_t = \frac{v_t}{v}, C_0 = \frac{C_L}{C_A\sqrt{3C_D}}, Ga = \frac{gD^3}{v^2}$$

where the - and + signs relate to upward and downward flow, respectively.

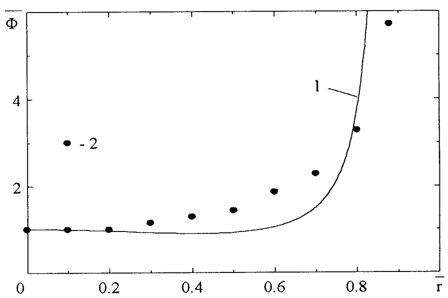

Figure 5. Comparison of simulated (1) and measured (2) void fraction distributions for upflow (Liu and Bankoff, 1993) at $J_f = 1.391$ m/s and $J_g = 0.347$ m/s.

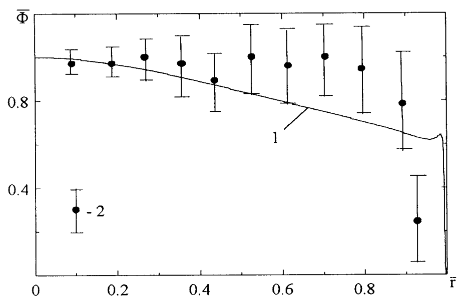

Figure 6. Comparison of simulated (1) and measured (2) void fraction distributions for downflow (Wang et al., 1987) at $J_f = 0.71$ m/s and $J_g = 0.10$ m/s.

Equation (21) represents the void fraction distribution as a product of two factors, the first of which describes the profile forming due to the lift force, and the second results from turbulent migration. The influence of the lift force on void fraction profiles depends on the flow direction, namely, the lift force causes an accumulation of bubbles near the wall in upward flow, whereas it tends to shift bubbles towards the pipe center in downward flow. As mentioned before, small and large bubbles migrate in opposite directions. So small bubbles accumulate in regions of high turbulence energy, whereas large bubbles locate in regions of low turbulence energy. On the whole, it is evident that the shape of actual void fraction profiles formed under the action of both forces is a function of the bubble size as well as the flow direction.

Figures 5 and 6 present comparisons of calculated radial distributions of the void fraction with experimental data obtained respectively for upward and downward flows. Simulations have been performed for real conditions that were realized in these experiments. The lift coefficient in (21) is of a mean value recommended in Wang et al. (1987), $C_L = 0.05$. It is evident a qualitative accord between calculated and measured void fraction distributions. The lack of good quantitative agreement can be apparently explained by failure to take account of the 'back-effect' of bubbles on fluid turbulence.

SUMMARY

A statistical model based on a kinetic equation for the PDF for predicting transport of arbitrary-density particles in turbulent shear flows has been developed. The proposed kinetic equation is used to derive the continuum governing equations that represent the conservation of mass, momentum, and particle turbulent stresses.

A turbulent migration force acting on a particle by nonuniform fluid turbulence has been specified. The action direction of this migration force is revealed to be dependent on both the particle density and its size.

The model predicts that the velocity fluctuation intensities of heavy particles in sheared flow can exceed the fluid turbulence intensity in the streamwise direction but are smaller in the transverse direction. The opposite tendency is revealed for bubble flow, and so the mean velocity shear decreases the streamwise bubble velocity fluctuations, while the transverse bubble fluctuating velocities can elevate over the fluid ones.

The model proposed gives plausible description of measured void fraction distributions in vertical pipe flows, in particular, predicts a peak of void fraction profile near the wall in upward flow and its shift towards the pipe center in downward flow.

REFERENCES

Csanady, G. T., 1963, "Turbulent Diffusion of Heavy Particles in the Atmosphere," *J. Atmos. Sci* , Vol. 20, pp. 201-208.

Derevich, I. V., and Zaichik, L. I., 1988, "Particle Deposition from a Turbulent Flow," *Fluid Dynamics*, Vol. 23, pp. 722-729.

Hinze, J. O., 1959, "Turbulence. An Introduction to Its Mechanism and Theory," McGraw-Hill, New York etc.

Liu, T. J., and Bankoff, S. G., 1993, "Structure of Air-Water Bubbly Flow in a Vertical Pipe," *Int. J. Heat Mass Transfer*, Vol. 36, pp. 1049-1072.

Reeks, M. W., 1991, "On a Kinetic Equation for the Transport of Particles in Turbulent Flows," *Phys. Fluids A*, Vol. 3, pp. 446-456.

Reeks, M. W., 1992, "On the Continuum Equation for Dispersed Particles in Nonuniform Flows," *Phys. Fluids A*, Vol. 4, pp. 1290-1303.

Tavoularis, S., and Corrsin, S., 1981, "Experiments in Nearly Homogeneous Turbulent Shear Flow with a Uniform Mean Temperature Gradient," *J. Fluid Mech.*, Vol. 104, pp. 311-347.

Wang, L. P., and Stock, D. E., 1993, "Dispersion of Heavy Particles in Turbulent Motion," *J. Atmos. Sci.*, Vol. 50, pp. 1897-1913.

Wang, S. K., Lee, S. J., Jones, O. C., and Lahey Jr., R. T., 1987, "3-D Turbulence Structure and Phase Distribution Measurements in Bubbly Two-Phase Flows," *Int. J. Multiphase Flow*, Vol. 13, pp. 327-343.

Zaichik, L. I., 1997, "Modelling of the Motion of Particles in Nonuniform Turbulent Flow Using the Equation for the Probability Density Function," *J. Appl. Math. Mech.*, Vol. 61, pp. 127-133.