## STRUCTURE-BASED TURBULENCE MODELING FOR WALL-BOUNDED FLOWS

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## **ABSTRACT**

The performance of Reynolds Stress Transport (RST) models in non-equilibrium flows is limited by the lack of information about two dynamically important effects: the role of energy-containing turbulence structure (dimensionality) and the breaking of reflectional symmetry due to strong mean or frame rotation. Both effects are fundamentally nonlocal in nature and this explains why it has been difficult to include them in one-point closures like RST models. Information about the energy-containing structure is necessary if turbulence models are to reflect differences in dynamic behavior associated with structures of different dimensionality (nearly isotropic turbulence vs. turbulence with strongly organized two-dimensional structures). Information about the breaking of reflectional symmetry is important whenever mean rotation is dynamically important (flow through axisymmetric diffuser or nozzle with swirl, flow through turbomachinery, etc.). Here we present a new one-point model that incorporates the needed structure information, and show a selection of results for homogeneous and inhomogeneous flows.

## 1. INTRODUCTION

Reynolds-averaged turbulence models are the primary tool for the engineering analysis of complex turbulent flows, but their performance in flows that must be computed in order to advance technology is at best inconsistent. Dynamically important features of the turbulence structure are inherently nonlocal in nature, and thus difficult to emulate in one-point closures, yet they cannot be completely ignored in models that are designed for use in complex flows. This lack of crucial information is now recognized as one of the primary challenges facing turbulence modeling.

Consider for example the case of Reynolds Stress Transport (RST) models where the Reynolds stresses  $R_{ij}$  are used

for closing the unknown terms in their own transport equations.  $R_{ij}$  carries information about the componentality of the turbulence (the relative strengths of different velocity components), but not about its dimensionality (the relative uniformity of the structure in different directions). Thus RST models cannot possibly satisfy conditions associated with the dimensionality of the turbulence, or reflect differences in dynamic behavior associated with structures of different dimensionality (nearly isotropic turbulence vs. turbulence with strongly organized two-dimensional structures). Similarly, well known limitations of RST models in predicting flows with strong rotation can be, at least partly, traced to the lack of dimensionality and other information.

The issues outlined above, and discussed in more detail in Reynolds & Kassinos (1995) and Kassinos, Reynolds & Rogers (1999), let us to introduce a set of one-point turbulence structure tensors that contain key information missing from standard one-point closures. Here we outline the construction of a one-point model based on the transport of one of these tensors, and show a selection of results for homogeneous and inhomogeneous flows.

#### 2. DEFINITIONS

We introduce the turbulent stream function  $\Psi'_i$ , defined by

$$u_i' = \epsilon_{its} \Psi_{s,t}' \quad \Psi_{i,i}' = 0 \quad \Psi_{i,nn}' = -\omega_i', \quad (1)$$

where  $u_i'$  and  $\omega_i'$  are the fluctuating velocity and vorticity components. The Reynolds stress tensor is given by

$$R_{ij} = \overline{u_i' u_j'} = \epsilon_{ipq} \epsilon_{jts} \overline{\Psi_{q,p}' \Psi_{s,t}'}, \qquad (2a)$$

and the associated nondimensional and anisotropy tensors are

$$r_{ij} = R_{ij}/q^2$$
  $\tilde{r}_{ij} = r_{ij} - \frac{1}{3}\delta_{ij}$ . (2b)

Here  $q^2=2k=R_{ii}$ . Using isotropic tensor identities (Mahoney 1985), we can write (2a) as

$$R_{ij} + \underbrace{\overline{\Psi'_{k,i}\Psi'_{k,j}}}_{D_{ij}} + \underbrace{\overline{\Psi'_{i,k}\Psi'_{j,k}}}_{F_{ij}} - \underbrace{(\overline{\Psi'_{i,k}\Psi'_{k,j}} + \overline{\Psi'_{j,k}\Psi'_{k,i}})}_{C_{ij} + C_{ji}} = \delta_{ij}q^{2}.$$

$$(3)$$

The constitutive equation (3) shows that one-point correlations of stream-function gradients, like the Reynolds stresses, are dominated by the energy-containing scales. These correlations contain independent information that is important for the proper characterization of non-equilibrium turbulence. For example, the  $D_{ij}$  tensor reveals the level of two-dimensionality (2D) of the turbulence, and  $F_{ij}$  describes the large-scale structure of the vorticity field.

For homogeneous turbulence  $C_{ij} = C_{ji} = 0$ , and the remaining tensors in (3) have equivalent representations in terms of the velocity spectrum tensor  $E_{ij}(\mathbf{k})$  and the vorticity spectrum tensor  $W_{ij}(\mathbf{k})$ . These are given below.

Structure dimensionality tensor:

$$D_{ij} = \int \frac{k_i k_j}{k^2} E_{nn}(\mathbf{k}) d^3 \mathbf{k}$$
$$d_{ij} = D_{ij}/q^2 \quad \tilde{d}_{ij} = d_{ij} - \frac{1}{3} \delta_{ij}$$
(4)

Structure circulicity tensor:

$$F_{ij} = \int \mathcal{F}_{ij}(\mathbf{k}) d^3 \mathbf{k}$$

$$f_{ij} = F_{ij}/q^2 \qquad \tilde{f}_{ij} = f_{ij} - \frac{1}{3} \delta_{ij} . \qquad (5)$$

Here  $\mathcal{F}_{ij}(\mathbf{k})$  is the circulicity spectrum tensor, which is related to the vorticity spectrum tensor  $W_{ij}(\mathbf{k}) = \overline{\hat{\omega}_i \hat{\omega}_j^*}$  through the relation

$$\mathcal{F}_{ij}(\mathbf{k}) = \frac{W_{ij}(\mathbf{k})}{k^2}.$$

We define the third rank tensor

$$Q_{ijk} = -\overline{u'_j \Psi'_{i,k}} \qquad q_{ijk} = Q_{ijk}/q^2.$$
 (6)

For homogeneous turbulence,  $Q_{ijk}$  is

$$Q_{ijk} = \epsilon_{ipq} M_{jqpk} \tag{7}$$

where  $M_{ijpq}$  is

$$M_{ijpq} = \int \frac{k_p k_q}{k^2} E_{ij}(\mathbf{k}) \ d^3 \mathbf{k} \,. \tag{8}$$

The definition of the third-rank fully symmetric *stropholysis* tensor is given by

$$Q_{ijk}^* = \frac{1}{6}(Q_{ijk} + Q_{jki} + Q_{kij} + Q_{ikj} + Q_{jik} + Q_{kji}).$$
(9)

For homogeneous turbulence,  $Q_{ijk}$  and  $Q_{ijk}^{st}$  are bi-trace free

$$Q_{iik} = Q_{iki} = Q_{kii} = 0 Q_{iik}^* = 0. (10)$$

A decomposition based on group theory shows that  $Q_{ijk}$  and  $Q_{ijk}^*$  (here we use  $q_{ijk}^*=Q_{ijk}^*/q^2$ ) are related to each other and lower-rank tensors,

$$Q_{ijk} = q^2 \left[ \frac{1}{6} \epsilon_{ijk} + \frac{1}{3} (\epsilon_{ikm} r_{mj} + \epsilon_{jim} d_{mk} + \epsilon_{kjm} f_{mi}) + q_{ijk}^* \right],$$
(11)

and

$$r_{ij} = \epsilon_{imp}q_{mjp}$$
  $d_{ij} = \epsilon_{imp}q_{pmj}$   $f_{ij} = \epsilon_{imp}q_{jpm}$ . (12)

# 3. MODEL FORMULATION FOR HOMOGENEOUS TURBULENCE

The one-point structure-based model carries the transport equation for  ${\bf Q}$  and a model transport equation for the dissipation rate  $\epsilon$ . The formulation of the model is based on simplified nonlocal theory making use of structure modeling ideas. In Section 3.1 we outline this nonlocal theory and in Section 3.2 we show how it leads to the one-point model.

## 3.1 IPRM formulation

Kassinos & Reynolds (1994,1996) formulated a simplified nonlocal theory (Particle Representation Model or PRM) for the RDT of homogeneous turbulence. The original idea was to represent the turbulence by an ensemble of fictitious particles. A number of key properties and their evolution equations are assigned to each particle. Ensemble averaging produces a representation of the one-point statistics of the turbulent field, which is exact for the case of RDT of homogeneous turbulence. In essence, this approach represents the simplest theory beyond one-point methods that provides closure for the RDT equations without modeling.

The Interacting Particle Representation Model (IPRM) is a recent extension of the PRM formulation that includes the effects of nonlinear eddy-eddy interactions, important when the mean deformations are slow. Unlike standard models, which use return-to-isotropy terms, the IPRM incorporates nonlinear effects through the use of effective gradients. The effective gradients idea postulates that the background nonlinear particle-particle interactions provide a gradient acting on each particle in addition to the actual mean velocity gradient. An advantage of this formulation is the preservation of the RDT structure of the governing equations even for slow deformations of homogeneous turbulence. A detailed account of these ideas is given in Kassinos & Reynolds (1996, 1997) and will not be repeated here. To a large extent, the one-point Q-model is based on the IPRM formulation.

The governing equations for the conditional (cluster averaged) IPRM formulation are (see Kassinos & Reynolds 1996)

$$\dot{n}_{i} = -G_{ki}^{n} n_{k} + G_{kr}^{n} n_{k} n_{r} n_{i} \qquad (13)$$

$$\dot{R}_{ij}^{\mid \mathbf{n}} = -G_{ik}^{v} R_{kj}^{\mid \mathbf{n}} - G_{jk}^{v} R_{ki}^{\mid \mathbf{n}}$$

$$+ [G_{km}^{n} + G_{km}^{v}] (R_{im}^{\mid \mathbf{n}} n_{k} n_{j} + R_{jm}^{\mid \mathbf{n}} n_{k} n_{i}) \qquad (14)$$

$$- [2C_{1} R_{ij}^{\mid \mathbf{n}} - C_{2}^{2} R_{kk}^{\mid \mathbf{n}} (\delta_{ij} - n_{i} n_{j})].$$

Here  $n_i(t)$  is the unit gradient vector and  $R_{ij}^{\text{In}}$  is the conditional Reynolds stress tensor corresponding to a cluster of particles with a common  $n_i(t)$ . The effective gradients are

$$G_{ij}^{n} = G_{ij} + \frac{C^{n}}{\tau^{*}} r_{ik} d_{kj}$$
  $G_{ij}^{v} = G_{ij} + \frac{C^{v}}{\tau^{*}} r_{ik} d_{kj}$ , (15)

where  $G_{ij}$  is the mean velocity gradient. The constants  $C^v$  and  $C^n$  are taken to be  $C^n = 2.2C^v = 2.2$ . The different values for these two constants account for the different rates of return to isotropy of  $D_{ij}$  and  $R_{ij}$ .

The turbulent time scale  $\tau^*$  is chosen so as to produce the proper dissipation rate. The rate of dissipation of the turbulent kinetic energy  $k=\frac{1}{2}q^2$  produced by the IPRM equation (14) is

$$\epsilon^{\text{PRM}} = q^2 \frac{C^v}{\tau^*} r_{ik} d_{km} r_{mi} \,. \tag{16}$$

We choose the time scale  $\tau^*$  so that  $\epsilon^{PRM} = \epsilon$ . This requires

$$\tau^* = (\frac{q^2}{\epsilon})C^v r_{ik} d_{km} r_{mi}. \tag{17}$$

To complete the IPRM we use the standard model equation for the dissipation rate  $(\epsilon)$  with a rotational modification to account for the suppression of  $\epsilon$  due to mean rotation,

$$\dot{\epsilon} = -C_0(\epsilon^2/q^2) - C_s S_{pq} r_{pq} \epsilon - C_\Omega \sqrt{\Omega_n \Omega_m d_{nm}} \epsilon.$$
(18)

Here  $\Omega_i$  is the mean vorticity vector, and the constants are

$$C_0 = 3.6$$
  $C_s = 3.0$  and  $C_\Omega = 0.01$ . (19)

The last term in (14) accounts for rotational randomization due to eddy-eddy interactions. We require that the rotational randomization model leaves the conditional energy unmodified. This requires that  $C_1 = C_2^2$ , and hence using dimensional considerations we take

$$C_1 = C_2^2 = \frac{8.5}{\tau^*} \,\Omega^* \, f_{pq} n_p n_q \tag{20}$$

where 
$$\Omega^* = \sqrt{\Omega_k^* \Omega_k^*}$$
 and  $\Omega_i^* = \epsilon_{ipq} r_{qk} d_{kp}$ .

# 3.2 The stropholysis equation

We consider general deformations of homogeneous turbulence. The most convenient method for deriving the  $\mathbf{Q}$  equation is to use the conditional (cluster averaged) IPRM formulation to obtain the evolution equation for  $\mathbf{M}$ , and then contract the  $\mathbf{M}$  equation with the alternating tensor  $\epsilon_{ijk}$  to extract the  $\mathbf{Q}$  equation. The PRM representation for  $\mathbf{Q}$  and  $\mathbf{M}$  is

$$Q_{ijk} = -\langle V^2 v_j s_i n_k \rangle \qquad M_{ijpq} = \langle V^2 v_i v_j n_p n_q \rangle$$
(21)

where  $s_i = \epsilon_{ikz} V_k n_z / V$  is the unit stream function vector (see Kassinos & Reynolds 1997). Hence using (13) and (14) and the definitions (7) and (21), one obtains

$$\begin{split} \frac{dQ_{ijk}}{dt} &= -G^v_{jm}Q_{imk} - G^n_{mk}Q_{ijm} - G^v_{sm}\epsilon_{its}M_{jmtk} \\ &- G^n_{mt}\epsilon_{its}M_{jsmk} + [G^n_{wq} + G^v_{wq}]Q_{iqwjk} \\ &+ 2G^n_{qr}Q_{ijkqr} - \frac{8.5}{\tau^*}\,\Omega^*\,f_{rs}[Q_{ijkrs} + Q_{jikrs}]\,. \end{split}$$
 Using the PRM representation,  $Q_{ijkqr} = \langle V^2v_js_in_kn_qn_r \rangle$ .

## 3.3 Closure of the stropholysis equation

Closure of (22) requires a model for the tensor  $Q_{ijkpq}$  in terms of  $Q_{ijk}$ . Once such a model has been specified, it effectively provides a model for  $M_{ijpq}$  in terms of  $Q_{ijk}$  since  $\mathbf{M}$  can be obtained from  $Q_{ijkpq}$  by a contraction with  $\epsilon_{ijk}$ . For small anisotropies, one can write an exact representation of  $Q_{ijkpq}$  in terms of  $Q_{ijk}$  that is linear in  $Q_{ijk}$ . Other tensors, like  $R_{ij}$ ,  $D_{ij}$  and  $F_{ij}$ , can be expressed in terms of  $Q_{ijk}$  [see (12)] and need not be included explicitly in the model. Definitions (contractions and continuity) determine all the numerical coefficients in the linear model. Thus the linear model contains no adjustable parameters.

In the presence of mean rotation, rotational randomization is an important dynamical effect that must be accounted for in the model. Rotational randomization, a strictly nonlocal effect that is lost in the averaging procedure generating the one-point statistics, is caused by the differential action of mean rotation on particle velocity vectors (Fourier modes) according to the alignment of the corresponding gradient (wavenumber) vectors with the axis of mean rotation. The main impact of Fourier randomization on one-point statistics is the damping of rotation-induced adjustments; here this effect is added explicitly through the simple model,

$$\frac{DQ_{ijk}}{Dt} = \dots - \gamma_1 (Q_{ijk} - Q_{ijk}^{rf}) 
- \gamma_2 \epsilon_{ijm} (R_{mk} - D_{mk}) - \gamma_3 \epsilon_{ikm} (F_{mj} - D_{mj}).$$
(23)

The first term accounts for the rotational randomization effects in rotation dominated flows while the remaining two terms account for the modification of these effects due to the combined action of mean strain and rotation.  $\mathbf{Q}^{rf}$  is the limiting state of  $\mathbf{Q}$  under rapid rotation. Here  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$  are scalar functions of the invariants of the mean strain and rotation and are determined from simple test cases. A detailed discussion of these models will appear separately.

The new one-point model produces excellent results for general irrotational deformations of homogeneous turbulence. A particularly interesting example is shown in Figure 1 where we consider the case of irrotational axisymmetric expansion (axisymmetric impingement). The mean velocity gradient tensor in this case is

$$S_{ij} = \frac{2}{\sqrt{3}} S \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}, \qquad S = \sqrt{S_{ij} S_{ij} / 2},$$
(24)

and the total strain

$$C = \int_0^t |S|t'dt' \tag{25}$$

is used as the horizontal axis in Figure 1. As was discussed in Kassinos & Reynolds (1996, 1997), the axisymmetric expansion flows exhibit a paradoxical behavior, where a slower mean deformation rate produces a stress anisotropy that exceeds the one produced under RDT for the same total mean strain. This effect is triggered by the different rates of return to isotropy in the  $\tilde{\mathbf{r}}$  and  $\tilde{\mathbf{d}}$  equations, but it is dynamically controlled by the rapid terms. The net effect is a growth of  $\tilde{\mathbf{r}}$  in expense of  $\tilde{\mathbf{d}}$ , which is strongly suppressed. The one-point model (see Figure 1) is able to capture these effects well and also predicts the correct decay rates for the normalized turbulent kinetic energy  $k/k_0$  and dissipation rate  $\epsilon/\epsilon_0$ . The predictions of the one-point Q-model are comparable to those of the nonlocal IPRM

The predictions of the one-point Q-model for the case of homogeneous shear (where the mean gradient is  $G_{12}=S$ ) are shown in Figure 2. Comparison is made to the DNS results of Rogers & Moin (1987). Note that the model produces satisfactory predictions for the components of  $r_{ij}=R_{ij}/q^2$ ,  $d_{ij}=D_{ij}/q^2$ , and  $f_{ij}=F_{ij}/q^2$ . A fully-developed stage was reached in the simulations for  $St\geq 10$ , and in this range both the Q-model and the IPRM predict the correct level for the dimensionless ratio of production over dissipation,  $P/\epsilon$ .

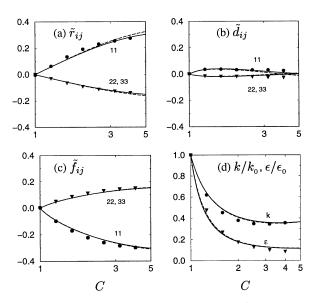


FIGURE 1. Comparison of the one-point Q-model predictions (dashed lines) with IPRM results (solid lines) and the 1985 DNS of Lee & Reynolds (symbols) for the axisymmetric expansion case EXO  $(Sq_0^2/\epsilon_0=0.82)$ . (a)-(c) evolution of the Reynolds stress, dimensionality, and circulicity anisotropies; 11 component ( $\bullet$ ), 22 and 33 components ( $\blacktriangledown$ ). (d) evolution of the normalized turbulent kinetic energy ( $\bullet$ ) and dissipation rate ( $\blacktriangledown$ ).

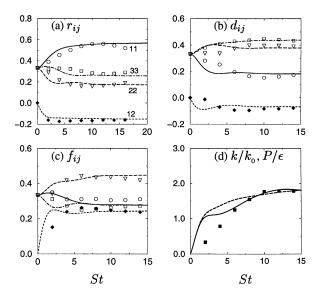


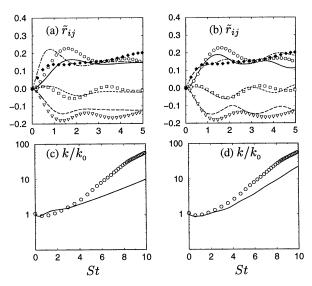
FIGURE 2. Comparison of Q-model predictions (lines) and the 1986 DNS of Rogers & Moin (symbols). (a)-(c) evolution of the Reynolds stress, dimensionality, and circulicity components in homogeneous shear with  $Sq_0^2/\epsilon_0=2.36$ : 11 component, (----,  $\bigcirc$ ); 22 component, (----,  $\bigcirc$ ); 33 component, (----,  $\bigcirc$ ); 12 component, (----,  $\spadesuit$ ). (d) evolution of production over dissipation rate  $(P/\epsilon)$ : model, (----); IPRM, (----); DNS ( $\blacksquare$ ).

A challenge for one-point models is provided by the elliptic streamlines flows (see Figure 3),

$$G_{ij} = \begin{pmatrix} 0 & 0 & -\gamma - e \\ 0 & 0 & 0 \\ \gamma - e & 0 & 0 \end{pmatrix} \qquad 0 < |e| < |\gamma|$$
(26)

which combine the effects of mean rotation and plane strain and emulate conditions encountered in turbomachinery. (Note that the case e=0 corresponds to pure rotation while the case  $|e|=|\gamma|$  corresponds to homogeneous shear).

Direct numerical simulations (Blaisdell & Shariff 1996) show exponential growth of the turbulent kinetic energy in elliptic streamline flows, which analysis shows is associated with instabilities in narrow wavenumber bands in wavenumber space. Standard k- $\epsilon$  models, as well as most RST models, erroneously predict decay of the turbulence. As shown in Figure 3, the one-point Q-model is able to capture the main features of the oscillations observed in the components of the Reynolds stress anisotropy  $\tilde{r}_{ij}$ . Furthermore, the model is able to capture an exponential growth of the turbulent kinetic energy. Note however that the initial growth rate predicted by both the nonlocal IPRM and the Q-model falls short of the rate predicted by the DNS. At longer times, the growth rates predicted by both models compare more favorably to the growth observed in the DNS.



# 4. INHOMOGENEOUS TURBULENCE

The Q-model has been implemented in a 1D code and is being tested for fully-developed channel flow with and without system rotation. Inhomogeneous effects are incorporated in the  $Q_{ijk}$  and  $\epsilon$  equations through the addition of standard gradient diffusion models, accounting for turbulent transport, as outlined below

$$\frac{DQ_{ijk}}{Dt} = \dots + \frac{\partial}{\partial x_r} \left( \left[ \nu \delta_{rs} + \frac{C_{\nu}}{\sigma_Q} R_{rs} \tau \right] \frac{\partial Q_{ijk}}{\partial x_s} \right)$$
(27)

$$\frac{D\epsilon}{Dt} = \dots + \frac{\partial}{\partial x_r} \left( \left[ \nu \delta_{rs} + \frac{C_{\nu}}{\sigma_{\epsilon}} R_{rs} \tau \right] \frac{\partial \epsilon}{\partial x_s} \right). \tag{28}$$

The turbulent kinetic energy is obtained from  $k = \epsilon_{ikj} Q_{ijk}/2$ .

Wall proximity effects and boundary conditions are treated through an elliptic relaxation scheme based on the ideas of Durbin (1993). Terms in the transport equation for  $Q_{ijk}$  that are assumed to represent nonlocal effects are lumped together into a term  $\wp_{ijk}$  m which is then replaced by a new tensor,  $kf_{ijk}$ , obtained through an elliptic relaxation scheme

$$L^2 \nabla^2 f_{ijk} - f_{ijk} = -\wp_{ijk}/k. \tag{29}$$

Here L is a characteristic length scale for the elliptic relaxation. The elliptic relaxation scheme allows the imposition

of boundary conditions that produce the correct near-wall behavior for various components of  $Q_{ijk}$ . Away from the wall (29) allows one to recover the homogeneous model. This is in analogy to the elliptic relaxation scheme applied to RST models by Durbin.

## 4.1 Representative results

Preliminary results obtained with the Q-model for fully developed channel flow are encouraging. The model was implemented in a 1D-code using elliptic relaxation, as outlined above, and with no wall-function treatment. A comparison of the Q-model predictions with DNS data (Mansour 1998) for fully developed channel flow at  $Re_{\tau}=395$  is shown in Figure 4.

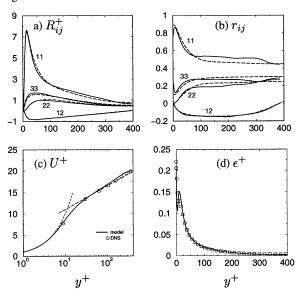


FIGURE 4. Comparison of model predictions with DNS (Mansour, 1998) for fully developed channel flow at  $Re_{\tau}=395$ . (a) components of the Reynolds stress tensor, (b) components of the Reynolds stress tensor normalized by its trace: model, (——); DNS (----). (c) mean velocity profile, (d) dissipation rate profile: model, (——); DNS, ( $\square$ ).

The Reynolds stress components (nondimensionalized with the wall shear velocity  $u_{ au}$ ) are shown in Figure 4a. The agreement between the model predictions (dashed lines) and the DNS (solid lines) is satisfactory. The model slightly overpredicts the peak in the streamwise component  $R_{11}^+$  that occurs at about  $y^+ \approx 15$ . The components of the normalized Reynolds stress tensor  $r_{ij} = R_{ij}/q^2$  are shown in Figure 4b. The agreement between the model predictions and the DNS results is again reasonable. The agreement in the case of the shear stress  $r_{12}$  is noteworthy. The mean velocity profile is shown Figure 4c. The model prediction is in good agreement with the DNS profile, the most notable difference being in the value of the mean velocity in the log region. Finally, the model profile of the dissipation rate  $\epsilon$  is shown in Figure 4d. The model is again in good agreement with the DNS, but has a larger wiggle near the wall than the data show. This difference

depends on the model transport equation for  $\epsilon$ , and we are currently exploring alternative formulations that aim at taking full advantage of the structure information carried in the new model.

The Q-model has also been tested for fully-developed Poiseuille flow with system rotation. Here we consider rotation about the spanwise axis and compare results with the LES of Kim (1983) for the case  $Re_{\tau}\equiv \overline{u}_{\tau}h/\nu=640$  and  $Ro\equiv 2h\Omega/U_b=0.068$ . The mean friction velocity  $\overline{u}_{\tau}$  is computed using the wall shear stress averaged on the two walls, h represents the channel half-width,  $U_b$  is the bulk mean velocity across the channel, and  $\Omega$  is the frame rotation rate.

A comparison of the model predictions for the turbulent intensities with the corresponding LES results is shown in Figure 5. The fully-developed case with no system rotation is also included [Fig. 5(a)] as a reference case. The agreement between the model predictions and the LES results for this reference case is acceptable. In the rotating case, the model is able to capture the characteristic asymmetry in the turbulent intensity profiles induced by the system rotation and overall agreement with the LES predictions is acceptable. The model correctly predicts that the wall normal intensity is significantly higher on the unstable (pressure) side than on the (stable) suction side of the channel. Near the channel centerline the model is able to capture the reversal of the stress anisotropy  $(u^+)$  becoming higher than  $v^+$ ) due to mean rotation.

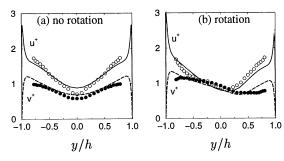


FIGURE 5. Fully-developed Poiseuille flow at  $Re_{\tau} = 640$  with (a) no rotation (Ro = 0) and (b) with spanwise rotation (Ro = 0.068). Comparison of model predictions (lines) for the streamwise ( $u^+$ ) and wall-normal ( $v^+$ ) turbulence intensities with results from the LES (symbols) of Kim (1983).

## 5. CONCLUSIONS

The turbulence structure affects the dynamics in nonequilibrium turbulence and its effects must be emulated by engineering models that are designed for use in complex flow regimes. This poses a challenge to traditional turbulence models which completely neglect turbulence structure. Here we outlined the construction of a new type of model that captures structure information missing from traditional one-point models. The model has been validated successfully for a wide range of deformations of homogeneous turbulence. Results obtained for simple wall-bounded flows are encouraging. We are currently evaluating the model in more complex flows.

## **ACKNOWLEDGEMENTS**

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