A LOW-MACH-NUMBER MODEL OF THE COUNTER-GRADIENT DIFFUSION IN A TURBULENT SWIRLING FLAME

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1 ABSTRACT

It is found by an a priori test that the counter-gradient diffusion of the turbulent heat flux observed in a non-premixed swirling flame can be quantitatively explained by a new turbulence model of first-order closure type. The new model has been derived by applying the two-scale direct-interaction approximation to the low-Mach-number equations with the assumption that the variation of density is relatively small. It can be applied to general turbulent flows with the density variation, including the case where the Boussinesq approximation can not hold. The model expression of turbulent heat flux involves the term related to the mean pressure gradient in addition to the usual term of gradient diffusion type. This expression can also reproduce the vertical turbulent heat flux of the natural convection along a heated vertical plate, which can not be explained by the standard turbulence model of gradient diffusion type.

2 INTRODUCTION

In incompressible turbulent flows, turbulent heat flux $\mathbf{H}$ can be usually well approximated by the eddy-diffusivity representation of gradient diffusion type (for example see Bradshaw et al., 1981):

$$\mathbf{H} \equiv -\left\langle \mathbf{u}'T' \right\rangle = \kappa_e \nabla < T >, \tag{1}$$

where $\mathbf{u}'$, T', $\langle T \rangle$, and κ_e are the fluctuating part of velocity, that of temperature, the mean part of temperature, and the eddy diffusivity, respectively. However, there is an experimental observation (Hardalupas et al., 1996). that the turbulent heat flux across a non-premixed swirling flame is in the direction opposite to that predicted by (1). This phenomenon is called the counter-gradient diffusion, and has been observed in a number of experiments (Moss, 1980; Shepherd et al., 1982; Cheng and Shepherd, 1991; Armstrong and Bray, 1992) for the turbulent flux of the mean reaction progress variable of premixed turbulent flames. The counter-gradient diffusion is generally related to the differential effect of mean presure gradients on hot, light products and on cold, heavy reactants (Libby and Bray, 1981).

In the case of counter-gradient diffusion, the standard model of gradient diffusion type based on the eddy diffusivity concept as (1) can not be used and alternative models must be sought. One of such alternative models is the Bray-Moss-Libby model (Libby and Bray, 1981) which is a second-order closure based on a bimodal probability density function for the reaction progress variable and is found to be consistent with the direct numerical simulation (Veynante et al., 1997).

The objective of the present paper is to propose a turbulence model of first-order closure type, which is much simpler than the Bray-Moss-Libby model and can explain the counter-gradient diffusion. It is also shown that the proposed model can reproduce the vertical turbulent heat flux of the natural convection along a heated vertical plate, which can not be expressed by the standard turbulence model of gradient diffusion type.

In Sec. 2 a new model is presented with a brief description of the derivation, and in Sec. 3 it is shown that the present model can reproduce the counter-gradient diffusion of the turbulent heat flux across a non-premixed swirling flame by an a priori test based on the experimental database (Hardalupas et al., 1996). Finally, in Sec.4 the conclusion of the present study is given.

3 A LOW-MACH-NUMBER MODEL

The turbulence model proposed in the present paper is derived from the low-Mach-number equations (Rehm and Baum, 1978). The fluctuating flow fields are expanded in powers of a compressibility parameter a, and the two-scale direct-interaction approximation (TSDIA) (Yoshizawa, 1984) is applied to the systems of equations of each order of a by using a scale parameter δ. The result of order up to $O(a\delta)$ is the present model given by

$$\mathbf{H} = \kappa_t \nabla T + T_0 \tau_0 \nabla < P >, \tag{2}$$

where T_0 and $< P >$ are the representative mean temperature and the mean pressure, respectively. In (2), κ_t and τ_0 are defined by

$$\kappa_t = 0.14\frac{K^2}{\epsilon}\left(1 + 0.27\frac{K}{\epsilon}\right) Q_*, \tag{3}$$

$$\tau_0 = C_\tau \frac{1}{T_0^2}\frac{K_T^2}{\epsilon_T}, \tag{4}$$

where K, ϵ, K_T, ϵ_T, Q_*, and C_τ are respectively the turbulent kinetic energy, the dissipation rate of K, the temperature variance, the dissipation rate of K_T, the normalized heat source, and a model constant whose value is theoretically estimated at 0.32. With the aid of the transport equations for the velocity fields, the model expression (2) is rewritten as

$$H^\alpha = \kappa_t \frac{\partial \langle T \rangle}{\partial x^\alpha} + < T > \tau \left(g^\alpha + \frac{\partial R^{\alpha a}}{\partial x^a} - \frac{DU^\alpha}{Dt}\right), \tag{5}$$

where

$$\tau = C_\tau \frac{1}{< T >^2}\frac{K_T^2}{\epsilon_T}, \tag{6}$$

and g^α, $R^{\alpha\beta}$, U^α, and D/DT denote the gravitational acceleration, the Reynolds stress, the mean velocity, and the Lagrange derivative, respectively. The details are referred to Shimomura (1998).

For buoyant flows, Yoshizawa (1983) and Okamoto (1996) proposed the turbulence models based on the Boussinesq approximation. In the case of steady flows under a large gravitational force (DU^α/Dt, $\partial R^{\alpha a}/\partial x^a << g^\alpha$), the expression (5) is almost identical to the model of Okamoto with the only exception of the expression for τ. Fig.1 shows the predictions of four models (the present, the Yoshizawa, the Okamoto, and the standard gradient diffusion models) in comparison with the experiment of Tsuji and Nagano (1988) for the vertical turbulent heat flux of the natural convection along a heated vertical plate. Though the standard turbulence model of gradient diffusion type cannot give rise to the vertical heat flux, the other three models can reproduce it. The present and the Okamoto models agree better with the experiment than the Yoshizawa model. Namely, the present model is a general model for turbulent flows of density variation, involving the appropriate model for the flows described by the Boussinesq approximation.

4 AN A PRIORI TEST IN A SWIRLING FLAME

Now, we put an a priori test to the present model (2), or (5) by applying to the turbulent heat flux across a non-premixed swirling flame based on the experimental database (Hardalupas et al., 1996). The mothod of the a priori test is as follows:

1. Interpolate the experimental data points by the cubic spline.
2. Evaluate the eddy viscosity by applying the least square method to the Reynolds stress
3. Evaluate the eddy diffusivity by the eddy viscosity.
4. Evaluate τ by applying to the least square method to the heat fluxes.
5. Calculate the heat fluxes by (5) with the evaluated τ.

Figs.2 and 3. show the predictions for the turbulent heat fluxes of the present model and the standard turbulence model of gradient diffusion type, comparing with the experiment of Hardalupas et al. (1996). The turbulent heat flux along the flame is shown in Fig.2

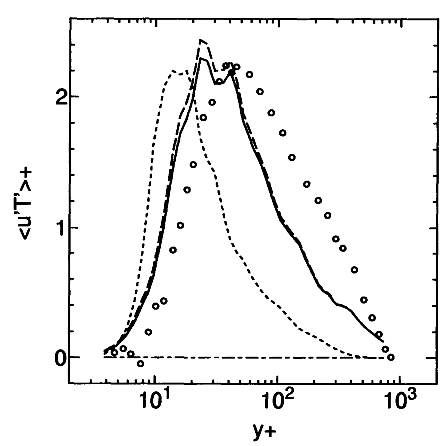

Fig.1 Vertical turbulent heat flux of the natural convection along a heated vertical plate: ○, the experiment of Tsuji and Nagano (1988); ———, the present model; - - - -, the Yoshizawa model (Yoshizawa, 1983) ; — — — the Okamoto model (Okamoto, 1996); — - — - — -, the standard gradient diffusion model.

and that across the flame in Fig.3.

In Fig.2, we notice that the present model agrees with the experimental data better than the standard model though both results are not satisfactory. We may say both models qualitatively predict the experimental data. However, in Fig. 3, it is noted that the standard model of gradient diffusion type produce the negative values contrary to the positive ones of experiment. On the other hand, it is found that the present model reproduces well with the experimental observation to quantitatively explain the counter-gradient diffusion.

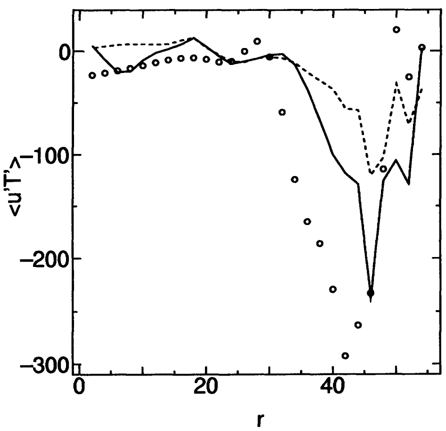

Fig.2 Turbulent heat flux along a non-premixed swirling flame: ○, the experiment of Hardalupas et al. (1996); ———, the present model; - - - - -, the standard gradient diffusion model.

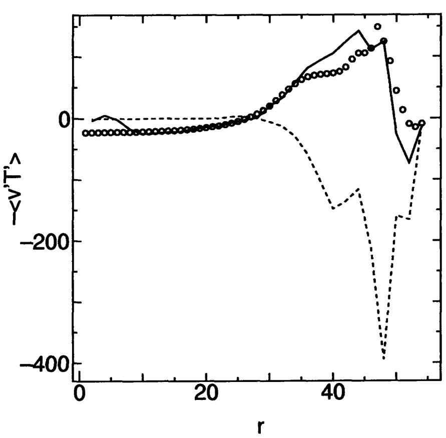

Fig.3 Turbulent heat flux across a non-premixed swirling flame: ○, the experiment of Hardalupas et al. (1996); ———, the present model; - - - - -, the standard gradient diffusion model.

5 CONCLUSION

In the present paper, a new turbulence model of first-order closure has been proposed from a theoretical calculation. The model is applicable to general turbulent flows of the density variation at low Mach numbers, involving the flows where the Boussinesq approximation holds. In fact, it has been found that the model can quantitatively reproduce the turbulent heat flux across a non-premixed swirling flame and can explain the counter-gradient diffusion. The model is much simpler than the Bray-Moss-Libby model of second-order closure type, and is worth investigating by a posteriori tests in future.

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