EFFECTS OF THE VELOCITY RATIO ON VORTICITY DYNAMICS AND MIXING IN COAXIAL JET FLOWS

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ABSTRACT

The effect of the ratio between the outer and inner velocities on the vorticity dynamics and mixing in the initial stage of development of coaxial jet flows is analyzed through axisymmetric numerical simulations. For relatively low values of the velocity ratio (in all cases greater than unity), the vorticity dynamics is dominated by the large positive startup vortex forming from the external shear-layer. As the velocity ratio increases, the roll-up of the internal negative vorticity layer produces a vortical structure comparable in intensity and size with the positive outer vortex. Thus, the mutual induction becomes more important in the dynamics of the startup vortices. The local mixing process also noticeably changes with the velocity ratio. Favorable features are observed for high velocity ratio, and, in particular, high mixing occurs between the streams near the jet outlet. Three-dimensional simulations carried out for a low value of the velocity ratio, indicate that azimuthal instabilities do not have a significant effect on the vorticity dynamics in the initial stages of the flow development.

INTRODUCTION

The coaxial jet configuration is used to obtain efficient mixing between two streams in many aerospace, industrial and environmental applications. Although the configuration is apparently simple from the geometrical point of view, the simultaneous presence of two shear layers leads to complex vorticity dynamics, characterized by non linear interactions between the structures forming from the instabilities of the two jets. As mixing is strictly connected with the vorticity dynamics, the mechanisms leading to mixing are consequently complex. Furthermore, the dynamics of coaxial jets is influenced by many parameters, like the velocity and diameter ratios, the Reynolds number, the outlet geometry, and the outlet flow conditions.

The velocity ratio between the two streams is clearly one of the fundamental parameters in coaxial jet flows. In recent experimental investigations (Rehab et al., 1997) it has been found that the variation of the velocity ratio leads, in the final statistically steady regime, to different dynamics of the vortical structures and different features of the mixing between the two jets.

However, the characterization of vorticity dynamics and of mixing in the initial phases of flow development, after the startup, is also interesting for combustion and pulsating jet applications.

In these initial phases of flow development measurements are very difficult, so that experimental investigations are limited to flow visualizations. Conversely, although restricted to low Reynolds numbers, numerical simulation can provide simultaneously the time evolution of scalar concentrations and of the vorticity field, so that the effects of the dynamics of vortical structures on the mixing processes can be studied. This may also give important indications on the basic mechanisms of mixing in the formation and evolution of vortical structures. Moreover, in numerical simulation the jet outlet conditions can be controlled more easily than in experiments and their influence on the flow dynamics and mixing can be singled out and better understood.

The aim of the present paper is to study by direct numerical simulation the effect of the velocity ratio on the vorticity dynamics and mixing in the initial stages of flow development, after a nearly-impulsive startup. The influence of the jet outlet conditions are also investigated. The investigation is carried out mainly by axisymmetric numerical simulations. Although it is known that, in the final regime after the initial transient, three-dimensional mechanisms play a fundamental role even in the near-field of coaxial jets, the axisymmetry assumption seems to be reasonable during the initial stages, in which the azimuthal instabilities do not have enough time to develop.

However, the effect on vorticity dynamics and mixing of 3D instabilities, forced by azimuthal perturbations of the inlet velocity profile, are also investigated by means of 3D simulations.

NUMERICS AND PROBLEM FORMULATION

The study has been carried out by numerical simulation of the incompressible Navier-Stokes equations in cylindrical coordinates and in primitive variables. The transport equations for two passive scalars have also been considered.

Spatial discretization is performed by central finite differences and a fractional-step algorithm is used to time march the equations. The numerical method is second-order accurate both in space and in time, and more details can be found in Salvetti et al. (1996) and Verzicco and Orlandi (1996).

The governing equations and all the quantities presented in the following are nondimensionalized using the radius of the internal jet, R, as the reference length, and the external velocity, U_e, as the reference velocity (U_e is held constant in all the simulations). Different values of the ratio between the outer and the inner velocities, r_u, are obtained by decreasing the internal velocity; r_u is always greater than 1, since we are interested, in particular, in propulsion and combustion applications. In the present simulations the ratio between the external and internal radius is $R_e/R = 1.93$. For all the simulations, the Reynolds number, based on U_e and R, is equal to 1500 and the Schmidt number is set equal to the unity for both scalars.

At the inlet of the computational domain (corresponding to the jet outlet), the axial velocity is set to zero at $t = 0$, and evolves to a prescribed profile $Q_z(r)$ in a time τ, as follows:

$$q_z(t,r) = f(t)Q_z(r) \;\; t < \tau \; ; \; f(t) = 3\left(\frac{t}{\tau}\right)^2 - 2\left(\frac{t}{\tau}\right)^3 \qquad (1)$$

The law for $f(t)$ in Eq. (1), together with a small value of τ, permits the reproduction of the transient generated by the opening of a valve, or the time required for a motor to reach its final speed in practical applications. Thus, the initial stages of the flow development, after a nearly-impulsive startup, can be studied. A study of the sensitivity to the value of τ is presented in the following.

After the end of the transient, a random perturbation of zero mean value is superposed on the velocity $Q_z(r)$. Details on its definition may be found in Salvetti et al. (1996). This perturbation allows the fluctuations of the inlet velocity profile due to the turbulence in the nozzle to be accounted for, even if in an approximate manner, and was shown to provide, in the final regime, a flow field whose general features are in better agreement with experiments (Salvetti et al., 1996). The effects of this perturbation on the vorticity dynamics in the initial transient phase of the flow development are studied in the following.

The radial velocity component is set equal to zero at the computational inlet and the inlet concentration of the first scalar is 1 in the internal jet and zero elsewhere, while the other one has a concentration at the inlet equal to unity in the external jet and equal to zero elsewhere.

At the outflow, radiative boundary conditions are applied to each variable, which permits the simulation of spatially evolving flows (Salvetti et al., 1996).

In the axisymmetric simulations, the computational grid is uniform in the axial z direction and non-uniform in the radial r direction, with points clustered in high-shear regions (the coordinate transformation can be found in Salvetti et al. (1996)); 385 points have been used in the axial direction and 305 in the radial one. The axial and radial dimensions of the computational domain are, respectively, $16R$ and $10R$. The grid independence of the results has been checked; the same evolution of the vorticity and scalar fields has been obtained in additional simulations carried out on grids with 769×305 and 385×611 points respectively. The results are not presented here for sake of brevity.

In the 3D simulations, three-dimensional instabilities are triggered by an azimuthal perturbation of the inlet velocity profile. This perturbation is obtained by considering that the internal and the external radii at the jet outlet vary with the azimuthal coordinate, θ, as for a corrugated nozzle:

$$R(\theta) = R + \varepsilon cos(n\theta) \quad R_e(\theta) = R_e + \varepsilon cos(n\theta) \qquad (2)$$

where $\varepsilon = 0.05R$. By varying n, different modes of azimuthal instability can be studied. In these simulations, the computational grid is uniform in both z and θ directions and non-uniform in the radial direction, with the same coordinate transformation as for the axisymmetric case. The axial and radial dimensions of the computational domain are also the same as in the axisymmetric case. Because of the symmetry of the flow, a sector corresponding to an angle of $2\pi/n$ has been considered and periodic boundary conditions are used in the azimuthal direction.

SENSITIVITY TO THE INLET CONDITIONS

Before studying the effects of the velocity ratio, the sensitivity of the numerical solution to the inlet conditions is investigated. This investigation is carried out for a coaxial jet configuration characterized by a velocity ratio $r_u = U_e/U_i = 4$.

After the impulsive startup, the roll-up of the positive and negative vorticity layers of the external jet produce two vortical structures of opposite sign. Since at this velocity ratio the intensity of the positive vortex is higher than that of the negative one, this last tends to be advected around the positive vortex; the two vortical structures move initially away from the axis in radial direction and then upwind in axial direction. Later on, because of the interaction with other positive vortical structures formed from the roll-up of the external shear layer, the positive startup vortex moves downwind away from the negative one. This dynamics is illustrated by the time evolution of the radial and axial locations of the startup vortices shown in Fig. 1. To investigate the effect of the random perturbation of the inlet velocity profile, the time evolutions of the location of the startup vortices, obtained with and without this perturbation, are compared in Figs. 1(a) and 1(b).

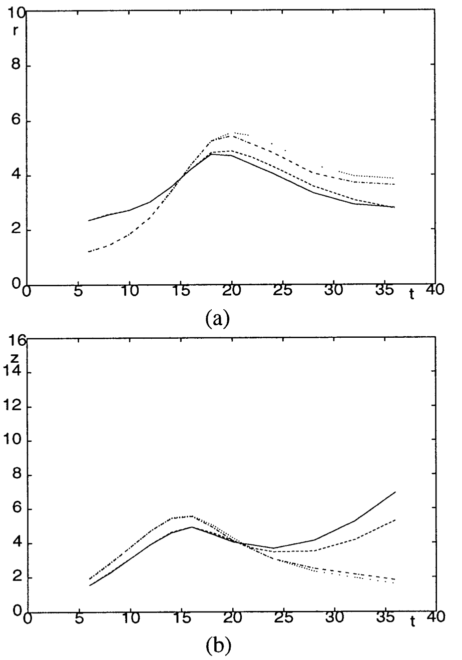


Figure 1: Time evolution of the radial (a) and axial (b) locations of the maximum (—— perturbed, – – – unperturbed) and minimum of vorticity (· · · perturbed, – · – unperturbed).

The inlet perturbation only slightly affects the vorticity dynamics in this initial phase; the most significant difference is the higher axial speed of the positive vortex observed after the nondimensional time $t \simeq 24$ in the perturbed case. This is due to the fact that, as observed also in the final regime (Salvetti et al., 1996), in the perturbed case the successive roll-ups of the external shear-layer occur closer to the jet outlet than in the unperturbed case and, hence, the induction of the formed vortical structures on the positive startup vortex is stronger. Moreover, after $t \simeq 24$, in the perturbed case the startup vortices are more farther apart also in the radial direction and, hence, the mutual induction is weaker.

The local maximum and minimum values of the vorticity have been used in Fig. 1 to identify the location of the centers of the positive and negative startup vortices, respectively; it has been checked that, in the transitional flow analyzed here, which is dominated by the evolution of a few large vortical structures, this criterion always gives locations that are in good agreement with other methods of vortex identification, as, for instance, the one proposed by Chong et al. (1990).

The nearly-impulsive startup of the jets is simulated by letting the axial inlet velocity evolve from zero to a prescribed profile in a time τ, which is small compared to the time required to reach the statistically steady regime. This transient time depends on different parameters, such as the velocity ratio or the Reynolds number, but it is usually larger than 100 nondimensional time units. To study the sensitivity of the solution to the value of τ, the time evolution of the radial and axial locations of the startup vortices obtained with $\tau = 0.8$, 1.6 and 4 is reported in Fig. 2. As the previous results show that the inlet velocity perturbation does not affect strongly the vorticity dynamics in the initial stages of the flow development, this investigation has been carried out without any perturbation to better isolate each single effect. Conversely, in the simulations presented in the next sections, carried out to study the effect of the velocity ratio, the random perturbation has been superposed to the inlet velocity profile, because, as discussed previously, this reproduces in a more realistic way the situation actually encountered in experiments or in practical applications.

Initially, the dynamics is not significantly affected by the value of τ, except for a slight decrease of the velocity of the two vortical structures as τ increases. More significant effects are observed after $t \simeq 24$ in the axial trajectory of the positive startup vortex: as τ increases, it moves downwind faster. This is again due to the interaction with other vortical structures formed from the successive roll-ups of the external shear-layer; when τ is large less structures are formed, but they are larger and more intense, so that their induction on the positive startup vortex is stronger. In the simulations in the next Sections τ is set to the value of 1.6.

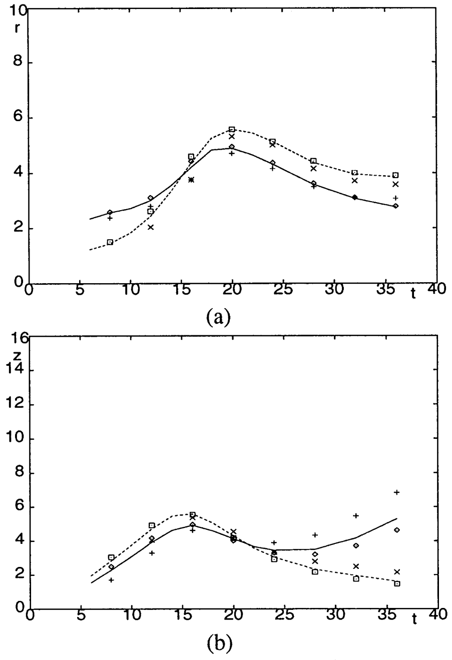
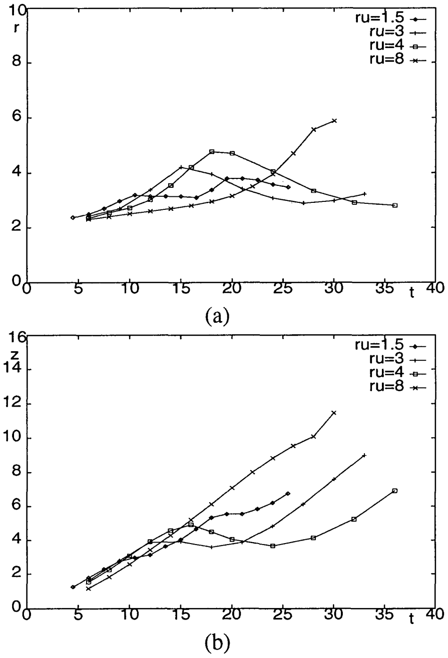


Figure 2: Time evolution of the radial (a) and axial (b) locations of the maximum (—— τ=1.6, ◇ τ=0.8, + τ=4) and minimum of vorticity (– – – τ=1.6, □ τ=0.8, × τ=4).

EFFECTS OF THE VELOCITY RATIO

Vorticity dynamics

For low values of the ratio between the external and the internal jet velocities ($r_u < 3$), the initial stage of flow development is dominated by the large startup vortex, formed from the external shear layer. After the startup, the roll-up of the negative vorticity layer produces a vortex which is noticeably smaller and is immediately advected around the large positive one. Other small negative structures form and are wrapped around the positive vortex. The latter initially moves downwind and then begins to interact with another positive vortex formed from the successive roll-up of the external shear-layer.



(a)

(b)

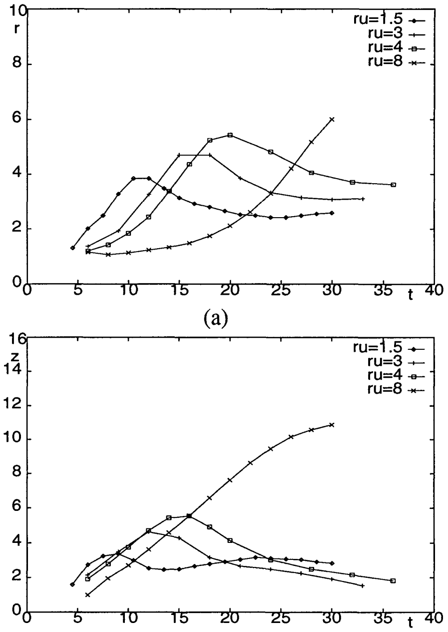
Figure 3: Time evolution of the radial (a) and axial (b) locations of the positive startup vortex (local maximum of vorticity).

As the velocity ratio increases, the roll-up of the internal negative vorticity layer produces a vortical structure that eventually becomes comparable in intensity and size to the one rolled-up from the external shear-layer. For $r_u \geq 3$, the induction of the negative vortex on the positive one becomes significant and the dynamics of the flow is significantly different.

For moderate velocity ratios, as for instance for $r_u = 4$, the negative vortex is still noticeably weaker and then it tends to be advected around the positive one; however, because of the mutual induction, both vortical structures move initially away from the axis in radial direction and then upwind in axial direction. Later on, because of the induction of other positive vortical structures formed from the roll-up of the external shear layer, the positive startup vortex is advected downwind, while the negative one keeps moving upwind.

As the velocity ratio further increases, the dynamics of the two startup vortices tends to become more close to that of a dipole. At $r_u = 8$, for instance, both vortices initially move downwind in the axial direction at an almost constant speed, and then they start also to move away from the axis in the radial direction.

These observations are summarized in Figs. 3 and 4, in which the radial and axial trajectories of the positive and negative startup vortices are reported for different values of r_u. Note that for $r_u = 8$ both vortices reach a distance from the jet outlet in the axial direction and from the axis in the radial one, which is significantly larger than for all other velocity ratios. In particular, the negative startup vortex is found to move upwind towards the jet outlet for all velocity ratios except for $r_u = 8$.



(a)

(b)

Figure 4: Time evolution of the radial (a) and axial (b) locations of the negative startup vortex (local minimum of vorticity).

Thus, it appears that significantly different vorticity dynamics can be obtained already in the initial stages of the flow development, depending on the mutual induction between the positive and negative startup vortices and on their interactions with the other vorticity layers and with the structures forming from the successive roll-ups of these shear layers. Indeed, the main effect of the velocity ratio is to change the circulations of the internal and the external startup vortices, which can be assumed to be proportional to $U_e - U_i$ and U_e respectively.

Mixing evolution

Let us now analyze the effect of the velocity ratio on the mixing process.

In many previous studies, the mixedness f was used as a quantitative measure of mixing. For non reacting flows the mixedness f_i can be defined, for each scalar i, as:

$$f_i = \frac{4}{V(A)} \int_A \xi_i(1-\xi_i)dA \quad i=1,2 \tag{3}$$

where ξ_i is the concentration of one of the inert scalars seeded in the internal (ξ_1) and external (ξ_2) jets and $V(A)$ is the volume of the domain A over which the integration is performed. In the present axisymmetric simulations $V(A) = \pi L_r^2 L_z$, where L_r and L_z are set equal to $6R$ and $16R$ respectively. Thus, the mixedness represents a quantitative indication of the global mixing between each scalar and the surrounding fluids in a given domain.

For all velocity ratios, in this initial phase of flow development, most of the mixing occurs between the scalar injected in the external jet and the surrounding air: indeed the values of f_1 have been found to be always much lower than those of f_2. As the velocity ratio increases, both f_1 and f_2 slightly decrease (not shown here for sake of brevity). The reasons of this behavior can be understood by analyzing the local mixing mechanisms. Indeed, the mixedness, because of its definition, can only give global information on the mixing processes; conversely, local indications on mixing can be obtained by the values of the integrand function in Eq. (3).

In Figs. 5-7 the coordinates of the points at which the values of $\xi_2(1-\xi_2)$ are greater than 0.23 are reported at representative time instants and different values of the velocity ratio. Note that this set of points represent the region in which the mixing between the scalar seeded in the external jet and the surrounding fluids is almost complete (the maximum value of $\xi_2(1-\xi_2)$ is 0.25).

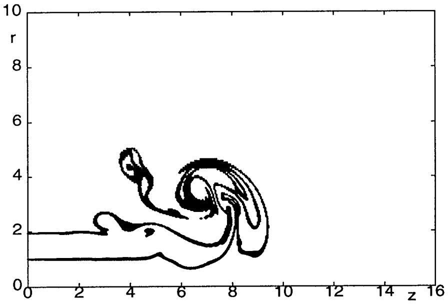


Figure 5: Points at which $\xi_2(1-\xi_2)$ is greater than 0.23 obtained with $ru = 1.5$ at t=25.5.

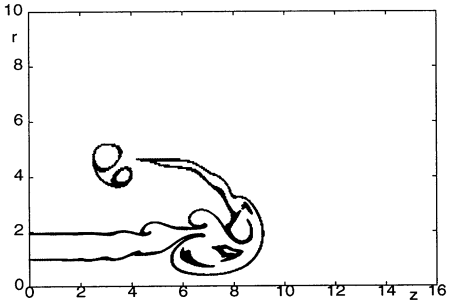


Figure 6: Points at which $\xi_2(1-\xi_2)$ is greater than 0.23 obtained with $ru = 4$ at t=24.

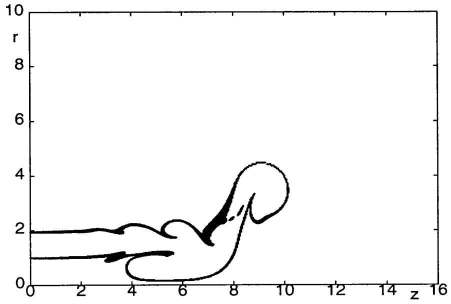


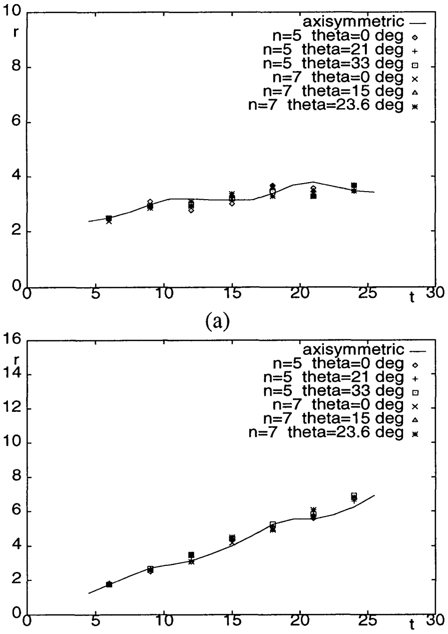
Figure 7: Points at which $\xi_2(1-\xi_2)$ is greater than 0.23 obtained with $ru = 8$ at t=24.

It appears that the main mechanism of mixing is the stretching of the shear-layers due to the formation and interaction of vortical structures and the consequent increase in the contact area between the scalars. Indeed, high mixing is always found in the shear-layers surrounding vortical structures, due to the high velocity gradients experienced in these regions by the passive scalars. The slight decrease in global mixedness observed as r_u increases can then be explained by the fact that the contact area between the scalars appears to be larger for low values of r_u. However, the local distribution of mixing is clearly completely different for the various velocity ratios. In particular, for $r_u = 8$ some favorable features appear. First of all, high values of $\xi_2(1-\xi_2)$ are found near the axis quite close to the jet outlet, and, from the comparison with the values of $\xi_1(1-\xi_1)$ (not shown here), it is clear that high mixing occurs in this zone between the two scalars seeded in the internal and external jets. In a previous experimental investigation (Rehab et al., 1997), for $r_u \geq 8$ an efficient mixing between the streams was found in the final regime near the jet outlet because of the presence of a recirculating bubble. The present study indicates that high mixing can be obtained near the jet outlet already in the first stages of the flow development. In practical applications this could imply, for instance, a more efficient combustion or a reduced length of combustion chambers. Moreover, the front of mixing appears much less jagged for r_u=8 than in the other cases. This could be an interesting feature, for instance, if dispersion of pollutants has to be avoided.

EFFECTS OF THREE-DIMENSIONAL INSTABILITIES

In order to study the effects of three-dimensional instabilities on the dynamics and mixing in the initial phases of flow development, 3D simulations have been carried out for r_u=1.5 and Re=750. The Schmidt number is equal to unity for both scalars. The 3D instabilities are triggered by azimuthal perturbations of the inlet velocity profile following Eq. (2). Simulations for n=5 and 7 have been performed. As previously mentioned, the radial and axial dimensions of the computational domain are the same as in the axisymmetric case, while in the azimuthal direction, because of the periodicity of the flow, only a sector of width $2\pi/n$ has been considered. The computational grid has 24×153×193 points in the azimuthal, radial and axial directions respectively. Previous axisymmetric simulations have shown that, at the present Reynolds number, this provides an adequate resolution, at least in the radial and axial directions.

As in the axisymmetric case, also in 3D simulations the initial phase of development of the flow is dominated by the evolution of the large start-up vortex. At the considered velocity ratio, the three-dimensionality of the flow has been found to have only a slight effect on the dynamics of this vortex; this can be seen, for instance, by comparing the time evolution of the radial and axial vortex locations on different azimuthal planes in 3D simulations with those obtained in the axisymmetric case (Figs. 8(a) and 8(b)). Since most of the mixing occurs in correspondence to this large vortex, the global mixing mechanisms are similar to those found in the axisymmetric case. However, in the 3D simulations the mixedness is strongly increased in the central part of each sector of width $2\pi/n$, where vorticity concentrates, while it is lower than in the axisymmetric case on the external (z,r) planes. A more detailed analysis of 3D effects on mixing is given in Salvetti and Lombardi (1998).



(a)

(b)

Figure 8: Time evolution of the radial (a) and axial (b) position of the startup vortex in 3D simulations $(r_u = 1.5)$.

CONCLUDING REMARKS

The effects of the velocity ratio, r_u, on the dynamics and mixing in the initial phases of development of coaxial jet flows have been studied by axisymmetric numerical simulations. Only coaxial jet configurations characterized by an external jet velocity higher than the internal one ($r_u > 1$) have been considered. For relatively low r_u values (approximately $r_u < 3$), the initial dynamics is dominated by the evolution of the large positive startup vortex formed from the roll-up of the external shear-layer. As the velocity ratio increases, the size and the intensity of the startup negative vortex become comparable to those of the positive one and, hence, the mutual induction between the two vortices becomes significant. As a result, from the very initial stages of development, the dynamics of the flow changes noticeably with the velocity ratio. Since the main mechanism of mixing between the streams has been found to be the stretching of the shear-layers due to the formation and interaction of vortical structures, the local mixing process also significantly changes with r_u. In particular, favorable mixing properties have been observed for high velocity ratios ($r_u = 8$), even in the first stages of the flow development. Thus, the present simulations indicate that significantly different vorticity dynamics, and hence mixing properties, can be obtained even in the initial stages of flow development, depending on the the mutual induction between the positive and negative startup vortices. This is a consequence of the fact that the main effect of the velocity ratio is to change the circulations of these vortices.

Conversely, the perturbation of the inlet velocity profile and the duration of the startup transient have a more limited impact on the vorticity dynamics in the initial stages of flow development.

Finally, the dynamics of the flow has been found to be not significantly affected by 3D instabilities, although local mixing mechanisms are changed.

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